

M.E. ELECTRICAL ENGINEERING FIRST YEAR SECOND SEMESTER
EXAMINATION, 2024

ADVANCED DIGITAL SIGNAL PROCESSING

Full Marks 100

Time: Three hours

(50 marks for each part)

Use a separate Answer-Script for each part

PART I

Answer any TWO questions

- 1.(a) The output $y[n]$ and the input $x[n]$ of a discrete-time linear time-invariant (DTLTI) system are related through the difference equation

$$y[n] - 0.7y[n-1] + 0.1y[n-2] = 5x[n] + x[n-1]$$

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Let the system transfer function be $G(z)$.

- (i) Draw the fully labeled pole-zero diagram for $G(z)$.
- (ii) Determine the causal unit impulse response $g_1[n]$ of the system.
- (iii) Determine the anti-causal unit impulse response $g_2[n]$ of the system.
- (iv) Determine the bicausal (two-sided) unit impulse response $g_3[n]$ of the system.
- (v) In each of (ii), (iii) and (iv), what is the region of convergence of $G(z)$?
- (vi) Which, if any, of $g_1[n]$, $g_2[n]$ and $g_3[n]$ represent(s) a stable system?

Derive the condition for stability, utilized for solving the problem.

- (b) The difference equation relating the output $y[n]$ and the input $x[n]$ of a DTLTI system is given by

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$$y[n] = bx[n] + ay[n-1], \text{ where } a \text{ and } b \text{ are real coefficients.}$$

Show that the system is an IIR system. Examine with the help of relevant sketches, the effect of a and b on the nature of the impulse response.

OR

[Turn over

1. (a) Design a second order low pass digital Butterworth filter with a dc gain of 0 dB, and an attenuation of 3 dB at 675 Hz. Use bilinear transformation preceded by frequency pre-warping. Consider a sampling frequency of 3 kHz. 12+3
+3
Obtain the difference equation relating the output and the input sequences of the filter.
Prove the warping formula used.
Explain why the technique is free from 'Characteristic Aliasing'.
- (b) Point out the important demerits of the impulse-invariant technique of designing digital IIR filters. 7
2. (a) Consider an LTI system with wide-sense stationary (WSS) input random process $X(t)$. the system has an impulse response $g(t) = e^{-3t} u(t)$, and the input process has an autocorrelation function $R_X(\tau) = e^{-2|\tau|}$. Define an error process $\varepsilon(t) = X(t) - Y(t)$. 8
Determine the following:
(i) The autocorrelation function $R_Y(\tau)$.
(ii) The power spectral density function $S_Y(\omega)$.
(iii) $E[\varepsilon^2(t)]$.
Derive the expressions used.
- (b) Consider a random process, where the sample functions are randomly phased sinusoids. Examine whether or not the random process is wide-sense stationary (WSS) and ergodic. 5
- (c) A DC signal is contaminated by a noise that is a sample realization of an ergodic random process with an autocorrelation function of $R_X(\tau) = e^{-|\tau|}$ W. The corrupted signal is digitized and then processed by a running linear averager with a window length of 50 samples. Determine the RMS value of the noise component of the output. Derive the expression used. 12
3. (a) It is desired that for a bipolar Nyquist ADC, the quantization noise power level should be at least 100 dB below the unclipped full scale 8

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sinusoidal signal power level. Find the minimum number of bits required for the ADC. Derive the expression used.

- (b) A ten bit, bipolar analog-to-digital converter (ADC) with a full-scale of $\pm 12\text{V}$, is used to digitize a random signal with a mean-square value of 4 V^2 and a bandwidth of 100 Hz. Determine the signal-to-quantization noise ratio when the sampling frequency is 400 Hz and also when the sampling frequency is 5 kHz. Comment on the difference in the result for the two cases, *with the help of mathematical analysis and illustration*. What happens to the effective number of bits? 12+5

Other than improving the signal-to-quantization noise ratio, what other benefit does oversampling have ? What is the demerit of oversampling? Explain.

4. Write notes on any two of the following.
- (a) Designing digital IIR filters by backward-difference transformation. 12 ½
+12 ½
- (b) Sigma delta modulator ADC.
- c) Concept of power spectral density and Wiener-Khintchine-Einstein theorem.
- (d) White noise.

[Turn over

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No. of Questions	PART II	Marks
	<i>Answer all questions.</i>	
1.	<p>How can discrete convolution summation be employed to realize a digital filter? What are the practical realization problems associated with implementing FIR digital filters? How can one such realization problem be overcome by employing a delayed and truncated real and symmetric impulse sequence for the filter?</p> <p align="center">OR</p> <p>How can a circular complex convolution integral be employed to derive the frequency response of a digital FIR filter? Hence, derive in detail the frequency response of Hamming window.</p> <p align="right">(CO1-K1,K2)</p>	10
2.	<p>Write a short note on <i>any one</i> of the following:</p> <p>i) Laplacian based second derivative spatial image filters.</p> <p>ii) Frequency domain methods based high pass image filters.</p> <p align="right">(CO2-K2)</p>	10
3.	<p>Explain the importance of regularization parameter and forgetting factor in general time-varying performance criterion employed in RLS algorithm. Explain in detail how a Kalman gain vector is employed to derive an RLS algorithm. Explain in this context the significance of and the difference between <i>a priori</i> error and <i>a posteriori</i> error.</p> <p align="center">OR</p> <p>Why and how a decorrelation delay can be employed for adaptive cancellation of noise in absence of an external reference source? Explain in detail how can Widrow-Hoff LMS algorithm be employed for adaptive digital filtering purposes. Comment on the computational burden of an LMS adapted <i>M</i>-th order FIR filter, in terms of additions and multiplications.</p> <p align="right">(CO3-K3)</p>	16

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4.	<p>The running estimate of the average power of the reference signal for a <i>fifth</i> order LMS adaptive FIR filter is 3.29. The step size of the LMS algorithm is so designed that the algorithm can achieve 80% of the maximum permissible speed of convergence. What will be the theoretical minimum value of mean square error, if the excess mean square error of the system is 0.4? Also determine the learning curve time constant, if the magnitude of the average Eigen value of the reference correlation matrix R is 1.97 for this system.</p> <p align="right">(CO4-K4)</p>	08
5.	<p>“The optimal filter gain in a Wiener FIR filter always varies between 0 and 1 and the filter gain at a particular frequency becomes more than 0.5 when strength of noise is greater than the strength of the original signal and vice versa.” – Justify or correct the statement citing suitable reasons.</p> <p align="center">OR</p> <p>“In the method of steepest descent employed to adapt digital filters, the gradient matrix is directly proportional to the vector signifying the difference between the filter weight vector and the Wiener solution vector and the gradient matrix is inversely proportional to the reference autocorrelation matrix.” – Justify or correct the statement citing suitable reasons.</p> <p align="right">(CO5-K4)</p>	06