BETCE EXAMINATION, 2024
(2nd Year 1st Semester)
Signals and Systems

Full Marks: 100 **Time: 3 hours**

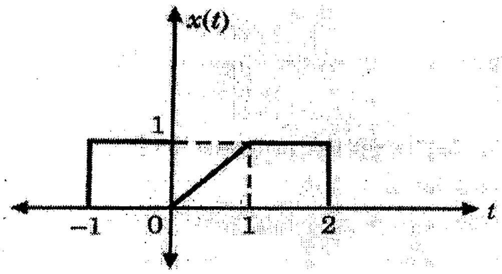
Answer all parts & subparts of a question under a unit serially in the same place

CO-1 (Marks: 25)

Q1. a) Find whether the signal $x(t) = 2\cos(10t+1) - \sin(4t-1)$ is periodic or not. Also, find the time period, if it is periodic. [4]

b) Check whether the given signal $g(t) = e^{j\left(2t+\frac{\pi}{4}\right)}$ is energy or power signal. [4]

c) For the given signal $x(t)$ as shown below, sketch $x(1-t/2)$ following necessary steps. [5]



d) Find the odd and even components of the signal defined by $g(t) = e^{jt}$ [2]

OR

Q1. a) Sketch the following signal:

$x(t) = r(-t+1)u(t) + 2r(t-1) - r(t-2) - u(t-2)$

where, $u(t)$ and $r(t)$ represents the unit step and unit ramp function respectively. Draw each segment separately and finally combine them to obtain $x(t)$. [6]

b) Determine the energy of the signal $x(t) = e^{-a|t|}$ *for* $a > 0$ [4]

c) Let $x_1(t)$ and $x_2(t)$ be two unit energy signals orthogonal over an interval from $t = t_1$ to t_2 and can be represented by two unit length, orthogonal vectors $(\vec{x}_1, \vec{x}_2)$. Consider a signal defined by

$g(t) = c_1 x_1(t) + c_2 x_2(t), \quad t_1 \le t \le t_2$

This signal can be represented as a vector $\vec{g}$ by a point (c_1, c_2) in the $x_1 - x_2$ plane. [3+2]

[Turn over

i) Determine the vector representation of the following signals in two-dimensional vector space:

$$g_1(t) = 2x_1(t) - x_2(t)$$
$$g_2(t) = -x_1(t) + 2x_2(t)$$
$$g_3(t) = x_1(t) + 2x_2(t)$$

ii)Also, identify one pair of mutually orthogonal and another pair of mutual non-orthogonal vectors.

Q2.a) Consider an LTI system having two signals as follows:

$$x(t) = u(t+1)$$

$$h(t) = u(t-2)$$

Find out the output response of the system by calculating the area of the overlapped regions between $x(\tau)$ and the various time shifted versions of the folded sequence $h(-\tau)$. [6]

b) Discuss in brief, the significance of the Energy Spectral Density (ESD) of a signal. How is it related to the auto-correlation function of the signal? [2+2]

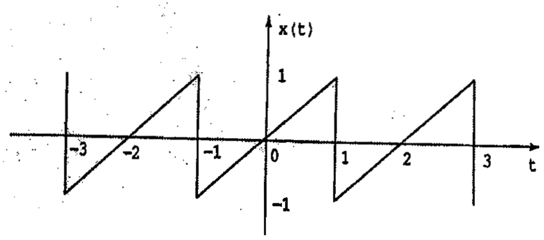
OR

Q2. a) Consider an arbitrary signal $g(t)$ having an energy of E_g.The Fourier transform of this signal is represented by $G(f)$ or $G(\omega)$. Calculate E_g from $G(f)$ or $G(\omega)$ using Parseval's theorem. [5]

b) Mathematically prove that time domain convolution between two signals leads to the frequency domain multiplication of the same signals. [5]

CO-2 (Marks: 15)

Q3. a) Find the trigonometric Exponential Fourier Series of the following periodic signal: [7]

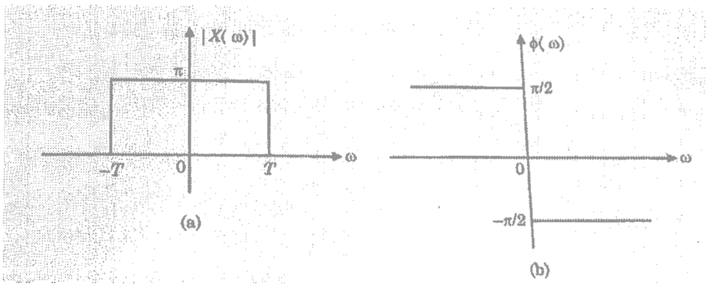


b) Find the Fourier transform of the given signal, $g(t) = u(t+1) - u(t-1)$, where, $u(t)$ represents unit step function. [5]

c) Mention the relationship between Trigonometric and Exponential Fourier spectra in terms of both magnitude and phase response. [3]

