Ex/ME/MATH/5/T/121/4/2024

B.Mechanical(Evening). Examination, 2024
(1ST YR, 2ND SEM)
MATHEMATICS
PAPER - IV

Full Marks : 100 **Time: Three hours**

Part - I

Answer any four questions **$12.5 \times 4 = 50$**

1.(a) If x_1 and x_2 are two positive values of variate, prove that their geometric mean is equal to the geometric mean of their arithmetic and harmonic means.

(b) Find the arithmetic mean from the frequency distribution.

Weight in kg.:	50	55	60	65	70
No. of men:	15	20	25	30	10

2.a Find the median and the median class of the data given below:

Class boundary:	15 - 25	25 - 35	35 - 45	45 - 55	55 - 65	65 - 75
Frequency:	4	11	19	14	0	2

2b. Measurements of the length in feet of 50 iron rods are distributed as follows :

Class boundary	Frequency
2.35 - 2.45	1
2.45 - 2.55	4
2.55 - 2.65	7
2.65 - 2.75	15
2.75 - 2.85	11
2.85 - 2.95	10
2.95 - 3.05	2

Find the value of mode and modal class.

3. What is standard deviation of a set of observations? From the following distribution of scores, calculate standard deviation :

Scores :	6 - 10	11 - 15	16 - 20	21 - 25	26 - 30
Frequency:	4	6	20	7	3

4.a State the axioms of probability.
If A and B are two events which may or may not be mutually exclusive, then prove that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(b) A random variable has the following probability distribution:

x:	4	5	6	8
p(x):	0.1	0.3	0.4	0.2

Find the expectation and variance of the random variable.

5. (a) Define independent events. Let A and B are two independent events. Show that A^c and B^c are also independent.

(b) If A and B are two events in a sample space S such that

$$P(A) = 0.3,\ P(B^c) = 0.4,\ P(A \cup B) = 0.8.$$

Find

$$(i)\ P(A \cap B).\quad (ii)\ P(A^c \cap B^c).\quad (iii)\ P(A^c \cup B^c)$$

[A^c is complement of A]

Part - II

Answer any four questions **12.5 $\times$ 4 = 50**

1. (a) Express

$$\begin{bmatrix} -3 & 2 & 3 \\ 3 & 4 & -5 \\ 5 & -6 & 7 \end{bmatrix}$$

as the sum of a symmetric and a skew symmetric matrix.

(b) Define orthogonal matrix. Show that transpose of an orthogonal matrix is orthogonal.

2. Find all eigen values and corresponding eigen vectors for the matrix

$$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

3. What do you mean by subspace of a vector space? State the necessary and sufficient condition for a non empty subset W of a vector space V(F) be a subspace of V. Show that intersection of two subspaces is also a subspace. Show also, the union of two subspaces is not a subspace.

Let S= {(x,y,z) / 3x-y+z=0}. Show that S is a subspace. Find also a basis for S.

4. What do you mean by inner product space? Define norm of a vector. Show that for any two vectors α, β,

$$(i) \ | (\alpha, \beta) | \leq \| \alpha \| \| \beta \| \quad (ii) \ \| \alpha + \beta \| \leq \| \alpha \| + \| \beta \|$$

5. Define linear mapping and kernel of a linear mapping. Show that $T : R^2 \to R^3$ defined by

$$T(x, y, z) = (x + y + z, 2x + y + 2z, x + 2y + z)$$

is a linear mapping. Find Kernel T and dimension of Kernel T.