Ref. No. : Ex/PG/ETCE/T/127C/2024

M.E. ELECTRONICS AND TELE-COMMUNICATION ENGINEERING FIRST YEAR SECOND SEMESTER - 2024

Stochastic Control

Time: 3 hours | Full Marks: 100

Attempt any 4 Questions.

1. a) Prove that for any random variable X, the variance of X, Var(X), and expectation of X, E(X), satisfies the following relation:

$$Var(X) = E(X^2) - E^2(X).$$

b) Given that the spinning pointer in a gambling game supports the following probability density function:

$$f_X(x) = \frac{1}{2\pi}, \quad 0 \le x \le 2\pi$$

$$= 0, \quad otherwise.$$

Evaluate $E(X)$ and $E(X^2)$, and hence evaluate $Var(X)$.

c) Show that for a Gaussian probability distribution of the form:

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\overline{x})^2}{2\sigma^2}\right]$$

has $mean = E(X) = \overline{x}$ and $Var(X) = \sigma^2$.

d) Consider an input process having an exponential autocorrelation function, given by $R_f(\tau) = \sigma^2 e^{-\beta|\tau|}$ and a first order low pass filter of the form $G(s) = \frac{1}{1+Ts}$. Compute the output spectral function and the mean-square value of the output: $E(x^2)$. The following integral, if needed, may be used.

[Turn over

$$\frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\frac{c(s)c(-s)}{d(s)d(-s)}ds = \frac{c_1^2 d_0 + c_0^2 d_2}{2d_0 d_1 d_2}$$

where $c(s) = c_1 s + c_0$ and $d(s) = d_2 s^2 + d_1 s + d_0$. [5+5+7+8]

2. a) What is the difference between probability distribution and fuzzy membership function ?

b) For a 2-rule fuzzy system, given by

$Rule\ 1 : IF\ x\ is\ A_1\ AND\ y\ is\ B_1\ , then\ z\ is\ C_1$

$Rule\ 2 : IF\ x\ is\ A_2\ AND\ y\ is\ B_2\ , then\ z\ is\ C_2$

Given the measurements $x = x'$ and $y = y'$; obtain the fuzzy inference graphically by Mamdani approach.

c) Given the Takagi-Sugeno fuzzy rules

$Rule\ 1 : IF\ x\ is\ A_1\ AND\ y\ is\ B_1\ , then\ control\ signal\ is\ u_1$

$Rule\ 2 : IF\ x\ is\ A_2\ AND\ y\ is\ B_2\ , then\ control\ signal\ is\ u_2$

Find the composite control signal u at the operating point $x = x'$ and $y = y'$.

d) How to handle the above problem probabilistically? [5+ 8+ 8+4]

3. a) State the importance of Sliding Mode Control (SMC).

b) For a given non-linear plant dynamics

$$\vec{x}^{(n)} = f(\vec{x}) + b(\vec{x})u$$

where $\vec{x} = [\,x_1\ x_2\ \dots\ x_n\,] = [\,x\ \dot{x}\ \ddot{x}\ \dots\ x^{(n-1)}\,]$

Obtain the sliding function $s(\vec{x}, t)$ for $n = 2$ and $n = 3$.

c) Define and justify the sliding condition ($s\dot{s} \leq 0$).

d) Give an interpretation of the sliding condition.

e) Given a plant dynamics

$$J\ddot{\theta}(t) = u(t) + d(t) \ , \ |d(t)| \leq \eta$$

Compute $s(t)$ for a given $\theta(t)$ and $\theta_d(t)$. Also check that with d(t) = 0 ,

$$u = J(-c\dot{e}(t) + \ddot{\theta}_d(t) - \frac{1}{J}(ks + \eta sgn(s))$$ [4+5+6+5+5]

4. a) Draw a schematic block diagram to control the rolling angle ϕ of a quadrotor.

b) State the dynamics of a quadrotor for the rolling problem.

c) Justify the importance of ESO and LRDC in the quadrotor system.

d) Show by Leibnitz's Theorem

$$\frac{d}{dt}[\int_0^t k_w \ \delta \ sat(\hat{x}_3(\tau), \delta) \ d\tau] = k_w \ \delta \ sat(\hat{x}_3(t), \delta)].$$

e) Compute the following integral used in LRDC design numerically

$$\int_0^t k_w \ \delta \ sat(\hat{x}_3(\tau), \delta) \ d\tau$$

by dividing the time in $[0, t]$ into n intervals of $\Delta\tau$ i.e., $t = n\Delta\tau$ and presuming that the expression within the integration does not change within the intervals $i(\Delta\tau)$, where $i = 0, 1, 2, \dots, (n-1)$.

f) What is $sat(\hat{x}_3(\tau), \delta)$?

g) Given the ESO dynamics

$$\dot{\hat{x}}_1(t) = \hat{x}_2(t) + \frac{\alpha_1}{\varepsilon}(x_1(t) - \hat{x}_1(t))$$

$$\dot{\hat{x}}_2(t) = \overline{u}_1(t) + \hat{x}_3(t) + \frac{\alpha_2}{\varepsilon^2}(x_1(t) - \hat{x}_1(t))$$

$$\dot{\hat{x}}_3(t) = \frac{\alpha_3}{\varepsilon^3}(x_1(t) - \hat{x}_1(t)).$$

How will you obtain $\hat{x}_1(t), \hat{x}_2(t), \hat{x}_3(t)$ over time t from the ESO dynamics?

[2+3+4+3+5+4+4]

5. a) Let $X(t)$ and $Y(t)$ be two random processes. Compute the autocorrelation of $Z(t) = X(t) + Y(t)$, presuming that the cross-correlation between X and Y is zero.

b) Consider a system G(S)= 1/ (1 +ST) with input signal spectrum $S_s = \frac{2\sigma^2\beta}{-s^2+\beta^2}$ and a noise spectrum $S_n = A$. Derive a mean-square error formulation, to compute T of the filter for optimal noise-free spectral power.

c) Write a note to digital filter and stochastic digital filter. [10+10+5]