Ref. No. Ex/EE/5/Math/T/113//2024

Bachelor of Engineering (Electrical) Examination, 2024

(1st Year, 1st Semester)

MATHEMATICS IIF

Time : Three hours | Full Marks : 100

(Symbols/ Notations have their usual meanings)

*Answer **any five** questions*
All questions carry equal marks

1.(a) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{bmatrix}.$$

(b) Find the rank of the matrix

$$\begin{bmatrix} 2 & 0 & 4 & 2 \\ 3 & 2 & 6 & 5 \\ 5 & 2 & 10 & 7 \\ 0 & 3 & 2 & 5 \end{bmatrix}.$$

(c) Solve the system of linear equations by matrix inversion method or Cramer's rule:

$$\begin{aligned} x + 2y - 3z &= 1 \\ 2x - y + z &= 4 \\ x + 3y &= 5 \end{aligned}$$

7+6+7

2(a) Define symmetric and skew-symmetric matrices with examples.

(b) State Cayley Hamilton theorem. Use this theorem to find the inverse of the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 2 & 3 & 2 \end{bmatrix}.$$

(c) Find the eigen values and the corresponding eigen vectors of the matrix

$$A = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}.$$

4+8+8

[Turn over

3. (a) Solve the differential equations:

(i) $(xy^2+2x^2y^3)dx+(x^2y-x^3y^2)dy=0$

(ii) $(2x\cos y+3x^2y)dx+(x^3-x^2\sin y-y)dy=0$

(b) Using method of variation of parameters, solve

$$\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=\frac{e^{3x}}{x^2}$$

(6+6)+8

4. (a) Solve the following differential equations:

(i) $\frac{d^2y}{dx^2}+4y=e^x+\sin 3x+x^2$

(ii) $\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=e^{-2x}\sin 2x$

(b) Solve the Cauchy-Euler equation

$$x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+2y=x\log x$$

(6+6)+8

5(a) Solve the following differential equations:

(i) $x^2y\,dx-(x^3+y^3)dy=0$

(ii) $(D^2-5D+6)y=e^{3x}$ where $D\equiv\frac{d}{dx}$

(b) (i) Show that $(\vec{a}\times\vec{b})^2=a^2b^2-(\vec{a}.\vec{b})^2$

(ii) Suppose that $\vec{\alpha},\vec{\beta},\vec{\gamma}$ are three vectors satisfying the condition $\vec{\alpha}+\vec{\beta}+\vec{\gamma}=\vec{0}$. If $|\vec{\alpha}|=3, |\vec{\beta}|=4, |\vec{\gamma}|=5$ then show that $\vec{\alpha}.\vec{\beta}+\vec{\beta}.\vec{\gamma}+\vec{\gamma}.\vec{\alpha}=-25$.

(5+5)+(5+5)

6.(a) Find the constant m such that the vectors $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{c}=3\hat{i}+m\hat{j}+5\hat{k}$ are coplanar.

(b) Show that $[\vec{\alpha}+\vec{\beta},\vec{\beta}+\vec{\gamma},\vec{\gamma}+\vec{\alpha}]=2[\vec{\alpha}\vec{\beta}\vec{\gamma}]$

(c) $\vec{f}=x^2y\hat{i}-2xz\hat{j}+2yz\hat{k}$ then find div curl $\vec{f}$.

7+6+7

7. (a) Prove that

$$\begin{vmatrix} b+c & a & a \\ c & c+a & a \\ b & a & a+b \end{vmatrix} = 4abc.$$

(b) Show that the vector

$\vec{f}=(y\sin z-\sin x)\hat{i}+(x\sin z+2yz)\hat{j}+(xy\cos z+y^2)\hat{k}$ is irrotational.

(c) Find the direction cosines of a line which is perpendicular to the lines whose direction ratios are (1, -1, 2) and (2,1,-1).

6+7+7