Ref. No. : Ex/ME(M2)/PE/B/T/316F/2024

B.E. MECHANICAL ENGINEERING THIRD YEAR FIRST SEMESTER - 2024

ELEMENTS OF COMPUTATIONAL FLUID DYNAMICS

Time : 3 hours **Maximum marks: 100**

(Answer **any five** questions. Write all pertinent assumptions. Assume any missing data. Please be to the point. Verbose sentences and expressions will lead to the deduction in marks.)

1. (a) Write the second-order accurate central difference finite difference approximations at the point (i, j) of the following expression:

$$ux\frac{\partial^2 u}{\partial x^2} + u^2 xy\frac{\partial^2 u}{\partial x \partial y} + uy\frac{\partial^2 u}{\partial y^2} + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + uxy + 7$$

$u(x, y)$ is the dependent variable and (x, y) are the independent variables. **(10)**

(b) Show that one may obtain a second order central difference expression of a function $y = f(x)$ at any point $x = x_i$(with $h = x_{i+1} - x_i$), $\left(\frac{d^2y}{dx^2}\right)_{x=x_i} = \frac{y_{i+1}-2y_i+y_{i-1}}{h^2}$. **(10)**

2. Consider the cross-section of a nuclear fuel rod as shown in the Figure 1. Nuclear energy at a uniform rate of q''' W/m^3 is generated in the rod. The surrounding coolant is at temperature T_∞ and the heat transfer coefficient is h.

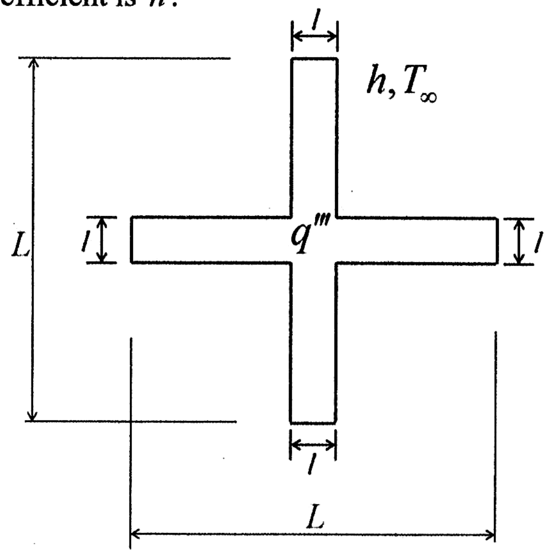

Figure 1.

Formulate the problem to obtain the steady state dimensionless temperature distribution $\theta(X, Y)$ in the rod using the finite-difference method. Follow the following steps: (i) Problem

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formulation: It will contain all assumptions for formulating the problem including governing equations, boundary conditions, etc. (ii) Computational method: Here you need to detail about the computational method you are employing for the solution to the problem, discretized equations and boundary conditions, computational geometry etc. (iii) Method of solution: You need to detail the numerical method for the solution. Use symmetry (1/4th of the domain) **(20)**

3. The dimensionless form of the governing equation with the initial and boundary conditions for an one dimensional transient heat conduction equation is given as follows:

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial X^2}$$

$$IC: \ at \ \tau = 0, \ \theta = 1, \ for \ all \ X$$

$$for \ \tau > 0,$$

$$BC1: \ at \ X = 0, \ \theta = 0$$

$$BC2: \ at \ X = 1, \ \frac{\partial \theta}{\partial X} = 0$$

Here, the dimensionless space coordinate, temperature, and time are X, θ, and τ, respectively. Discretize the above equation using the explicit method and obtain the stability condition employing the von Neumann Stability Analysis. **(20)**

4. (a) What are the types of errors in numerical solutions? What is the grid independence test and why it is required? **(6)**

   (b) Derive the Streamfunction-Vorticity method for the solution of incompressible Navier-Stokes equations. Derive boundary conditions for stream function and vorticity for (i) a stationery wall and (ii) when the wall is moving with a velocity U_∞. **(14)**

5. (a) What is pressure-velocity decoupling? How the pressure-velocity coupling is achieved? **(5)**

   (b) Derive the SIMPLE algorithm for the numerical solution of incompressible Navier Stokes equations. Express all steps in semi-discretized form. Write the step by step solution algorithm. **(15)**

6. Consider a two-dimensional flow of Newtonian, incompressible fluid through a channel, as shown in Figure 2. The length of the channel is L, whereas its height is $2h$. At the inlet, a uniform velocity, $u(x, y) = U_\infty, v(x, y) = 0$, and temperature $T(x, y) = T_\infty$ are prescribed. The channel walls are maintained at a constant temperature T_W, where $T_W > T_\infty$. At the exit, the flow is fully developed and the pressure is atmospheric. Formulate the problem by the primitive variables approach using **Projection Scheme** to obtain the velocity, pressure, and temperature

fields. Using the finite-difference method, show detailed discretizations at all known points. Assume temperature independent thermophysical properties of the fluid.

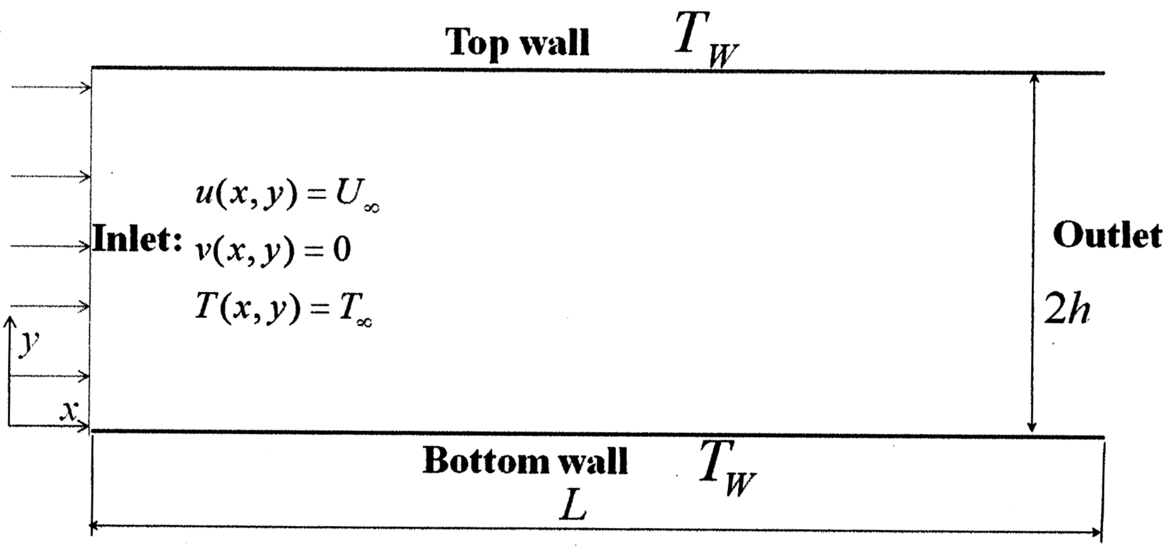

Figure 2.

(20)

7. Write short notes: (a) SMAC algorithm. (b) Upwinding scheme. (c) Proper choice of setting outlet boundary for a fluid flow problem. **(10+5+5)**