B.E. MECHANICAL ENGINEERING FOURTH YEAR FIRST SEMESTER EXAM.– 2023-2024

Subject: Finite Element Method for Non-Structural Applications

Time: 3 Hours **Full Marks: 100**

USE SEPARATE ANSWERSCRIPT FOR EACH PART

PART 1 (Marks 50)

Answer all the questions.

Q1. Find the functional for the ordinary differential equation: $y'' + y' + x = 0$, $0 < x < 1$; subject to the boundary conditions $y(0) = y(1) = 0$. **[10]**

Q2. Consider a wall (as shown in **Fig. Q2)** having thickness l=**150 mm**, area of cross section A = **0.1m²**, thermal conductivity of the material K = **10 W/mK**. The temperature of the surface at the left end of the wall is **20°C** and that of the surface at the right end of the wall is **−10°C**. The rate of heat generation within the wall is $g_0 = 5 \times 10^7$ **W/m³**.

(a) Obtain the differential equation for the one dimensional steady-state heat transfer through the wall considering internal heat generation. Also mention the boundary condition. **[5]**

(b) Obtain finite element equations for the one dimensional steady-state heat transfer through the wall using **Galerkin's weighted residual technique**.

Use three equal finite elements with **linear shape function**. Highlight the following features in your answer.

(i) Expression of the domain residue for a representative element.

(ii) Weak form of weighted residual statement for each element.

(iii) Element level equations for each element in matrix form.

(iv) Assembled finite element equations for heat conduction in matrix form before and after applying boundary conditions.

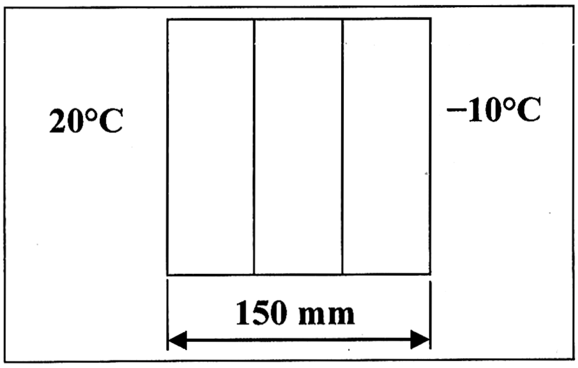

Fig. Q2

[4 + 6 + 15 + 5 = 30]

(c) Solve the above equation to obtain the nodal values of steady state temperature. **[5]**

[Turn over

PART 2 (Marks 50)

Answer all the questions.

Q3. Consider **axisymmetric quasiharmonic** heat conduction problem discretized using 3 node triangular elements. Assume the material to be **isotropic**. Consider an element with convective boundary at one edge. Starting from the governing equations, derive the expressions for stiffness matrix and force vector. **[20]**

Q4. Consider the bar (**Fig. Q2**) subjected to an axially distributed time-dependent load. Derive the governing partial differential equation. The bar is discretized using two 2-node bar elements. Find out the finite element matrices. **[10]**

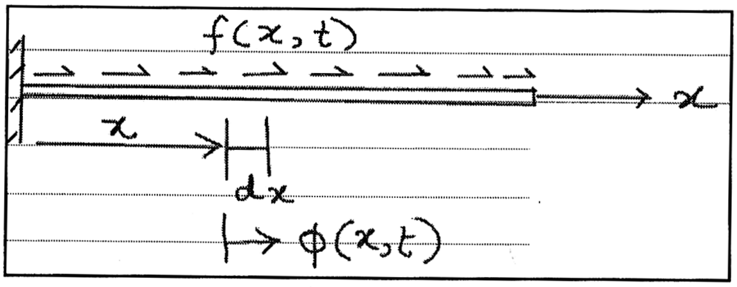

Fig. Q4

Q5. Describe the mode superposition method for solving first order equations of the following form:-

$$[K_t]\{\dot{\varphi}\} + [K]\{\varphi\} = \{R(t)\}$$

Also describe an implicit finite difference algorithm for solution of the same equation. **[20]**