Ex/ME/5/MATH/T/221/2024

B.Mechanical. Examination, 2024
(2nd YR, 2nd SEM)
MATHEMATICS
PAPER - IV

Full Marks : 100 **Time: Three hours**

(Use separate answer scripts)

Part - I

Answer any five questions **$10 \times 5 = 50$**

1. What is standard deviation of a set of observations? From the following distribution of scores, calculate standard deviation :

Scores :	6 - 10	11 - 15	16 - 20	21 - 25	26 - 30
Frequency:	4	6	20	7	3

2. (a) Define independent events. Let A and B are two independent events. Show that A^c and B^c are also independent.

(b) If A and B are two events in a sample space S such that

$$P(A) = 0.3,\ P(B^c) = 0.4,\ P(A \cup B) = 0.8.$$

Find

$$(i)\ P(A \cap B),\ \ (ii)\ P(A^c \cap B^c),\ \ (iii)\ P(A^c \cup B^c)$$

[A^c is complement of A]

3. (a) Define conditional probability.

(b) If A and B are two events in a sample space S such that

$$P(A \cap B^c) = \frac{1}{3} \quad , \quad P(A \cap B) = \frac{2}{3}.$$

Find P(B).

(c) Prove that for any frequency distribution, standard deviation is not less than mean deviation from mean.

4. (a) Two unbiased dice are thrown together at random. Find the expected value of the total number of points shown up.

(b) A random variable has the following probability distribution:

x:	1	2	3
p(x):	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

Find the expectation and variance of the random variable x.

5.(a) If x_1 and x_2 are two positive values of variate, prove that their geometric mean is equal to the geometric mean of their arithmetic and harmonic means.

(b) Measurements of the length in feet of 50 iron rods are distributed as follows :

Class boundary	Frequency
2.35 - 2.45	1
2.45 - 2.55	4
2.55 - 2.65	7
2.65 - 2.75	15
2.75 - 2.85	11
2.85 - 2.95	10
2.95 - 3.05	2

Find the value of mode and modal class.

6. (a) In a given business venture a man can make a profit of Rs. 2,000 with probability 0.4 or incur a loss of Rs. 1000 with probability 0.6. Calculate his expectation of profit.

(b) Find the mean and central moments of arbitrary order n for the normal distribution with parameter μ and σ . Also find the coeffient of skewness and kurtosis for this distribution.

Ref. No.: Ex/ME(M2)/BS/B/MATH/T/221/2024

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Part - II (50 Marks)

Answer any five questions $10 \times 5 = 50$

1.(a) Find the Z-Transformations of the following functions:

$$(i)\ f(n) = \cosh nx \qquad (ii)\ f(n) = na^n$$

(b) Find inverse Z transformation of $F(z)$ where

$$F(z) = \frac{z}{z^2 - 6z + 8}.$$

2. (a) Solve the equation using Z-Transformation

$$f(n+1) + 2f(n) = n, \quad given:\ f(0) = 1.$$

(b) Find Laplace transformation of $\sin\sqrt{t}$.

3.(i) Find $L[F''(t)]$, where L stands for Laplace Transformation.

(ii) Solve the equation using Laplace Transformation:

$$y'' + 4y = 0\ , \quad given:\ y(0) = 0,\ y'(0) = 3.$$

4. a. Find Laplace transformation of f(t) where

$$f(t) = t \quad , \quad 0 \leq t \leq \frac{1}{2},$$
$$= t - 1 \quad , \quad \frac{1}{2} \leq t \leq 1,$$
$$= 0 \quad , \quad t > 1.$$

b. Find the Fourier Transformations of the following function

$$(i)\ f(x) = e^{-|x|}.$$

5. State Dirichlet's conditions for convergence of a Fourier series. Find the Fourier series of the function

$$f(x) = 0, \quad when \quad -\pi < x \leq 0,$$
$$= \frac{3x}{4}, \quad when \quad 0 \leq x \leq \pi.$$

6. (a) Find the Fourier series of the function

$$f(t) = 0, \quad when \quad -2 < t < -1$$
$$= 100, \quad when \quad -1 < t < 1$$
$$= 0, \quad when \quad 1 < t < 2.$$

(b) Show that

$$\int_s^\infty f(u)du = L\left(\frac{F(t)}{t}\right),$$

where

$$L(F(t)) = f(s).$$