Ref. No.: Ex/EE/5/T/512A/2024

BACHELOR OF ENGINEERING (ELECTRICAL ENGINEERING)

5TH YEAR 1ST SEMESTER EXAMINATION, 2024

Subject: DIGITAL CONTROL TECHNIQUES **Time: Three Hours** **Full Marks: 100**

Use a separate Answer-script for each Part

Part I (50 marks)

Question No.	Question 1 is compulsory Answer *Any Two* questions from the rest (2×20)	Marks
Q1	Answer ***any Two*** of the following:	
(a)	Classify the types of signals associated with discrete-time systems based on sampling and quantization and draw their time-domain representations.	5
(b)	Obtain z-Transform of unit step functions that is delayed by 1 sampling period.	5
(c)	State and prove the "*Initial Value Theorem*" in respect of Discrete-time Systems.	5
(d)	Show how the left half of the s-plane will be mapped into the z-plane. Briefly discuss the concepts of "*Primary Strip*" and "*Complementary Strip*" in respect of the mapping from s-plane to z-plane.	5
Q2 (a)	What is an Impulse Sampler? Why is it also referred to as an *Impulse Modulator*?	2+4
(b)	(i) What is a Data Hold Circuit? Show how Zero Order Hold can be used to reconstruct analog signals from their sampled versions.	2+2
	(ii) Derive the transfer function of the Zero Order Hold circuit assuming an Impulse function at t=0 as the input.	6
(c)	Draw the output response of a real sampler with First Order Hold circuit for a unit step input.	4
Q3 (a)	(i) Define Convolution Summation for discrete-time systems?	4
	(ii) Derive the expression for Pulse Transfer Function for a discrete-time system.	4
(b)	Obtain the Pulse Transfer Function of a continuous-time system given by $$G(s) = \frac{1}{s+a}$$ Assume an Impulse Sampler at the input of the continuous-time system.	4
(c)	Show how the followings will be mapped from the left half of the s-plane to the z-plane: (i) constant frequency (ω) loci, (ii) constant damping ratio (ζ) loci.	3+5

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Q4 (a) State and prove the "*Final Value Theorem*" in respect of Discrete-time Systems. 6

(b) Determine, using the Final Value theorem, the value of $x(\infty)$ for $X(z)$ given as 4

$$X(z) = \frac{1}{1 - z^{-1}} - \frac{1}{1 - e^{-aT}z^{-1}}, \quad a > 0$$

Justify your answer by obtaining the value of $x(t)$ as $t \to \infty$. 2

(c) Obtain, with the help of starred Laplace Transform, the transfer function $C(z)/R(z)$ for the closed loop configuration shown in Figure Q4(c). 8

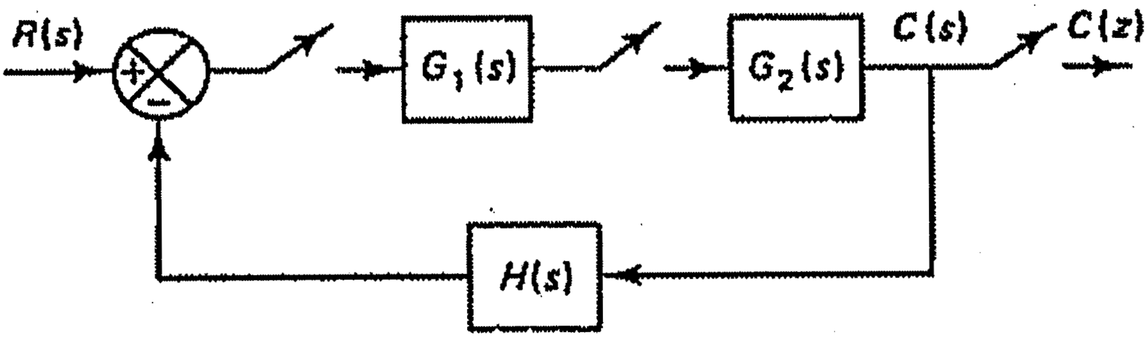

Figure Q4(c)

Q5 (a) (i) State Jury's Stability Test for a closed-loop discrete-time system. 5

(ii) Determine the stability of a discrete-time system having the characteristic equation 5

$$Q(z) = z^3 - 1.8z^2 + 1.05z - 0.20 = 0$$

(b) Derive the expressions for the *Static Position*, *Velocity* and *Acceleration Error Constants* for the discrete-time control system shown in Figure Q5(b). 10

