B.E. MECHANICAL ENGINEERING THIRD YEAR SECOND SEMESTER – 2024

Subject: MECHANICAL VIBRATION ANALYSIS **Time: Three Hours** **Full Marks: 100**

The value of the acceleration due to gravity (g) can be taken as 10 m/s², if it is not specified.

Any missing information may be suitably assumed with appropriate justification.

Q1. Answer any one question from this group.

Q1(a). A viscously damped system has a stiffness of **5,000 N/m**, critical damping constant of **0.2 Ns/mm**, and a logarithmic decrement of **2.0**. Determine the **natural frequency** and **damping ratio** of the system. If the system is given an initial velocity of **1 m/s** with zero initial displacement, determine the **maximum displacement** of the system during its free vibration. **[10]**

Q1(b). A uniform bar of length L and weight W is suspended **symmetrically** by two strings of length h, as shown in **Fig. Q1b**. Set up the differential equation of motion **for small angular oscillations** of the bar **about the vertical axis** ***O-O***, and determine its **natural period of oscillation**. Consider that **the oscillation of the bar is occurring in the horizontal plane only** and the moment of inertia of the bar about the vertical axis ***O-O*** is $WL^2/(12g)$. Show a clear free body diagram for this purpose. **[10]**

Q2. Answer any two questions from this group.

Q2(a). Consider a spring-mass-damper system which is excited by a harmonic motion ($y = Y\sin\omega t$) at its supporting base as shown in **Fig. Q2a**. Show that the **displacement transmissibility** of the system is given by

$$T_R = \left|\frac{X}{Y}\right| = \left[\frac{1+(2\varsigma r)^2}{(1-r^2)^2+(2\varsigma r)^2}\right]^{1/2}$$

where, X is the amplitude of the absolute displacement response of the mass m, ς is the damping ratio and $r = \dfrac{\omega}{\sqrt{k/m}}$. **[15]**

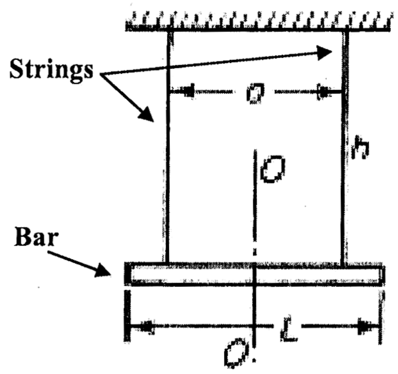

Fig. Q1b

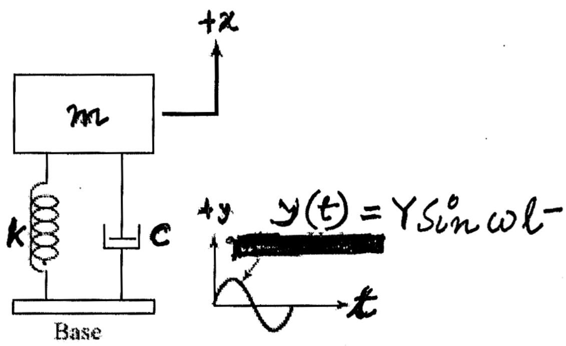

Fig. Q2a

Q2(b). The point of suspension of a simple pendulum is given a harmonic motion $x_0 = X_0 \sin \omega t$ along a horizontal line, as shown in **Fig. Q2b. Write the differential equation of motion of the pendulum** using the coordinates shown. Assume that the amplitude of oscillation of the pendulum mass m with respect to the vertical line is small. Determine **the steady-state solution** for x / x_0, and show that when $\omega = \sqrt{2}\, \omega_n$, the node of vibration is found at the midpoint of l, where ω_n is the natural frequency of the pendulum. **[15]**

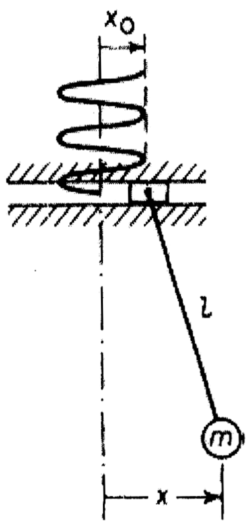

Fig. Q2b

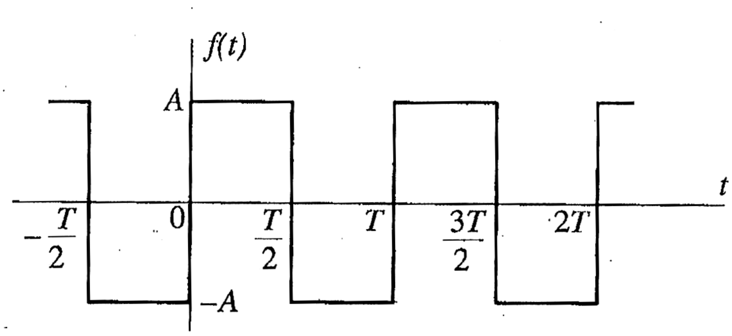

Fig. Q2c

Q2(c). An undamped single degree-of-freedom spring-mass system has the natural frequency ω_n and a mass of **1 kg**. The system is subject to a periodic excitation $f(t)$ as shown in **Fig. Q2c**. The fundamental frequency of the excitation signal is $\omega_0 = \frac{2\pi}{T} = \frac{\omega_n}{2}$. Obtain the expression for **the amplitude of the r^{th} harmonic of the excitation** as it is expanded in Fourier series. Also obtain **the amplitude of the steady-state vibration response** of the system due the r^{th} harmonic of the excitation. **[15]**

Q3. Answer any three questions from this group.

