Bachelor of Mechanical Engineering Examination, 2024

(2nd Year, 1st Semester)

MATHEMATICS – III

Time : Three hours Full Marks : 100

Use separate answer script for each Part.

Part – I (Marks : 50)

Answer ***any five*** questions. 10×5=50

1. Solve the equations :

   i) $xdy - ydx = \sqrt{y^2 + x^2 dx}$

   ii) $\frac{dy}{dx} = \sqrt{y - x}$

2. Find general solution and singular solution :

$$p = \ln(px - y), \text{ where } p = \frac{dy}{dx}$$

3. Find the general solution :

$$(D^2 + D - 6)y = x^2, \text{ where } D \equiv \frac{d}{dx}$$

4. Solve the following Legendre's differential equation about $x = 0$

$$(1 - x^2)\frac{d^2y}{dx^2} - 2xy\frac{dy}{dx} + n(n+1)y = 0$$

5. Define ordinary point and regular singular point of the differential equation

$$P_0(x)\frac{d^2y}{dx^2} + P_1(x)\frac{dy}{dx} + P_2(x)y = 0$$

Find the series solution near the ordinary point $x = 0$ of the equation

$$\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + 3y = 0$$

6. Solve the equation using the method of separation of variables

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2},$$

where,

$$u(0, t) = 0, \; u(L, t) = 0, \; u(x, 0) = f(x), \; 0 \le x \le L, \; t > 0$$

7. Solve:

i) $xyp + y^2 q = zxy - 2x^2$.

ii) $z(x+y)p + z(x-y)q = x^2 + y^2$.

where $p = \frac{\partial z}{\partial x}, \; q = \frac{\partial z}{\partial y}$.

8. Solve the equation by using method of variation of parameters

$$\frac{d^2 y}{dx^2} + a^2 y = \sec ax$$

Part – II (Marks : 50)

Answer any 5 questions $\qquad 10 \times 5 = 50$

1. i) Is the union of two subspaces of a vector space V always a subspace of V? Justify your answer.

   ii) Determine $L\{\alpha, \beta\}$ where $\alpha = (1,3,0)$, $\beta = (2,1,-2)$ in $\mathbb{R}^3$. Examine if $\gamma = (-1,3,2)$, $\delta = (4,7,-2)$ are in $L\{\alpha, \beta\}$.

   iii) For what real values of k does the set $S = \{ (k, 0,1), (1, k+1,1), (1,1,1)\}$ form a basis of $\mathbb{R}^3$. [(1+2) + (2+2) + 3]

2. i) Examine if the set $S = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathbb{R}_{2x2}: \det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = 0\right\}$ is a subspace of the vector space $\mathbb{R}_{2x2}$.

   ii) $U = \{(x, y, z): x + y + z = 0\}$ and $W = \{(x, y, z): x + 2y - z = 0\}$ are two subspaces in $\mathbb{R}^3$. Find dim U, dim W, dim $(U \cap W)$ and dim $(U + W)$.

   iii) Use Gram-Schmidt process to obtain an orthogonal basis of $\mathbb{R}^3$ with standard inner product from the basis set $\{ (1,0,1), (1,1,1), (1,3,4)\}$. [2+3+5]

3. i) Find a basis and the dimension of the subspace W of $\mathbb{R}^3$ where,

   $$W = \{ (x, y, z) \in \mathbb{R}^3: x + 2y + z = 0, 2x + y + 3z = 0\}$$

   ii) A mapping $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by,

   $$T(x_1, x_2, x_3) = (x_1 + x_2 + x_3, 2x_1 + x_2 + 2x_3, x_1 + 2x_2 + x_3)$$

   Show that T is a linear transformation. Find Ker$T$ and dimension of $KerT$. [4+6]

4. i) Find the eigen values and corresponding eigen vectors of [4+6]

   $$\begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & -1 \\ 3 & 2 & -2 \end{pmatrix}$$

   ii) Diagonalize the matrix

   $$\begin{pmatrix} 3 & 2 & 2 \\ 1 & 4 & 1 \\ -2 & -4 & -1 \end{pmatrix}$$

5. i) State and prove Cauchy-Schwartz inequality.
ii) If T is a linear operator on a complex inner product space V such that $\langle Tv, v\rangle = 0$, $\forall\, v \in V$. Then show that T = 0. [6+4]

6. A linear mapping $T: \mathbb{R}^3 \to \mathbb{R}^3$ is defined by
$T(x, y, z) = (x + y + z, x + z, x + y)$, $(x, y, z) \in \mathbb{R}^3$.
If the matrix of T relative to the ordered basis {(1,0,0), (0,1,0), (0,0,1) } of $\mathbb{R}^3$ *be* A and if the matrix of T relative to the ordered basis {(0,1,1), (1,0,1), (1,1,0) } of $\mathbb{R}^3$ *be* B, then show that A, B are similar matrices.

7. i) Let A be a (7 x 7), matrix over $\mathbb{R}$ with characteristic polynomial $= (t-2)^4\,(t-5)^3$ and minimal polynomial $= (t-2)^2(t-5)^3$. What will be the possible Jordon Canonical form(s) of A?
ii) Let V be the vector space over $\mathbb{C}$ of all polynomials in a variable X of degree at most 3. Let $D: V \to V$ be the linear operator given by the differentiation with respect to X. Let A be the matrix of D with respect to some basis for V. Check whether A is a diagonalizable matrix or not. [5+5]

8. Find rational canonical form of the matrix [10]

$$\begin{bmatrix} 0 & -4 & 85 \\ 1 & 4 & -30 \\ 0 & 0 & 3 \end{bmatrix}$$