Ref. No. : **Ex/PG/EE/T/113B/2024**

M.E. ELECTRICAL ENGINEERING FIRST YEAR FIRST SEMESTER - 2024

DIGITAL CONTROL THEORY (CS)

Time : 3 hours

Full Marks : 100
(50 marks for each part)

Use separate answer-scripts for each parts.

Part-I

Answer any three questions. Two marks reserved for neatness.

Answer all parts of a question in the sequential order.

1 a) Explain with neat sketches, the various dynamic characteristics observed in the Sample and Hold operations during the A/D conversion of a time-varying analog signal. In this context, define various time delays associated with this operation.

b) Derive the transfer function of Zero-order Hold.

[(8+4)+4=16]

2 a) The block diagram of a closed-loop negative feedback sampled data control system is shown in Figure P2. Derive an expression for the pulse transfer function of the overall system.

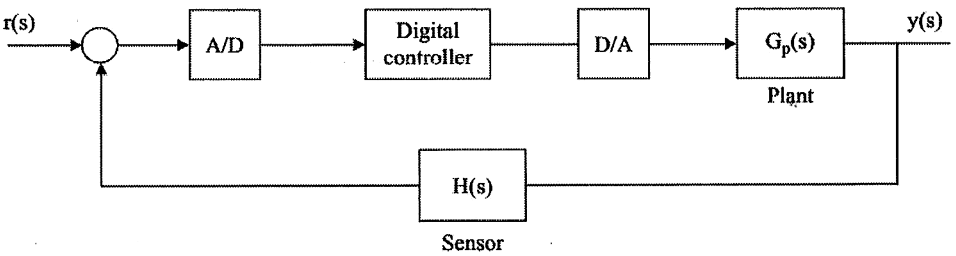

Fig. P2

The digital controller is given by a constant gain of 2.5 and the plant is given by the transfer function of $\dfrac{1}{s(s+2)}$. Sensor gain is unity. The sample time is 0.5 second.

b) Now, a unit step signal is applied to the sampled data digital system shown in Figure P2. Calculate and plot the output response of the system considering at least 10 instants of time.

[6+10=16]

[Turn over

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3. The feed-forward pulse transfer function of a unity gain feedback discrete-time control system is given as:

$$G(z) = \frac{Kz}{(z-0.5)} \cdot \frac{(1-e^{-T})}{(z-e^{-T})}$$

Where K is the variable gain of the controller. Draw the Root-locus diagram for the above system for T=0.1 sec. Comment on the nature of the system.

[16]

4. Consider the system described by following mathematical model:

$$y(k) - 0.75y(k-1) - 0.82y(k-2) + 0.69y(k-3) - 0.55y(k-4) = x(k)$$

Where *x(k)* is the input and *y(k)* is the output of the system. Test the stability of the system employing Jury's Stability test.

[16]

5. Write brief note on any two from the following:

a) Various types of signals encountered in a closed-loop digital control system.

b) Position Algorithm for Digital PID Controller Implementation.

c) Error Constants computed for various signals in discrete-time control systems of various types.

d) Mapping from s-domain to z-domain.

[8+8=16]

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MASTER OF ENGINEERING ELECTRICAL ENGINEERING

1ST YEAR 1ST SEMESTER EXAMINATION, 2024

Subject: DIGITAL CONTROL THEORY (CS) **Time: Three Hours** **Full Marks: 100**

<u>Part II</u> (50 marks)

Question No. — ***Question 1 is compulsory*** — Marks

Answer ***Any Two*** questions from the rest (2×20)

Q1 Answer ***Any Two*** of the following questions (2×5 = 10)

