Ref. No. : Ex/ME(M2)/BS/B/MATH/T/211/2024(S)

**B.E. MECHANICAL ENGINEERING (Supplimentary)**
**EXAMINATION 2024**
**Second Year First Semester**
**Mathematics -III**

**Full Marks -100** **Time : 3 hr**

**Use Separate Answer scripts for each group.**

## Group – A

**Answer any five from the followings.**

1. Solve the following differential equations:

(i) $(x^4y^2 - y)dx + (x^2y^4 - x)dy = 0$
(ii) $(\frac{2x^2}{y} + \frac{x}{y})dx + 2xdy = 0$ 5 + 5

2. Find the general solution of the differential equation

$\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = e^x + cosx$ 10

3. Solve the differential equation $\frac{d^2y}{dx^2} + y = cosec\ x$ by method of variation of parameter. 10

4. Solve the differential equation $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 2x^2$ 10

5. Find the series solution of the equation $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - y = 0$ near the ordinary point $x = 0$. 10

6. Solve the following differential equations where $p = \partial z/\partial x, q = \partial z/\partial y$ :

(i) $zxp + zyq = xy$
(ii) $xyp + y^2q = zxy - 2x^2$ 10

7. Solve the equation using method of separation of variables $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$ over $0 < x < 3, t > 0$ for the boundary conditions $u(0,t) = u(3,t) = 0$ and the initial condition $u(x,0) = 5\ sin\ 4\pi x$. 10

[ Turn over

## Group – B

Answer any 5 questions

10 x 5 = 50

1. i) Find the eigen values and corresponding eigen vectors of

$$\begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & -1 \\ 3 & 2 & -2 \end{pmatrix}$$

ii) Diagonalize the matrix

$$\begin{pmatrix} 3 & 2 & 2 \\ 1 & 4 & 1 \\ -2 & -4 & -1 \end{pmatrix}$$

[4+6]

2. i) Determine the subspace of $\mathbb{R}^3$ spanned by vectors

$$\alpha = (1, 2, 3), \beta = (3, 1, 0)$$

Examine if,

a) $\gamma = (2, 1, 3)$ is in the subspace

b) $\delta = (-1, 3, 6)$ is in the subspace

ii) Prove that the set $S = \{ (2, 1, 1), (1, 2, 1), (1,1,2) \}$ is a basis of $\mathbb{R}^3$.

[6+4]

3. i) Is the union of two subspaces of a vector space V always a subspace of V? Justify your answer.

ii) Determine $L\{\alpha, \beta\}$ where $\alpha = (1,3,0)$, $\beta = (2,1,-2)$ $in$ $\mathbb{R}^3$. Examine if $\gamma = (-1,3,2)$, $\delta = (4,7,-2)$ are in $L\{\alpha, \beta\}$.

iii) For what real values of k does the set $S = \{ (k, 0,1), (1, k + 1,1), (1,1,1)\}$ from a basis of $\mathbb{R}^3$. [(1+2) + (2+2) + 3]

4. i) State and prove Cauchy-Schwartz inequality.

ii) Write down the definition of unitary operators. Show that T in a unitary operator iff $T^*T = I$.

[6+4]

5. i) Let A be a (6 x 6), matrix over $\mathbb{R}$ with characteristic polynomial $= (x-3)^2\,(x-4)^4$ and minimal polynomial $(x-3)(x-2)^2$. What will be the possible Jordon Canonical form(s) of A?

ii) Let V be the vector space over $\mathbb{C}$ of all polynomials in a variable X of degree at most 3. Let $D: V \to V$ be the linear operator given by the differentiation with respect to X. Let A be the matrix of D with respect to some basis for V. Show that A is a nilpotent matrix.

[5+5]

6. i) Let A be a (7 x 7), matrix over $\mathbb{R}$ with characteristic polynomial $= (t-2)^4\,(t-5)^3$ and minimal polynomial $= (t-2)^2(t-5)^3$. What will be the possible Jordon Canonical form(s) of A?

ii) Let V be the vector space over $\mathbb{C}$ of all polynomials in a variable X of degree at most 3. Let $D: V \to V$ be the linear operator given by the differentiation with respect to X. Let A be the matrix of D with respect to some basis for V. Check whether A is a diagonalizable matrix or not. [5+5]

7. Suppose V be the subspace of $\mathbb{R}^5$ with basis,

$$u_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix};\ u_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \\ 1 \end{bmatrix};\ u_3 = \begin{bmatrix} 1 \\ 4 \\ -1 \\ 1 \\ -1 \end{bmatrix};\ u_4 = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

Apply Gram-Schmidt algorithm to find the orthogonal basis for V.