Ref. No.: Ex/PG/ETCE/T/111A/2024

Master of E. & Tel. E. Examination, 2024

(1st Semester)

STATISTICAL COMMUNICATION THEORY

Time: Three hours **Full Marks: 100**

Answer any five questions
Answer must be written at one place for each attempted question

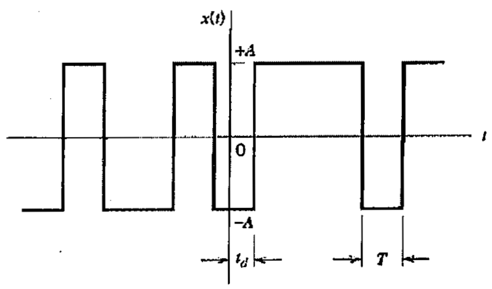
Q1 (a) Show the generation of a random process $x_i(n)$ from a sample space S in discrete time domain through experimental outcome w_i. Also show that the random process is the indexed sequence of variables.
Write expressions for mean, variance, auto covariance and autocorrelation for discrete time random process. When auto covariance and auto correlation functions are equal? 10

(b) In a digital data acquisition device suppose the recorded signal y(n) is given as the sum of signal x(n) and additive noise w(n). x(n) and w(n) are uncorrelated RPs. Find the autocorrelation expression for y(n). 05

(c) When is a random process x(n) called Wide Sense Stationary (WSS)? What is the autocorrelation value of a WSS with lag k=0? Prove that autocorrelation of RP is an even function. 05

Q2. (a) Consider a sinusoidal signal with random phase Θ, defined by,
$X(t) = A \cos(2\pi f_c t + \Theta)$, where A and f_c are constant, Θ is a random variable uniformly distributed over [$-\pi$ to $+\pi$), is the autocorrelation function $R_x(\tau)$ WSS? Draw $R_x(\tau)$ vs τ. What are the amplitude and frequency of $R_x(\tau)$? 06

(b) The random binary wave is given as below, establish that its autocorrelation function and explain. 05



(c) Define ensemble average and time average of a RP with expressions and then explain how ergodicity of the RP is defined. What is the significance of ergodicity? 05

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(d) Given the random process $X(t) = A \cos(nw_0t+\Theta) + v(n)$, where $v(n)$ is a zero mean white Gaussian noise with variance σ_v^2, if A is another random variable with zero mean and variance σ_A^2, find the autocorrelation of the process and determine if the process is ergodic. 04

Q3. (a) $x(n)$ is a WSS random process with mean m_x and autocorrelation $r_x(k)$. $x(n)$ is filtered with a stable linear shift invariant filter having a unit sample response $h(n)$ that produces an output $y(n)$. Establish the relationship between the input autocorrelation function $R_x(t)$ of $x(n)$, and output autocorrelation function $R_y(k)$. Find the variance and power spectrum of the output process $y(n)$. 10

(b) Let $x(n)$ be a RP that is generated by filtering white noise $w(n)$ with a first-order linear shift invariant filter having a system function $H(z) = 1 / (1-0.16 z^{-1})$, the variance of the white noise is 1. Write expression for power spectrum $P_x(z)$ of $x(n)$ and from power spectrum find the autocorrelation function $R_x(k)$ and draw the sequence. 06

(c) The autocorrelation function of a WSS random process is given by, $R_x(k) = 2^{-k^2}$, state with reason if the autocorrelation is valid, draw the power spectrum if the correlation is valid. 04

Q4. (a) Describe the spectral factorization process of a WSS random process. Define regular process, innovation representation and whitening filter. 12

(b) The power spectrum of WSS process $x(n)$ is given by,

$P_x(e^{jw}) = (16- 14 \cos w) / (26-10 \cos w)$, find the whitening filter $H(z)$ that produces unit variance white noise when the input is $x(n)$. 08

Q5. (a) In a problem of Wiener FIR filtering process, a signal $d(n)$ is to be estimated from a noise corrupted observation $x(n) = d(n) + v(n)$. The noise $v(n)$ is zero mean RP and uncorrelated with $d(n)$. Establish the Wiener-Hopf equation and find minimum mean square error expression. 10

(b) If $d(n)$ be an AR(1) process with an autocorrelation sequence $r_d(k) = (0.8)^{|k|}$, $d(n)$ is observed in presence of noise $v(n)$ with variance $\sigma_v^2 = 1$. Find the Wiener Filter $W(e^{jw})$ and $P_d(e^{jw})$ and mean square error ξ_{min}. 08

Q6. (a) The non-causal IIR Wiener filter problem, design the system function $H(z)$ and minimum mean square error without and with noise consideration during the observation process. 10

(b) The causal Wiener filter may be viewed as a cascade of a whitening filter with a cascade filter that produces the minimum mean square estimate of desired signal $d(n)$ from $\varepsilon(n)$ - whitened input for real process, the system function of cascade is ,

$H(z) = F(z)\, G(z) = (1/\sigma_0^2\, Q(z))\, [\, P_{dx}(z) / Q(z^{-1})\,]_+$ and the mean

Square error is $\xi_{min} = r_d(0) - \Sigma_{k=0}^{\infty} h(k)\, r_{dx}(k)$.

(i) If $r_{d\varepsilon}(k) = \delta(k)$ and $P_\varepsilon(z) = 4 / [(1 - 0.5z^{-1})(1 - 0.5 z)]$, find the unit sample response $h(n)$ of the causal Wiener filter. Find the mean square error. 10

Q7. (a) Describe the principle of spectrum estimation for any random signal. 05

(b). Define Periodogram and then describe the process of Bartllet's Windowing by Periodogram averaging process for spectrum estimation. 10

(c) How is frequency estimation done through the process of eigendecomposition method of autocorrelation matrix? 05

Q8. (a) Establish the average cost or risk factor $\overline{C}$ in binary hypothesis decision using Bayes criteria. Determine the optimum likelihood ratio $\Lambda(z)$ and threshold η in terms of cost and then define the decision rule. How is Minimax criterion established from Bayes decision? 10

(b) If $p(z|H_0)$ and $p(z|H_1)$ are the prior probabilities for a hypothesis testing in which
$P(z|H_1) = 1/(\sqrt{2\pi}) \exp(-z^2/2)$ and $p(z|H_0) = ½ \exp(-|z|)$

If for the Bayes test $C_{11} = C_{00} = 0$ and $C_{01} = C_{10} = 1$, and $P(H_0) = 1/4$, find probability of false alarm P_F and probability of Detection P_D. For the same assignment of costs, find the performance for minimax test. 10

Q.9 Write short notes on any two: 10x2 =20

(a) Neyman-Pearson Test

(b) Representation of narrowband noise in terms of envelope and Phase

(c) Multistep prediction problem