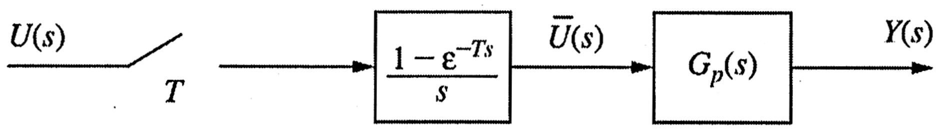
(a) Define State Transition Matrix for a discrete-time system. Derive the solution of the following discrete-time state equation in terms of the State Transition Matrix. 2+3

$$\boldsymbol{x}(k+1) = \boldsymbol{A}\,\boldsymbol{x}(k)$$

(b) Consider a sampled-data system with $T = 0.1$ sec and continuous-time plant

$$G_p(s) = \frac{10}{s(s+1)}$$

Obtain the discrete-time state-space model of the system. 5



(c) Obtain the state-space model of the system described by the difference equation 5

$$y(k+2) = u(k) + 1.7y(k+1) - 0.72y(k)$$

where, $u(k)$ is the input and $y(k)$ is the output of the system.

(d) Briefly discuss the principles of "Full-Order", "Reduced-Order" and "Minimum-Order" State Observers. 5

Q2 (a) Consider a continuous-time system

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

(i) Show that the system is completely state controllable and observable. 4

(ii) Obtain the state equation for the discrete-time system, assuming that the input to the system is held constant between consecutive sampling instants of $T = 1$ sec. 4

(iii) Show that the discrete-time system will remain completely state controllable and observable *if and only if* $T \neq n\pi$, where $n = 1, 2, 3, \ldots$ 4

(b) Define complete state controllability and observability for a discrete-time system. State the *Principle of Duality* for discrete-time systems. 2+2+4

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Q3 (a) Derive the Pulse Transfer Function for the n-th order state space representation 4

$$\boldsymbol{x}\,(k+1) = \boldsymbol{A}\,x(k) + \boldsymbol{B}\,u(k)$$
$$\boldsymbol{y}\,(k) = \boldsymbol{C}\,\boldsymbol{x}(k) + \boldsymbol{D}\,u(k)$$

(b) What is Similarity Transformation? Show that the Pulse Transfer Function is invariant under Similarity Transformation. 2+4

(c) Given a Pulse Transfer Function

$$\frac{Y(z)}{U(z)} = \frac{z+3}{z^2+3z+2}$$

(i) Obtain the discrete-time state-space representation of the system. 4

(ii) From the state equation obtain the values of $y(k)$ for the first 5 sample instants. 6

Q4 (a) Derive the Necessary and Sufficient condition for complete state controllability for an n-th order SISO Linear Time-Invariant Discrete-Time system. 8

(b) The state variable model of a servo motor is given by

$$\boldsymbol{x}\,(k+1) = \begin{bmatrix} 1 & 0.095 \\ 0 & 0.905 \end{bmatrix} \boldsymbol{x}(k) + \begin{bmatrix} 0.005 \\ 0.095 \end{bmatrix} \boldsymbol{u}(k)$$
$$y(k) = [1 \quad 0]\boldsymbol{x}(k)$$

with, $x_1(k)$: angular position of the rotor shaft, $x_2(k)$: shaft velocity.

Consider that the closed loop system is implemented with usual unity feedback.

(i) Obtain, via Ackermann's formula, the values of state feedback gain such that the overall system has closed loop characteristic equation 6

$$z^2 - 1.78\,z + 0.82 = 0$$

(ii) Design an observer for the plant with the observer considered as critically damped with the roots at $z = 0.82$. 6

Q5 (a) What is State Observer? Why is it necessary? 2+2

(b) With the help of a schematic block diagram obtain the state equation for a full-order state observer-based feedback control system. 8

Derive the expression for the error dynamics of the full-order state observer. 2

(c) What is Deadbeat Control? 2

Consider a SISO, Linear-Discrete-Time-Invariant (LDTI) system given by

$$\boldsymbol{x}\,(k+1) = \begin{bmatrix} 0 & 1 \\ -0.16 & -1 \end{bmatrix} \boldsymbol{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \boldsymbol{u}(k)$$

Show that the state feedback gain matrix $\boldsymbol{K} = [-0.16 \quad -1]$ would yield deadbeat response for any arbitrary initial condition $\boldsymbol{x}(0)$. 4