Ref. No.: Ex/ET/PC/B/T/216/2024

B.E. ELECTRONICS AND TELE-COMMUNICATION ENGINEERING

SECOND YEAR FIRST SEMESTER EXAM 2024

Subject: MATHEMATICS III

Time: 3 Hours **Full Marks: 100**

Answer Question No. 1 and any FIVE questions from the rest. Q.1 carries 20 marks and the rest are of 16 marks each. Answer in brief with proper justification. All the symbols used carry the standard meanings.

Q. No.		Marks
1.	***Answer with proper reasons. Attempt all the parts in one place.*** ***Section I: Select with reasons the correct option:*** i). The condition for independence of two events A and B is (a) $P(A \cap B) = P(A)P(B)$ (b) $P(A - B) = P(A)P(B)$ (c) $P(A \cap B) = P(A)P(B/A)$ ii). If $F(x)$ is the distribution function of a random variable, then (a) $F(x)$ is continuous at all points (b) $F(-\infty) = 1$ (c) $F(+\infty) = 1$ iii). For $f(z) = (z-1) + \sin\left(\frac{1}{z-1}\right)$, the point $z = 1$ is (a) a simple pole (b) a non-isolated singularity (c) an essential singularity ***Section II: Answer in brief.*** iv). If A and B are independent events such that $P(B) = \frac{2}{7}, P(A + \bar{B}) = 0.8$, then find $P(A)$. v). Find the smallest value of K in the Tchebycheff's inequality $P(\lvert X - \mu\rvert < K\sigma) \geq 1 - \frac{1}{K^2}$ for which the probability is at least 0.99. vi). Three different numbers are selected at random from the set $A = \{1,2,3,\ldots,11,12\}$. Find the probability that the product of two of the numbers is equal to the third. vii). Evaluate $\lim_{z \to -1+i} \frac{z^2+2z+2}{z^2+2i}$ viii). A random variable X has the following pdf: $f(x) = \begin{cases} k, & -2 < x < 2 \\ 0, & elsewhere \end{cases}$. Find the value of the constant k. ix). Show that $F = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative field. x). A random variable X has Poisson distribution such that $P(1) = P(2)$. Calculate the standard deviation of X.	$2 \times 10 =$ 20
2.	(a) Establish Poisson's Equation and hence Laplace's equation from the fundamental concept of Divergence and the basic equations of Electrostatics. (b) i) Show that a necessary and sufficient condition that $F_1dx + F_2dy + F_3dz$ be an exact differential is that $\nabla \times \boldsymbol{F} = \boldsymbol{0}$ where $\boldsymbol{F} = F_1\hat{i} + F_2\hat{j} + F_3\hat{k}$. ii) Show that $(y^2z^3 \cos x - 4x^3z)dx + 2z^3y \sin x\, dy + (3y^2z^2 \sin x - x^4)dz$ is an exact differential of a function ϕ and find ϕ.	$8 + 4 + 4$ $= 16$
3.	(a) A, B & C in order toss a coin. The first one to throw a head win. What are their respective chances of winning? Assume that the game can continue indefinitely. (b) In a normal distribution, 31% of the items are under 45 and 8% are above 64. Find the mean and standard deviation. [Given: $P(0 < Z < 1.405) = 0.42$, $P(-0.496 < Z < 0) = 0.19$] (c) Prove that for normal distribution, the mean deviation from the mean equals nearly $\frac{4}{5}$ times of the standard deviation.	$6 + 5 + 5$ $= 16$

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4.	(a) With the number of trials increased indefinitely and the probability of success in a single trial being very small, show that the probability mass function (p.m.f.) of Poisson variate can be obtained as a limiting case of the p.m.f. of Binomial variate. (b) Out of 800 families with 4 children each, how many families would be expected to have: i) 2 boys and 2 girls, ii) at least one boy, iii) no girl, iv) at most 2 girls, v) children of both sexes? Assume equal probabilities for boys and girls.	8 + 8 = 16
5.	(a) Let $X_1, X_2, \dots X_n$ be a sequence of independent random variables, each having mean m and variance σ^2. Show that $\frac{1}{n}\sum_{i=1}^{n} X_i \to m$ in mean square as $n \to \infty$. (b) A sign in an elevator reads *'capacity 2200 kg or 30 persons*. Assume a standard deviation of $15kg$ for the weight of a person drawn at random from all the people who might ride the elevator, and calculate approximately his expected weight. It is given that the probability that a full load of 30 persons will weigh more than 2200 kg is 0.33.[Given: $P(Z) > 0.044 = 0.33$, where Z is a standard normal variate.] (c) Let $f(z) = u(x, y) + iv(x, y)$ be an analytic function. If $u = 3x + 2xy$, then find v and express $f(z)$ in terms of z.	5 + 6 + 5 = 16
6.	(a) If $f(z)$ is analytic, where $z = z + iy$, prove that $\left\{\frac{\partial}{\partial x}\lvert f\rvert\right\}^2 + \left\{\frac{\partial}{\partial y}\lvert f\rvert\right\}^2 = \lvert f'\rvert^2$. (b) Find the Taylor series expansion of a function of complex variable $f(z) = \frac{1}{(z-1)(z-3)}$ about the point $z = 4$. Find its region of convergence. (c) Find the poles of $f(z) = \frac{(z^2-2z)}{(z+1)^2(z^2+4)}$ and residues at the poles which lie on imaginary axis.	5 + 6 + 5 = 16
7.	(a) Use Cauchy's integral formula evaluate $\int_{\lvert z\rvert=1} \frac{e^{3z}dz}{(4z-\pi i)^3}$ (b) Using Cauchy's residue theorem evaluate $\int_{-\infty}^{\infty} \frac{x^2\,dx}{(x^2+1)(x^2+4)}$ (c) Find the Laurent's series of $f(z) = \frac{5z+2}{z^3-z^2-2z}$ in the region $1 < \lvert z+1\rvert < 3$	6 + 6 + 4 = 16
8.	(a) Solve in series the equation $\frac{d^2y}{dx^2} + xy = 0$ (b) Verify Stokes' theorem for $\mathbf{A} = (2x - y)\,\hat{\imath} - yz^2\,\hat{\jmath} - y^2z\,\hat{k}$, where $\boldsymbol{S}$ is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and $\boldsymbol{C}$ is its boundary. (c) Evaluate $\int_{(0,0)}^{(2,1)} (10x^4 - 2xy^3)dx - 3x^2y^2dy$ along the path $x^4 - 6xy^3 = 4y^2$	6 + 6 + 4 = 16
9.	(a) Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ given that $u(0,t) = u(l,t) = 0, u(x,0) = f(x)$ and $\frac{\partial u}{\partial t}(x,0) = 0$ where $0 < x < l$. (b) Prove that $J_2'(x) = \left(1 - \frac{4}{x^2}\right)J_1(x) + \frac{2}{x}J_0(x)$ (c) Find the bilinear transformation which maps the points $z_1 = 2, z_2 = i, z_3 = -2$ into the points $w_1 = 1, w_2 = i, w_3 = -1$ respectively.	7 + 4 + 5 = 16