Ref. No. : Ex/ME(M2)/PE/B/T/316F/2024(S)

B. E. MECH. ENGG. THIRD YEAR FIRST SEMESTER SUPPLEMENTARY EXAM – 2024

Subject: ELEMENTS OF COMPUTATIONAL FLUID DYNAMICS **Time : 3 hrs** **Full Marks : 100**

Instructions : <u>Answer any five (5) questions</u>. Write all pertinent assumptions. Assume any missing data. Please be to the point. Verbose sentences and expressions will lead to the deduction in marks.

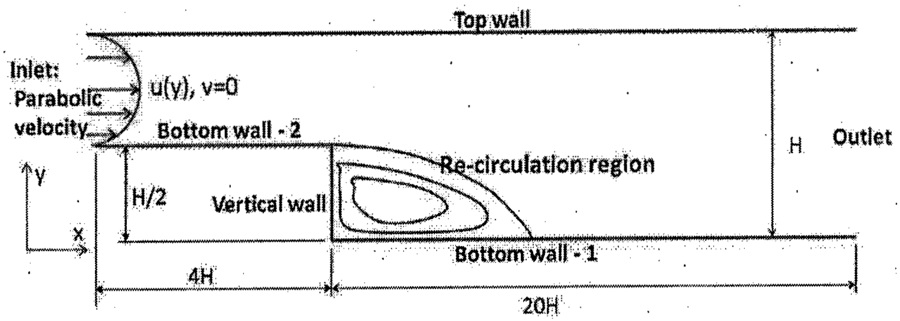
1. (a) Derive the second-order accurate central difference finite difference approximations at the point (i, j) of the following expression: $\frac{\partial^2 u}{\partial x^2}+y\frac{\partial^2 u}{\partial y^2}-e^{xy}$, where $u(x, y)$ is the dependent variable and, (x, y) are the independent variables, respectively.

(b) Employing von Neumann Stability Analysis, obtain the stability condition of a one-dimensional transient heat conduction equation using the Crank-Nicolson method. **(10+10)**

2. A copper plate of 20 cm thickness is initially at 550°C. It is suddenly plunged into cold water. As a consequence, the surface temperature drops to 70°C and remains at this value for the rest of the cooling process. Estimate the time elapsed for the middle plane temperature to cool down to 250°C by dividing the thickness of the plate into five equal divisions and applying BTCS finite difference method. Assume thermal conductivity, density, and specific heat of copper as: k_{Cu}=401 W/m.K, ρ_{Cu}=8850 kg/m^3, Cp_{Cu}=390 J/kg.K. **(20)**

3. Flow over a backward-facing step is a very popular benchmark problem against which CFD codes are tested for comparison and accuracy. As shown in the figure 1, the fluid (assume air) is entering into the backward-facing step with a parabolic inlet velocity and at the outlet it is fully developed.

Figure 1:



Formulate the problem by the *stream function-vorticity* method so that the velocity field can be obtained. Assume the expression of the inlet parabolic velocity profile as: $u(y)=\frac{2y}{H}\left(1-\frac{2y}{H}\right)$. (i) Write governing equations along with the boundary conditions. (ii) Formulate with the stream function-vorticity method. (iii) Discretize the formulated equations using the finite difference method. iv) Write the discretized form of the boundary conditions. (v) Write the step-by-step solution algorithm. **(20)**

4. Let us assume laminar, two-dimensional flow air past a circular cylinder through a horizontal channel as shown in the figure 2. At the inlet, the velocity profile is parabolic and is expressed by: $\frac{u(x,y)}{U_\infty}=\frac{-1}{225}\left(\frac{y}{D}\right)^2+\frac{2}{15}\left(\frac{y}{D}\right)$

At the exit, the flow is fully developed and the pressure is atmospheric. Formulate the problem by the primitive

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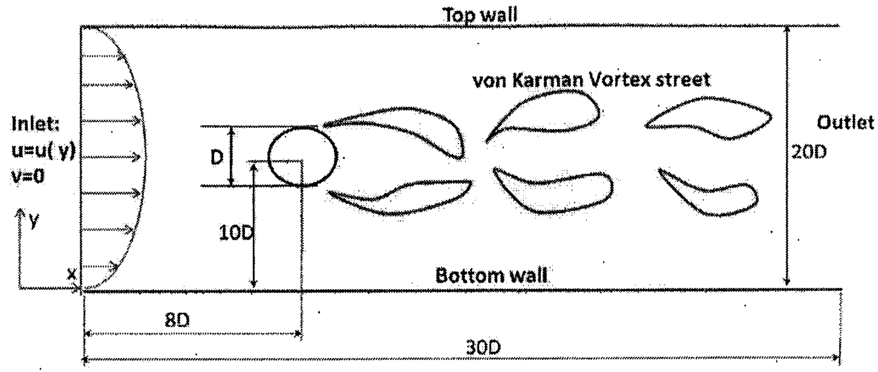
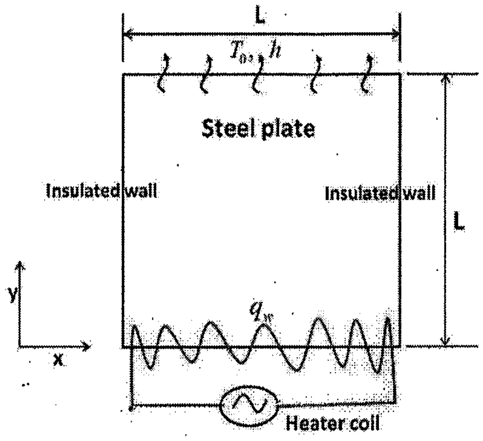
variables approach to obtain the velocity and pressure fields. Show detailed discretizations at all known points. Here, U_∞ is the reference velocity, and D is the cylinder diamater. (20)

Figure 2:



5. An electric heater is mounted at the base of a square steel plate of length L. The heater is continuously generating a heat flux of intensity q_w. The steel plate is kept in a room, where the ambient temperature is T_0 and the convective heat transfer coefficient of the air is h . A schematic representation of the problem is shown in the figure 3 below.

Figure 3:



Formulate the above problem for obtaining dimensionless temperature distribution using the finite difference method. Also, write the step-by-step solution algorithm. Clearly mention all the pertinent assumptions you choose. Assume any missing data. (20)

6. Write short notes on: a) Simple algorithm. b) Upwinding scheme. c) TDMA algorithm. d) Over relaxation and under relaxation. (5+5+5+5)