Ex/ME/5/Math/T/111/2024

B.Mechanical(Evening). Examination, 2024
(1ST YR, 1ST SEM)
MATHEMATICS
PAPER - III

Full Marks : 100 **Time: Three hours**

Part - I

Answer any four questions **$12.5 \times 4 = 50$**

1.(a) Find the Z-Transformations of the following functions:

$$(i)\ f(n) = 10n \qquad (ii)\ f(n) = 5^n$$

(b) Solve the equation using Z-Transformation

$$f(n+2) - 3f(n+1) + 2f(n) = 0\ ,\ \ given:\ \ f(0) = 1,\ \ f(1) = 2.$$

2. Find the Laplace Transformations of the following functions:

$$(i)\quad F'(t)\ \ and\ \ (ii)\quad F''(t).$$

(ii) Solve by using Laplace Transformation

$$y'' + 4y = 0,\ \ given:\ \ y(0) = 0,\ \ y'(0) = 1.$$

3. Find the Fourier Transformations of the following functions

$$(i)\ e^{-|t|}\quad (ii)\ f(t) = 7e^{-5t^2}$$

4. State Dirichlet's conditions for convergence of a Fourier series. Find the Fourier series of the function

$$f(x) = -3x, \quad when \quad -\pi < x \leq 0$$
$$= 3x \ , \quad when \quad 0 \leq x \leq \pi$$

5.(i) Find inverse Laplace Transformation of the function

$$F(z) = \frac{z}{z^2 - z + 8}$$

(ii) Find the Laplace Transformations of the following functions:

$$f(t) = \frac{10t}{T} \ , \quad 0 < t < T$$
$$= 6 \ , \quad t > T$$

Part-II

Answer any two questions: **15 $\times$ 2 = 30**

6. (i) Solve the equation:

$$\frac{dy}{dx} = \frac{x + 2y - 3}{2x + y - 3}$$

(ii). Find general solution and singular solution:

$$p = \ln(px - y), \quad where \quad p = \frac{dy}{dx}$$

7. Find the general solution:

$$(D^2 - D - 6)y = x, \quad where \quad D = \frac{d}{dx}$$

8. Solve the Bessel's differential equation.

Part-III

Answer the following questions: **10 $\times 2 = 20$**

11. Solve the equations:

$(i)\ (y+z)p + (z+x)q = x+y\ ,$ $\qquad (ii)\ z^2 - pz + qz + (x+y)^2 = 0$

$$\left[where\ \ p = \frac{\partial z}{\partial x}\ ,\ \ q = \frac{\partial z}{\partial y}\right]$$

11. Solve the equation using the method of separation of variables.

$$\frac{\partial^2 u}{\partial x^2} + 6\frac{\partial^2 u}{\partial y^2} = 0$$