OR

Q3. a) The magnitude $|X(\omega)|$ and phase spectra $\Phi(\omega)$ of a signal $x(t)$ are shown in fig(a) and fig(b) respectively. [2+1+4]



i) Write down the expressions of $|X(\omega)|$ and $\Phi(\omega)$ for different ranges of ω.

ii) Hence write down the expression of $X(\omega)$ for different ranges of ω.

iii) Finally, using the above information and following the definition of the Inverse Fourier transform, calculate the value of $x(t)$.

b) An arbitrary signal $g(t)$ is multiplied by a sinusoid $\cos\omega_o t$. [3+2]

i) What will be the impact of this time domain multiplication in frequency domain? Explain necessary diagrams.

ii) Does this process have any special significance in communication system?

c) Find the Fourier transform of the signal, $e^{-(t-t_o)}u(t-t_o)$. Use the properties of Fourier transform wherever necessary. [3]

CO-3 (Marks: 10)

Q4. A signal $g(t)$ band-limited to B Hz is sampled by a periodic pulse train $\delta_{T_S}(t)$ having a period of T_S to have the sampled signal $\overline{g}(t)$. Show that $g(t)$ can be recovered from $\overline{g}(t)$, if sampling frequency f_S meets the requirement $f_S \geq 2\text{B}$. Use necessary diagrams and mathematical expressions. [10]

OR

Q4. a) Consider that a periodic pulse train is modulated by a sinusoidal signal. Sketch the waveforms of Pulse Width Modulated (PWM) and Pulse Position Modulated (PPM) signals with reference to the above signals. [3+3]

b) A signal $x(t) = 1 + \cos(10\pi t) + \cos(30\pi t)$ is sampled by a periodic pulse train having sampling period of 0.04 sec to obtain its sampled version $\bar{x}(t)$. Is it possible to recover the signal $x(t)$ from its sampled version $\bar{x}(t)$? Justify your answer. [4]

CO-4 (Marks: 15)

Q5. a) Check whether the system described by $y(n) = \log_{10}|x(n)|$ is [1+1+4+3]

i) static or dynamic
ii) causal or non-causal
iii) linear or non-linear
iv) time variant or time-invariant

b) Two LTI systems having impulse responses $h_1(n) = \{\underset{\uparrow}{1}, 2\}$ and $h_2(n) = \{\underset{\uparrow}{3}, 4\}$ are connected in cascade. Calculate the overall response of the system by computing the convolution of above signals using graphical approach. [10]

CO-5 (Marks: 10)

Q6. The joint probability Distribution Function (PDF) of two random variables X and Y is defined by [4+4+2]

$$f(x,y) = x + y \quad 0 < x \le 1; \quad 0 < y \le 1$$
$$= 0 \quad elsewhere$$

Calculate the following:

a) Covariance of X and Y i.e. $Cov(X,Y)$
b) Variance of X and Y i.e. σ_X^2 and σ_Y^2
c) Correlation coefficient between X and Y i.e. ρ_{XY}

CO-6 (Marks: 25)

Q7. Consider that noise is a random process and it can be represented as the superposition of noise spectral components. Also consider that the spectral component associated with the kth frequency interval is given by:

$n_k(t) = a_k \cos(2\pi k\Delta ft) + b_k \sin(2\pi k\Delta ft)$, where symbols have their usual meanings. [5+5+5]

a) Show that a_k and b_k are Gaussian random variables.

b) Suppose, a filter having transfer function $H(f)$ in placed in a noisy communication environment. Derive the relation between the input and output Power Spectral Density (PSD) of this filter.

c) Using above relation, calculate the noise power at the output of an RC low-pass filter when white noise is present at the input of this filter.

Q8. a) Consider a cascade of two amplifier stages where A_1 is the gain and R_1 is the total input noise resistance of the first stage; A_2 is the gain and R_2 is the total input noise resistance of the second stage and R_3 is the output resistance. Calculate the equivalent resistance of the cascaded amplifier. [7]
b) How does a resistor act as noise generator? [3]

OR

Q8. a) In a cascade of two amplifiers, the first stage has a power gain of G_1 and Noise Figure of F_1; whereas, for the second stage, these values are G_2 and F_2 respectively. If the input and output noise power of this cascade is N_i and N_O respectively, calculate the overall Noise Figure of this cascade. [7]
b) How are transit-time noise and flicker noise generated within an amplifying device? [3]