Ref. No. : Ex/PG/ETCE/T/129E/2024

M.E. ELECTRONICS AND TELE-COMMUNICATION ENGINEERING FIRST YEAR SECOND SEMESTER - 2024

Adaptive and Optimal Control

Time: 3 Hours | Full Marks: 100

Attempt any 4 Questions.

1. Given the error signal $E(s) = \frac{s+100K_1K_2}{s^2+100K_1K_2s+100K_1}$, the input signal $R(s) = \frac{1}{s}$ and the control signal $U(s) = \frac{sK_1}{s^2+100K_1K_2s+100K_1}$.

   a) Determine the performance index $J_e = \int_0^{\infty} e^2(t)dt$ and $J_u = \int_0^{\infty} u^2(t)dt$, where $e(t)$ and $u(t)$ are the error and control signals respectively. The following integral, if needed, can be used

   $$\frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \frac{c(s)c(-s)}{d(s)d(-s)} ds = \frac{c_1^2 d_0 + c_0^2 d_2}{2d_0 d_1 d_2}$$

   where $c(s) = c_1 s + c_0$ and $d(s) = d_2 s^2 + d_1 s + d_0$.

   Construct a resulting performance index $J = J_e + \lambda J_u$ and hence show that the performance index is optimized, if $\lambda = 0.25, K_1 = 2, K_2 = 0.1$.

   b) Also construct the Hessian matrix $\mathbf{H}$ and show that the Hessian is positive definite for $K_1 = 2, K_2 = 0.1$.

   c) What do J_e and J_u physically signify ?

   d) Also define the Parseval's theorem used in optimizing J_e, J_u. [10+5+5+5]

[Turn over

2. a) Given the plant dynamics $\dot{\boldsymbol{x}}(t) = \boldsymbol{a}(\boldsymbol{x}(t), \boldsymbol{u}(t), t)$ and the performance index given by $\boldsymbol{J} = h(\boldsymbol{x}(t_f), t_f) + \int_{t_0}^{t_f} g(\boldsymbol{x}(\tau), \boldsymbol{u}(\tau), \tau) d\tau$. Derive the Hamiltonian $\mathcal{H}$ and the Hamilton-Jacobi-Bellman equation.

   b) A first order system is described by the differential equation

$$\dot{x}(t) = x(t) + u(t).$$

   The performance index is given by $J = \frac{1}{4}x^2(T) + \int_0^T \frac{1}{4}u^2(t)dt$, where T is the final time. Use Hamilton-Jacobi-Bellman equation to determine the time-varying optimal control law $u^*(t)$. [15+10]

3. a) Develop a recurrent relationship between the performance measures $J_{N-2,N}(\boldsymbol{x}(N-2), \boldsymbol{u}(N-2), \boldsymbol{u}(N-1))$ and $J_{N-1,N}(\boldsymbol{x}(N-1), \boldsymbol{u}(N-1))$ for the system described by the state equation $\dot{\boldsymbol{x}}(t) = \boldsymbol{a}(\boldsymbol{x}(t), \boldsymbol{u}(t))$, whose performance index is given by

$$\boldsymbol{J} = h\left(\boldsymbol{x}(t_f)\right) + \int_0^{t_f} g(\boldsymbol{x}(t), \boldsymbol{u}(t))dt.$$

   b) Obtain the general form of $J^*_{N-K,N}(\boldsymbol{x}(N-K))$ in terms of $J^*_{N-K+1,N}\left(\boldsymbol{a}_D(\boldsymbol{x}(N-K), \boldsymbol{u}(N-K))\right)$. [13+12]

4. a) Define the principle of optimality and illustrate the principle in the context of a routing problem.

   b) Given $x(k+1) = x(k) + u(k)$ for $k = 0, 1$, where $u(0)$ and $u(1)$ are to be selected to minimize the cost $J = x^2(2) + 2u^2(0) + 2u^2(1)$, satisfying $0 \leq x(k) \leq 1.5$, for $k = 0, 1, 2$ and $-1 \leq u(k) \leq 1$, for $k = 0, 1$.
   Use the Dynamic Programming approach to obtain the optimal control applied at $k = 1$, i.e., $u^*(x(1), 1)$ for different possible values of $u(1)$ and $x(1)$. [10+15]

5. a) Find an extremal for the functional

$$J(x) = \int_0^{\frac{\pi}{2}} [\dot{x}^2(t) - x^2(t)]dt$$

with boundary conditions $x(0) = 0$ and $x\left(\frac{\pi}{2}\right) = 1$. Use the Euler equation to find the extremal.

b) Derive the Euler equation used to develop the above extremal. [10+15]

6. Write notes on the following :-
   a) Model Reference and Self-Tuning adaptive control.
   b) Testing linearity of a functional by homogeneity and additivity checking.

[12+13]