Ref. No. : Ex/ME(M2)/BS/B/MATH/T/121/2024

B.E. MECHANICAL ENGINEERING EXAMINATION 2024
FIRST YEAR SECOND SEMESTER
Mathematics -II

Full Marks -100 **Time : 3 hr**

Use Separate Answer scripts for each part.

Part -I

Answer Question no 1 any eight from the followings.

1. If $\lambda \neq 0$ be an eigen value of a non singular matrix A, find an eigen value of A^{-1}. 2

2. Reduce the matrix A to row reduced echelon form, where
$A = \begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$ -and hence determine its rank. 6

3. (a) Solve by matrix inversion method the following system of equations
$2x - 3y + 4z = -4$
$x \quad + z = 0$
$-y + 4z = 2$ 6

4. Find for what values of k, the following system of equations
x + y + z = 1,
2x + y + 4z = k ,
$4x + y + 10z = 2k$,
has (a) a unique solution, (b) infinitely many solutions, (b) no solution. 6

5. Find the eigen values and the corresponding eigen vectors of the matrix
$A = \begin{pmatrix} 2 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 0 \end{pmatrix}$. 6

6. Verify that the matrix $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ satisfies it's own characteristic equation. Hence find A^9. 6

7. Two eigen vectors of a square matrix A over a field F corresponding to two distinct eigen values of A are linearly independent. 6

8. Show that if the straight lines whose direction cosines are given by the relations $al + bm + cn = 0$ and $fmn + gnl + hlm = 0$ be parallel, then one of the relations $\sqrt{af} + \sqrt{bg} + \sqrt{ch} = 0$ is true.
Prove further that if the lines be at right angles, then $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$. 6

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9. A variable plane which is at a constant distance $p$from the origin meets the axes at A, B, C. Show that the locus of the centroid of the tetrahedron OABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{16}{p^2}$. 6

10. A variable line intersects the lines $y=0, z=c$; $x=0, z=-c$; and is parallel to the plane $lx+my+nz=p$. Prove that the surface generated by it is $lx(z-c)+my(z+c)+n(z^2-c^2)=0$. 6

11. A sphere of constant radius "r" passes through the origin and cuts the axes in A, B, C. Prove that the locus of the centroid of the triangle ABC is $9(x^2+y^2+z^2)^2=4r^2$. 6

12. Find the equation of shortest distance between the straight lines $\frac{x}{4}=\frac{y+1}{3}=\frac{z-2}{3}$ and $5x-2y-3z+6=0=x-3y+2z-3$. Find also the co -ordinates of the points where the line of shortest distance meets the given lines. 6

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B.E. MECHANICAL ENGINEERING EXAMINATION 2024
FIRST YEAR SECOND SEMESTER
Subject-MATHEMATICS II

Full Marks: 100 **Time: Three Hours**

Part-II (50 Marks)

Answer any five questions

(Symbols/Notations have their usual meanings)

1.a) Show by vector method $\cos(A+B) = \cos A\cos B - \sin A\sin B$ 3

b) If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors with the conditions $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, $|\vec{a}|=3, |\vec{b}|=4$ and $|\vec{c}|=5$ then show that $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=-25$. 3

c) Find the equations of the tangent plane and normal line to the surface $2x^2+y^2+2z=3$, at the point (2,1,-3). 4

2.a) Prove that if $(xyz)^q(x^p\hat{\imath}+y^p\hat{\jmath}+z^p\hat{k})$ be irrotational then $q=0$ or $p=-1$. 4

b) If $\vec{F}=(x+y+1)\hat{\imath}+\hat{\jmath}-(x+y)\hat{k}$, find $\vec{F}.(\vec{\nabla}\times\vec{F})$ 3

c) Show that the $\nabla^2(\ln r)=\frac{1}{r^2}$, where $\vec{r}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$ and $r=|\vec{r}|$ 3

3.a) Find the work done in moving a particle around a circle C in the $xy-$plane, if the circle has center at the origin and radius 2 unit and if the field is given by

$$\vec{F}=(2x-y+2z)\hat{\imath}+(x+y-z^2)\hat{\jmath}+(3x-2y-5z)\hat{k}$$ 5

b) Using Stokes' theorem, evaluate $\oint_C (xy\,dx + xy^2 dy)$ where C is the square in the $xy-$ plane with vertices (1,0), (-1,0), (0,1), (0,-1). 5

4.a) Find the directional derivative of $\varphi = 4xz^3 - 3x^2y^2$ at $(2,-1,2)$ in the direction $2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$. 3

b) Show that $\mathrm{curl}(\mathrm{grad}\, f) = \vec{0}$ 3

c) If $\mathrm{div}(\mathrm{grad}\, r^m) = 0$, then find the value of m, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = |\vec{r}|$. 4

5.a) If $\vec{F} = \vec{\nabla}\varphi$, where φ is a single-valued and has continuous partial derivatives, show that the work done in moving a particle from one point $P(x_1, y_1, z_1)$ in this field to another point $Q(x_2, y_2, z_2)$ is independent of the path joining the two points. 4

b) State Stokes' theorem and hence evaluate the surface integral for the function $\vec{f} = x^2\,\hat{\imath} + xy\,\hat{\jmath}$, integrated around the square in the plane $z = 0$, whose sides are along the lines $x = 0, x = a, y = 0$ and $y = a$. 6

6.a) Show that the vector field given by $(y + \sin z)\hat{\imath} + x\hat{\jmath} + (x\cos z)\hat{k}$ is conservative and hence find the scalar potential of this field. 4

b) Find the equation of the normal and osculating plane of the curve $r(t) = \cos t\,\hat{\imath} + \sin t\,\hat{\jmath} + t\hat{k}$ at the point $P(1,0,0)$. 6

7) Verify Gauss's Divergence theorem for vector function

$$\vec{F} = (2x - z)\hat{\imath} + x^2y\,\hat{\jmath} - xz^2\hat{k}$$

taken over the region bounded by $0 \le x \le 1,\ 0 \le y \le 1, 0 \le z \le 1$ 10