Ex/ME(M2)/PE/B/T/316A/2024

B.E. MECHANICAL ENGINEERING 3RD YEAR 1st SEMESTER EXAMINATION, 2024

SUBJECT: Introduction to Finite Element Method

Time: Three Hours | Full Marks 100

Answer any 5 questions

All questions carry equal marks. Assume appropriate values for missing data, if any.

Question 1

From minimization of potential energy (PE) derive the stiffness matrix of a 2 node, 2 degree of freedom bar element.

Write down the PE expression for a bar element taking into account thermal stress. How does it modify the force vector?

Starting from the bar element, how do you compute the stiffness matrix of a plane truss element? Mention how do you get the transformation matrix?

Question 2

Using minimization of PE derive the general expression for stiffness of a two-dimensional beam element

Using the shape functions are given below derive the expression for k11.

$$N_1 = \left(1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3}\right) \quad N_2 = \left(x - \frac{2x^2}{L} + \frac{x^3}{L^2}\right)$$

$$N_3 = \left(\frac{3x^2}{L^2} - \frac{2x^3}{L^3}\right) \quad N_4 = \left(\frac{x^3}{L^2} - \frac{x^2}{L}\right)$$

For a three-dimensional beam element the nodes are at (0, 0, 0) and (1m, 0, 0). The 3rd point is at (0.5m, 0.5m, 0.5m). Find out the transformation matrix.

[Turn over

Question 3

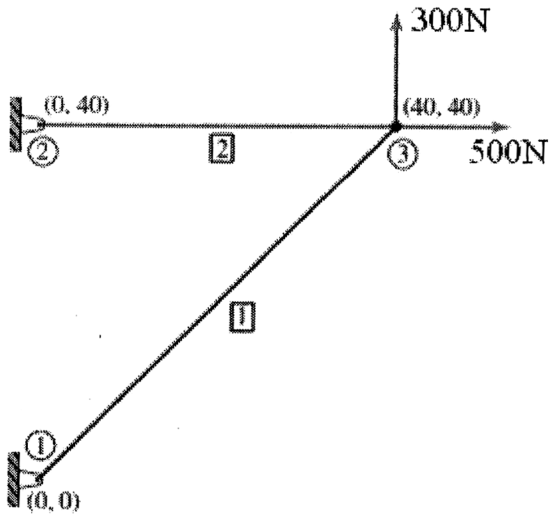

Figure Q3

Consider the truss structure shown in Figure Q3. The coordinates are given in centimeters. The modulus of elasticity and cross-sectional area are $2 \times 10^7\ N/cm^2$ and $2cm^2$ respectively.

The joint is pinned joint and the supports are hinges

a) Find out the displacements at node 3

b) Find out the stresses in the elements.

You may use the following relation:-

$$[K^e] = \frac{AE}{l}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ & s^2 & -cs & -s^2 \\ & & c^2 & cs \\ & sym & & s^2 \end{bmatrix}$$

Question 4

a) What do you mean by plane stress and plane strain problems? Give examples

b) Draw a triangular 3 node CST element and show the nodal degrees of freedom.

c) Starting from the expression of PE, derive the expression for stiffness matrix for such an element?

Take $N_i = \frac{1}{2\Delta}(a_i + b_i x + c_i y)$
Where, $a_1 = x_2 y_3 - x_3 y_2, b_1 = y_2 - y_3, c_1 = x_3 - x_2$

(Multiplication of [B] and [D] matrices is not required. But form of [B] and [D] matrices should be clearly shown. For [D] matrix consider either plane stress or plane strain)

Question 5

State the process of deriving the stiffness matrix, body and surface force vectors of a 4 node isoparametric quadrilateral element.

Two 4 node quadrilateral elements in physical space are shown in Figure Q5. Find out the Jacobian matrices for both the cases at $\xi = 0, \eta = 0$. Write your observation using a sentence.

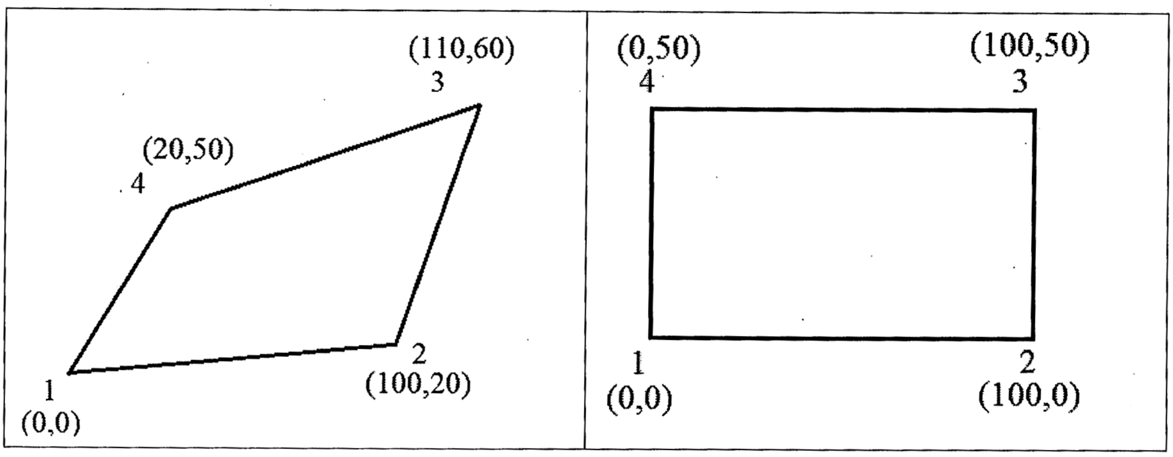

Figure Q5

Question 6

a. Determine the shape functions of a 9 node isoparametric quadrilateral element using Lagrange interpolation function

b. Sketch them

c. Evaluate the integral $\int_{-1}^{1}\int_{-1}^{1} r^3 s^3\,dr\,ds$. Use 2 point and 3 point Gauss quadrature rule. Use the data given in Table 1. Are the results same? Explain your answer.

Table 1. Data for 2 point and 3 point Gauss quadrature rule

Number of points	Locations	Weights
2	$\pm\frac{1}{\sqrt{3}}$	1
3	$\pm\sqrt{0.6}, 0$	$\frac{5}{9}, \frac{5}{9}, \frac{8}{9}$