Bachelor of Engineering (Electrical) Examination, 2024

(1st Year, 1st Semester Supplementary)

MATHEMATICS - IIF

Time : Three hours Full Marks : 100

(Symbols/ Notations have their usual meanings)

*Answer **any five** questions*
All questions carry equal marks

1.(a) Find the inverse of the matrix

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 0 & 1 & 4 \\ 1 & 5 & -2 \end{bmatrix}$$

(b) Find the rank of the matrix

$$B = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 2 & 4 & -3 \end{bmatrix}$$

(c) Solve the system of linear equations by Cramer's rule:

$$\begin{aligned} x + 3y &= 5 \\ -2x + 3y + z &= 1 \\ y + z &= -2 \end{aligned}$$

7+7+6

2(a) Define Minor and Cofactor of an element of a square matrix with example.

(b) State Cayley Hamilton theorem. Use this theorem to find the inverse of the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 2 & 3 & 2 \end{bmatrix}$$

(c) Find the eigen values and the corresponding eigen vectors of the matrix

$$A = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$$

4+8+8

3. (a) Solve the following differential equations:

(i) $1+e^{x/y}dx+e^{x/y}(1-x/y)dy=0$

(ii) $(x^2+y^2+1)dx-2xy\,dy=0$

(b) Using method of variation of parameters, solve

$$\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=\frac{e^{3x}}{x^2}$$

(6+6)+8

4. (a) Solve the following differential equations:

(i) $(D^2-3D+2)y=6e^{-3x}+\sin 2x\ ,\ D\equiv\frac{d}{dx}$

(ii) $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+5y=x^2\sin(\log x)$

(b) Solve the Cauchy-Euler equation

$$x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+2y=x\log x$$

(6+6)+8

5. (a) (i) Show that $(\vec{a}\times\vec{b})^2=a^2b^2-(\vec{a}.\vec{b})^2$

(ii) If $|\vec{\alpha}|=10, |\vec{\beta}|=1, \vec{\alpha}.\vec{\beta}=6$ then evaluate $|\vec{\alpha}\times\vec{\beta}|$.

(b) (i) Suppose that $\vec{\alpha},\vec{\beta},\vec{\gamma}$ are three vectors satisfying the condition $\vec{\alpha}+\vec{\beta}+\vec{\gamma}=\vec{0}$. If $|\vec{\alpha}|=3, |\vec{\beta}|=4, |\vec{\gamma}|=5$ then show that $\vec{\alpha}.\vec{\beta}+\vec{\beta}.\vec{\gamma}+\vec{\gamma}.\vec{\alpha}=-25$.

(ii) If $\vec{f}=x^2y\hat{i}-2xz\hat{j}+2yz\hat{k}$ then find curlcurl $\vec{f}$.

(5+5)+(5+5)

6.(a) If $\vec{a}$ is a constant vector and $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ then show that

(i) div $(\vec{a}\times\vec{r})=0$ and

(ii) curl $(\vec{a}\times\vec{r})=2\vec{a}$.

(b) Find the value of the constant λ so that the vector field defined by

$\vec{f}=(2x^2y^2+z^2)\hat{i}+(3xy^3-x^2z)\hat{j}+(\lambda xy^2z+xy)\hat{k}$ is solenoidal.

(c) Prove that $div(\phi\vec{A})=grad(\phi).\vec{A}+\phi\, div(\vec{A})$

8+6+6

7. (a) Prove that

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}=(a+b+c)^3$$

(b) Show that the vector

$\vec{f}=(x^2+yz)\hat{i}+(y^2+2x)\hat{j}+(z^2+xy)\hat{k}$ is irrotational.

(c) Suppose that A(1,8,4), B(0,–11,4) and C(2,–3,1) are three points and D is the foot of the perpendicular from A to BC. Find the coordinates of D.

7+6+7