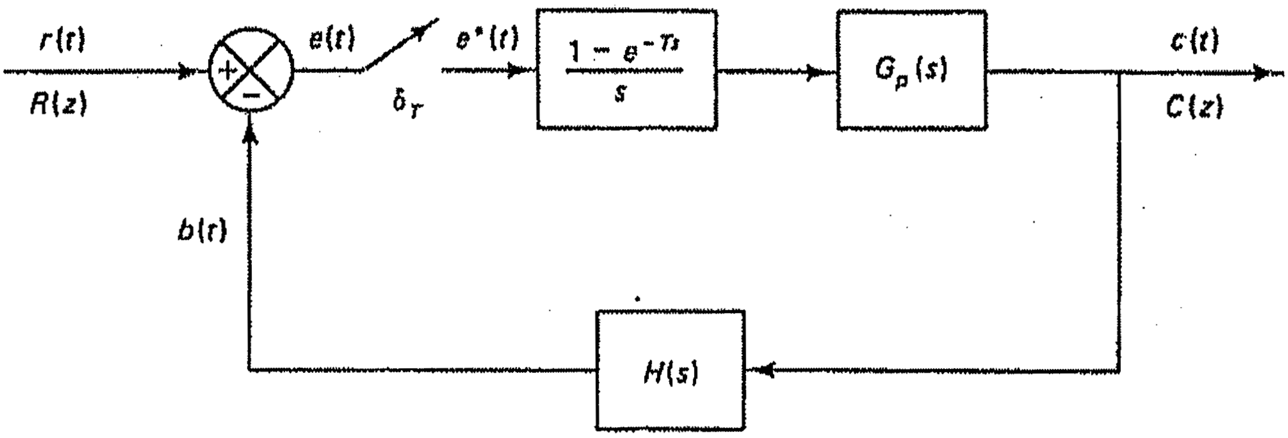

Figure Q5(b)

Ref. No.: Ex/EE/5/T/512A/2024

B.E. ELECTRICAL ENGINEERING 5TH YEAR 1ST SEMESTER EXAMINATION, 2024

Subject: DIGITAL CONTROL TECHNIQUES Part: II Full Marks: 50

Question No.	*Question 1* is compulsory Answer *Any Two* questions from the rest (2×20)	Marks
Q1	*Answer* ***anyTWO*** *Questions* (2 × 5=10)	
(a)	Given a pulse transfer of a discrete-time system, show that the state-space representation of the system is not unique.	5
(b)	Obtain the state-variable model of the system described by the difference equation $y(k+2)+2y(k+1)+0.25y(k)=u(k+1)+2u(k)$ where, $u(k)$ is the input and $y(k)$ is the output of the system.	5
(c)	Consider the following discrete time state model $\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix}=\begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}+\begin{bmatrix} 1 \\ 1 \end{bmatrix}u(k)$ Obtain the pulse transfer function. Consider an asymptotically stable continuous-time system.	5
(d)	Derive the solution of the linear-time-invariant discrete-time state equation $x(k+1)=Gx(k)$ in terms of the State Transition Matrix.	5
Q2 (a)	What is Similarity Transformation? Show that the Pulse Transfer Function is invariant under Similarity Transformation.	2+4
(b)	Obtain the state space representation of the following difference equation using direct programming method. $y(k+2)+y(k+1)+0.16y(k)=u(k+1)+2u(k)$ Where, $u(k)$ is the input and $y(k)$ is the output of the system.	
	(i) Obtain the state-variable model in Diagonal Canonical Form.	4
	(ii) Draw the corresponding simulation diagram.	2
(c)	Obtain the state transition matrix of the above discrete-time system.	8
Q3 (a)	Define complete state controllability and output controllability for a discrete-time system. State the necessary and sufficient condition for the complete state controllability and observability.	2+2+4
(b)	The discrete-time system is defined by $\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix}=\begin{bmatrix} 0 & 1 \\ -0.16 & -1 \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}+\begin{bmatrix} 1 \\ 0.5 \end{bmatrix}u(k)$ $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix}=\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is completely state controllable. Determine a sequence of control signal	4+4+4

$u(0)$ and $u(1)$ such that the state $x(2)$ becomes, $\begin{bmatrix} x_1(2) \\ x_2(2) \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

Q4 (a) Define State Transition Matrix of discrete-time system. — 2+6

Consider the following discrete-time system,

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.24 & -1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

$y(k) = [1\ 0] \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$. Assume that the following outputs are observed, $y(0) = 1$, $y(1) = 2$

The control signals are given by $u(0) = 2$, $u(1) = -1$. Determine the initial state $x(0)$. Also determine state $x(1)$ and $x(2)$.

(b) Consider a system given by the pulse transfer function — 4+4+4

$$G(z) = \frac{1 + 0.8z^{-1}}{1 - z^{-1} + 0.5z^{-2}}$$

Obtain the state and output equations of the system in the

(i) Controllable Canonical Form,

(ii) Observable Canonical Form and

(iii) Diagonal Canonical Form.

Q5 (a) Consider the following oscillatory system — 4+4

$\frac{y(s)}{u(s)} = \frac{\omega^2}{s^2 + \omega^2}$, Obtain the continuous-time state space representation of the system.

Then discretized the system and obtain the discrete-time state space representation of the system.

(b) Derive the expression for state feedback gain matrix, 'K', commonly called Ackermann's Formula. — 6

(c) Consider the system $x(k+1) = Gx(k) + Hu(k)$, where, — 6

$G = \begin{bmatrix} 0 & 1 \\ -0.16 & -1 \end{bmatrix}$, $H = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Assume that the following control scheme is used.

$u = -Kx$

Determine the state feedback gain matrix, 'K' such that the system will have closed loop poles at $z = 0.2 + j0.3$, $z = 0.2 - j0.3$.