Ref. No.: **Ex/ET/PC/B/T/323/2024**

B.E. ELECTRONICS AND TELE-COMMUNICATION ENGINEERING THIRD YEAR SECOND SEMESTER - 2024

Subject: DIGITAL CONTROL SYSTEMS   Time: 3 Hours   Full Marks: 100

All parts of the same question must be answered at one place only.

PART-A: Answer any ONE [CO1]

1. State and prove Nyquist sampling theorem. 10

2. For an open loop digital control system, derive the expression of the spectra of the flat-top sampled error signal. 10

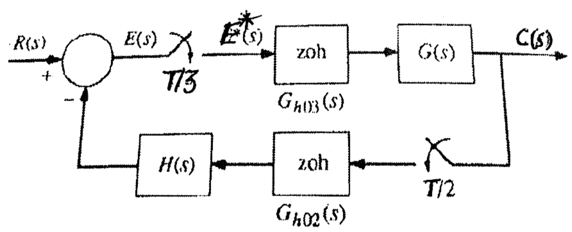
PART-B: Answer any TWO [CO2]

3. (a) A sampler cannot be represented by transfer function. Justify. 3

   (b) Derive the transfer function of a polygonal hold circuit. 7

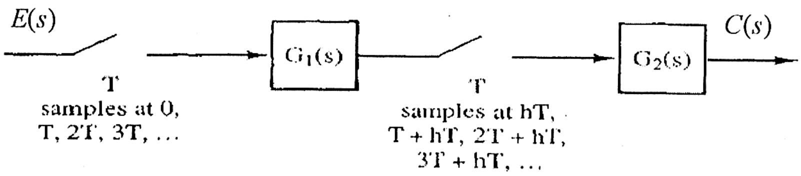
4. Determine the output response of a fast-slow sampling system by realizing a fast sampler (with sampling period *T/N*) using a slow sampler (of sampling period *T*). 10

5. Determine the closed loop transfer function for the following multi-rate control system. 10

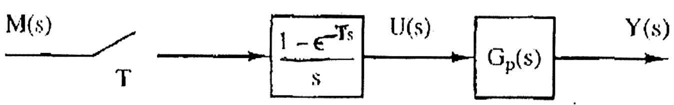


PART-C: Answer any TWO [CO3]

6. Determine $C(z)$ of the following system. 15



7. (a) Derive the expressions of the system matrix **A** and input matrix **B** of the following open loop digital control system in terms of the system and input matrices of the continuous plant $G_p(s)$. 9



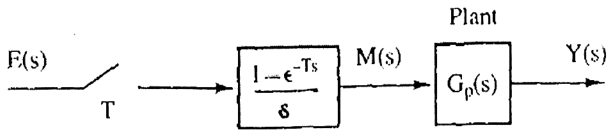
   (b) Hence determine **A** and **B** for 6

$G_p(s) = \frac{10}{}$

[Turn over

8. Consider the following system with the behavior of the plant described by the first order differential equation

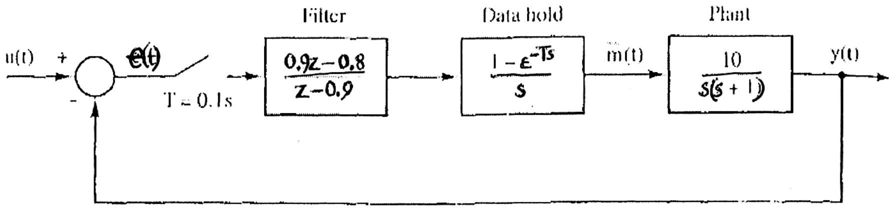
$$\frac{d^2y(t)}{dt^2}+0.15\frac{dy(t)}{dt}+0.005y(t)=0.1m(t).$$



(a) Draw a continuous-time simulation diagram for $G_p(s)$ and give the state equations. 7

(b) Use the state-variable model of part (a) to find a discrete state model for the entire system. 8

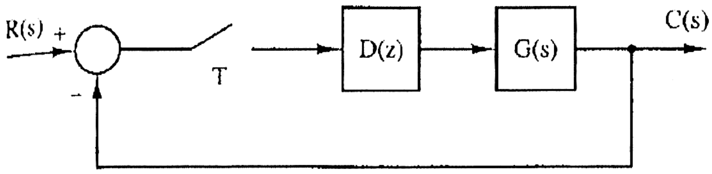
9. Derive the state-space representation of the following closed loop digital control system. 15



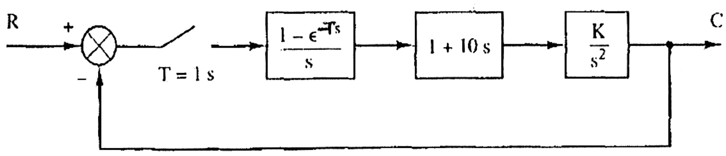
PART-D: Answer any TWO [CO4]

10. (a) What is bilinear transform? 4

(b) Design a digital controller D(z) to attain a steady state error less than 0.01 for unit ramp input and to ensure stability of the entire system with $G(s)=\frac{1-\exp(-Ts)}{s(s+1)}$ and T=0.1 sec. 6



11. Find the range of K for stability of the system from its root locus. Also determine the oscillating frequency for the marginal stability. 10



12. State and prove Nyquist stability criterion for digital control system. 10

13. (a) Explain the effect of addition of open loop poles on the closed loop stability using root locus. 6

(b) Using Nyquist stability criteria, comment on stability of a closed loop system with 4 open loop transfer function $\overline{GH}(z) = \dfrac{0.01kz}{(z-1)^2(z-0.905)}$.

PART-E: Answer any TWO [CO5]

14. For a plant described by 10

$$\vec{x}(k+1) = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix} \vec{x}(k) + \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix} u(k)$$

find the gain matrix K required to realize a damping ratio of 0.46 and a time constant of 0.5 s.

15. (a) Derive the state dynamics of a reduced order state observer. 7

(b) Derive the condition of observability. 3

16. Given a first order plant described by $x(k+1) = 0.9x(k) + 0.1u(k)$ with the cost function 10

$$J_3 = \sum_{k=0}^{3} \left(x^2(k) + 5u^2(k) \right)$$

calculate the optimal control inputs to minimize the cost function.

17. For a linear digital control system, state and prove the Lyapunov stability criterion. Hence show that 10

$$\vec{u}^o(k) = -(\mathbf{B}^T\mathbf{P}\mathbf{B})^{-1}(\mathbf{B}^T\mathbf{P}\mathbf{A})\vec{x}(k)$$

where the symbols carry their usual meaning.