Ref. No.: Ex/ME(M2)/BS/B/MATH/T/111/2024(S)

B.E. MECHANICAL ENGINEERING SUPPLEMENTARY EXAMINATIONS - 2024

FIRST YEAR FIRST SEMESTER

Mathematics-I

Time : Three hours **Full Marks:100**

(*Notations and symbols have their usual meanings.*)

GROUP- A

Answer any five questions from the following.

1. (a) Test wheather the following series converges or not
   (*i*) $\frac{1}{1.2} + \frac{1}{3.4} + \cdots \frac{1}{2n(2n-1)}$
   (*ii*) $\Sigma_{n=2}^{\infty} \frac{1}{\sqrt{n(n-1)}}$.
   (*b*) Find out all the asymptotes of the curve
   $$y = \frac{x^2 - 6x + 3}{x + 3}.$$
   3 + 3 + 4

2. (i) Define a monotone sequence and a bounded sequence.
   (ii)Show that the sequence $\{x_n\}$, where $x_n = \frac{4n+3}{n+2}$ is a bounded monotonic increasing sequence.
   (iii) Examine the convergence of following sequence $\{x_n\}$, where $x_n = \frac{(3n+1)(n-2)}{n(n+3)}$
   3 + 4 + 3

3. (i) State and prove Lagrange's Mean value theorem.
   (ii) Using Mean Value Theorem prove that $\frac{x}{1+x} < log(1 + x) < x$, for all $x > 0$.
   5 + 5

4. (i)(i) Suppose a function $f(x, y)$ defined by $f(x, y) = \frac{x^3+y^3}{x-y}, x \neq y$ and $f(x, y) = 0, x = y$. Is $f(x, y)$ continuous at $(0, 0)$?
   (ii) Using Lagrange's method of undetermind multiplier, find the extreme value of $x^2 + y^2 + z^2$ subject to the condition $ax + by + cz = p$.
   5 + 5

5. (i) If $y = tan^{-1}x$, deduce that
   $(1 + x^2)y_{n+2} + 2(n + 1)xy_{n+1} + n(n + 1)y_n = 0$.

[Turn over

(ii) If a function $f(x,y)$ is defined by $f(x,y) = xy\frac{x^2-y^2}{x^2+y^2}$, when $x^2+y^2 \neq 0$ and $f(x,y) = 0$, when $x^2+y^2 = 0$, show that $f_{xy}(0,0) \neq f_{yx}(0,0)$.

5 + 5

6. (i) State Euler's theorem of homogeneous function of two variables.
(ii) If $u = tan^{-1}\frac{x^3+y^3}{x-y}$ prove that
$x^2\frac{\delta^2 u}{\delta x^2} + 2xy\frac{\delta^2 u}{\delta x \delta y} + y^2\frac{\delta^2 u}{\delta y^2} = (1 - 4sin^2 u)sin2u$.

(b) Evaluate the limit $\lim_{x\to 0} cotx log\frac{1+x}{1-x}$. 2 +5 + 3

7. (i) If v be a function of r alone, where $r^2 = x^2 + y^2 + z^2$. Show that $\frac{\delta^2 v}{\delta x^2} + \frac{\delta^2 v}{\delta y^2} + \frac{\delta^2 v}{\delta z^2} = \frac{\delta^2 v}{\delta r^2} + \frac{2}{r}\frac{\delta v}{\delta r}$.
(ii) If $u = log(x^3+y^3+z^3-3xyz)$, then show that $\frac{\delta u}{\delta x} + \frac{\delta u}{\delta y} + \frac{\delta u}{\delta z} = \frac{3}{x+y+z}$.

5 + 5

GROUP- B

Answer Question Number 8 and any *four* questions from the rest.

8. Define Riemann Integration of a bounded function $f(x)$ in [a,b].

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9. (a) Express $\int_0^1 x^m(1-x^n)^p dx$ in terms of *Beta* function and hence evaluate $\int_0^1 x^5(1-x^3)^{10} dx$.
(b) Evaluate $\int_0^\infty 4x^4 e^{-x^4} dx$.

7 + 5

10. (a) Find the approximate value of $\int_0^1 \frac{dx}{1+x^2}$ by Simpson's $\frac{1}{3}$ Rule taking upto five decimal places.
(b) Suppose $f(x) = x$ and $g(x) = e^x$, verify the first Mean Value Theorem of Integral Calculus for the interval [-1,1].

5 + 7

11. Examine the convergence of following integrals (any two)
(a) $\int_1^\infty \frac{dx}{x^{\frac{1}{3}}(1+x^{\frac{1}{2}})}$
(b) $\int_0^1 \frac{dx}{\sqrt{x(1-x)}}$
(c) $\int_a^\infty e^{-x}\frac{sinx}{x^2} dx, a > 0$. 6 + 6

12. (a) Evaluate $\iint xy(x+y)dxdy$ over the area bounded by $y = x^2$ and $y = x$.

    (b) Evaluate $\int_0^{\pi} \int_0^{a(1+cos\theta)} r^3 sin\theta cos\theta d\theta dr$. 6 + 6

13. (a) Determine the length of one arc of the cycloid $x = a(\theta + sin\theta)$, $y = a(1 - cos\theta)$.

    (b) Find the area of the loop of the curve $x(x^2 + y^2) = a(x^2 - y^2)$. 6 + 6

14. (a) Find the surface of the solid generated by revolution of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ about the x-axis.

    (b) Show that $\int_0^{\frac{\pi}{2}} cos^4 x dx = \frac{3\pi}{16}$. 7 + 5