Q3(a). Determine the natural frequency and mode shape vectors of the torsional system shown in **Fig. Q3a**. In this process derive the **equations of motion** of the system with necessary free body diagram. Consider that $K_1 = K_2 = 100$ **N·m/rad** and $J_1 = 50$ **kg·m²**, $J_2 = 20$ **kg·m²**. **[15]**

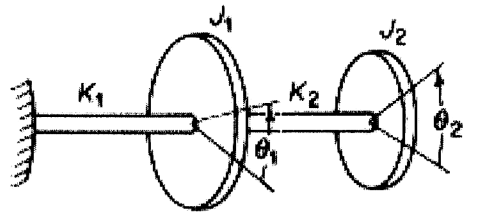

Fig. Q3a

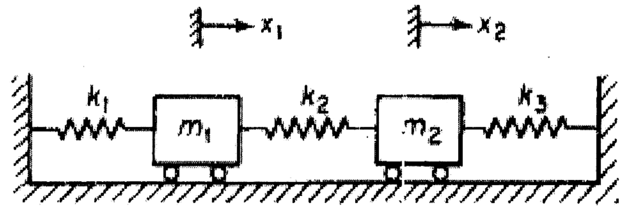

Fig. Q3b

Q3(b). Define the **flexibility influence coefficient. Prove Maxwell's Reciprocity Theorem** in relation to flexibility influence coefficients. Find the flexibility influence coefficients for the system shown in **Fig. Q3b**. Consider that $K_1 = K_2 = 100$ **N/m** and $K_3 = 200$ **N/m**. **[15]**

Q3(c). Consider a two-degrees-of-freedom undamped vibratory system has mass matrix and stiffness matrix given by $[M] = \begin{bmatrix} 9 & 0 \\ 0 & 1 \end{bmatrix}$ and $[K] = \begin{bmatrix} 27 & -3 \\ -3 & 3 \end{bmatrix}$ respectively. The system is subject to the initial conditions given by $x_1(0) = 1$; $x_2(0) = \dot{x}_1(0) = \dot{x}_2(0) = 0$. Find out **the natural frequencies of the system and the corresponding mode shape vectors**. Obtain a **general expression for the free vibration response** of the system due to the given initial conditions. **[15]**

Q3(d). Prove that **the governing differential equation** of a uniform, prismatic Euler-Bernoulli beam for its free transverse vibration is given by

$$EI\frac{\partial^4 y}{\partial x^4} + \rho A\frac{\partial^2 y}{\partial t^2} = 0,$$

where $y(x,t)$ is the deflection of any generic point on the beam on the elastic line at a distance x from the left end and at any instant t during vibration, EI is the flexural rigidity, A is the cross section area of the beam and ρ is the density of the beam material.

Show that if the beam is simply supported **the mode shape function** of the beam during free vibration in n^{th} normal mode is given by $Y(x) = \sin\left(\frac{n\pi x}{l}\right)$ where l is the length of the beam between supports. **[15]**

Q4. Answer any one questions from this group.

Q4(a). Consider the system shown in **Fig. Q4a**, where the mass $\boldsymbol{m_1}$ is subject to a harmonic force $\boldsymbol{F = F_0 \sin \omega t}$. Obtain the equations of motion of the system for forced vibration. Find out the expression of the **frequency response function matrix** for the system. If $\boldsymbol{m_1}$ **= 2 kg**, $\boldsymbol{m_2}$ **= 1 kg**, $\boldsymbol{K_1}$ **= 200 N/m** and $\boldsymbol{K_2}$ **= 100 N/m**, $\boldsymbol{F_0}$ **= 2 N**, determine the **steady state response amplitude** for both the masses at $\boldsymbol{\omega}$ **= 20 rad/s**. **[15]**

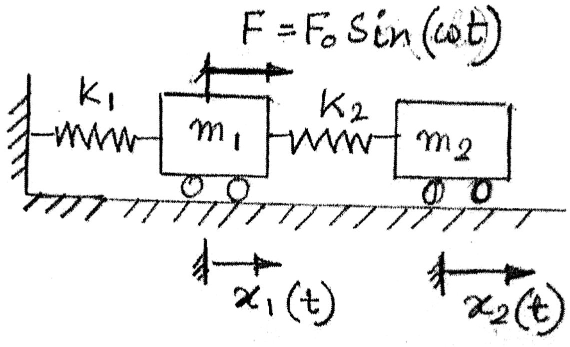

Fig. Q4a

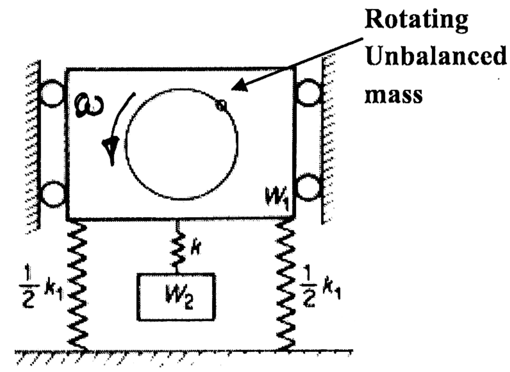

Fig. Q4b

Q4(b). For the system shown in **Fig. Q4b**, W_1 **= 200 N** and the absorber weight W_2 **= 50 N**. If W_1 is excited by a **2 kg·cm** unbalance rotating at $\boldsymbol{\omega}$ **= 1800 rpm**, determine the proper value of **the stiffness of the absorber spring** $\boldsymbol{k}$ for perfect vibration absorption. Also determine **the steady-state vibration amplitude of** $\boldsymbol{W_2}$. Write the equations of motion for this purpose. **[15]**