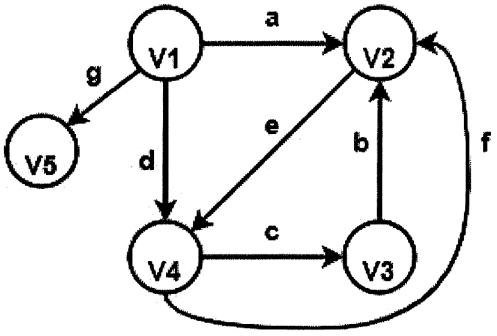
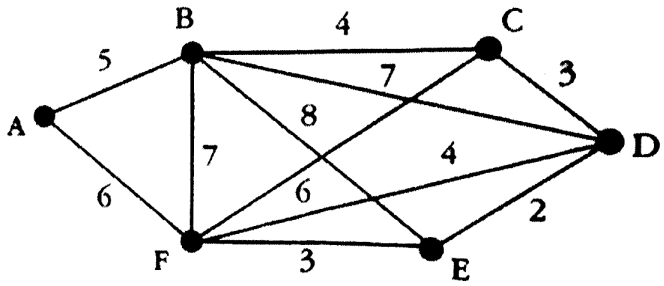
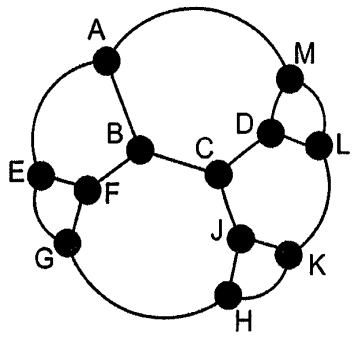
Bachelor Of Engineering In Information Technology
SECOND YEAR SECOND SEMESTER EXAM – 2024
Subject Name –Graph Theory & Combinatorics (IT/PC/B/T/224)
Full Marks=100

CO1 [10]	Q1. (i) Suppose G is a non-directed graph with 12 edges. If 6 vertices are each of degree 3 and rest are degree less than 3, what is the minimum number of vertices in G? (ii) Find the circuit matrix of the following graph G.  (iii) What are the degrees of vertices in a graph G with no adjacent edges? [4+4+2=10]
CO2 [20]	**Q2.** (i) In an electric network consisting of 300 nodes and 500 cables connecting these nodes, find the minimum number of cables that can be removed to reduce the network to a spanning tree. The goal is to remove the cables such that all nodes remain connected without creating isolated groups of nodes. (ii) Prove that "Every connected graph has a spanning tree". (iii) A rooted-tree has n vertices and k number of pendant vertices. Find the number of vertices having degree 3. (iv) Using Kruskal's algorithm find the minimal spanning tree of the following graph.  [4+4+5+7=20]
CO3 [20]	**Q3.** (i) Given a graph **G** with **n = 6** nodes and **e = 8** edges, labelled respectively as: Nodes: A,B,C,D,E,F Edges: $(A-B),(A-C),(A-D),(B-D),(B-E),(C-F),(D-E),(E-F)$ a. Identify all the cut edges in the graph, if any and explain your process.

	b. Determine if there are any cut vertices, and if so, identify them. c. Determine if there are any minimal cut set that would disconnect node A from node F. (ii) Justify this using a suitable example "The vertex connectivity of any graph G can never exceed the edge connectivity of G". (iii) Given two graphs **G and H**: **Graph G**: $V1=\{A,B,C,D,E,F\}$, $E1=\{(A,B),(A,C),(B,C),(B,D),(C,E),(D,F),(E,F)\}$ **Graph *H***: $V2=\{B,D,E,F,G\}$, $E2=\{(B,D),(D,E),(D,F),(E,F),(E,G)\}$ Compute the union of graphs G and H, denoted as $G \cup H$, and provide the resulting set of vertices and edges. [8+6+6=20]
CO4 [20]	**Q4.** (i) "A graph is **2-colorable** if it is bipartite and every cycle has an even length." Justify this with a suitable example. (ii) Find the **chromatic number** of the graph given below. (Mention all steps properly)  (iii) Consider a bipartite graph ***G*** with two sets of vertices: **Set U:** { A, B, C, D } **Set V:** { 1, 2, 3, 4 } The graph contains the following edges connecting the two sets: $(A,1)$; $(A,2)$; $(B,2)$; $(B,3)$; $(C,3)$; $(C,4)$; $(D,4)$ (a) Find the **maximum matching,** if any, in this bipartite graph. (b) Find a **maximal matching, if any,** in the graph and list the matching edges. (c) Determine if the graph has a **perfect matching**. If so, provide the set of edges that form the perfect matching. If not, explain why. [5+5+(3+3+4)=20]
CO5 [20]	**Q5.** (i) What is the minimum number of students required in a Graph Theory Class class to be sure that at **least six** will receive the same grade, if there are five possible grades, **A, B, C, D, and F**?

	(ii) Suppose a club has 25 members. How many ways are there to choose a **president, vice-president, secretary**, and **treasurer** of the club, where no person can hold more than one post? (iii) **Seven** women and **nine** men are on the faculty in the IT department at a University. How many ways are there to select a committee of **five** members of the department if at least **one woman** and at least **one man** must be on the committee? [6+7+7=20]
CO6 [10]	**Q6.** **(i)** Find the generating functions of the following sequences in closed form: **<4, 4, 4, 4, 4, ...>** (ii) Using generating functions, find a_n in terms of n for the case given below: $a_0 = 1,\ a_1 = 2$ and $a_{n+2} = 5a_{n+1} - 4a_n$ for $n \geq 0$ [5+5=10]

CO1: Explain and **discuss** the concept of different types of Graphs with fundamental properties and **express** different types of matrix representation. (K2)
CO2: Illustrate different types of trees such as (i) rooted tree (ii) spanning tree etc, and **explain** their properties. (K3)
CO3: Apply operations like Union, Deletion, and decomposition of graphs and **illustrate** Cut vertex and Cut edge and their properties. (K3)
CO4: Illustrate planar graph and their properties and Graph Coloring and Matching. (K3)
CO5: Apply and **evaluate** basic counting rules, pigeon-hole principle and principle of inclusion-exclusion. (K3)
CO6: Apply and **Solve** problems using Generating Function and Recurrence Relations. (K3)