Ref. No.: Ex/EE/PE/B/T/421A/2024

B.E. ELECTRICAL ENGINEERING FOURTH YEAR SECOND SEMESTER - 2024

ADVANCED CONTROL THEORY

Time: 3 Hours
Full Marks: 100

Answer both parts on the same answer script

Part-I

In this part
Answer Question no. 1 which is compulsory
AND
Answer any one question from Question nos. 2 and 3
AND
Any one question from Question nos. 4 and 5

1. a) Define the term "nonlinearity with memory". Give two examples of nonlinearity with memory. **CO1** **[4]**

   b) Correct or justify the statement "*A dead zone is an example of nonlinearity with memory*". **CO2** **[2]**

   c) Define the term "Equilibrium Point" of a nonlinear system. **CO1** **[2]**

   d) A nonlinear system is expressed as follows: **CO4** **[12]**

$$\dot{x}_1 = (1 - x_1)x_1 - \frac{2x_1x_2}{(1 + x_1)}$$
$$\dot{x}_2 = \left\{1 - \frac{x_2}{(1 + x_1)}\right\}x_2$$

   i. Determine the equilibrium points of the above system.

   ii. Linearize the above system about its equilibrium point at the origin.

   iii. Comment about the asymptotic stability of the system at x=0.

[Turn over

Ref. No.: Ex/EE/PE/B/T/421A/2024

2.	a) State Lyapunov's 2nd theorem.	**CO1**	**[5]**
	b) The dynamics of an unforced nonlinear system is described by	**CO4**	**[10]**

$$\dot{x}_1 = -x_2 - x_1^3$$
$$\dot{x}_2 = x_1 - x_2^3$$

Using the function $V = \frac{1}{2}(x_1^2 + x_2^2)$ as the Lyapunov function, investigate the stability of the system about its equilibrium point at the origin.

OR

3.	a) Define the term "phase portrait" for a nonlinear system.	**CO1**	**[5]**
	b) A satellite attitude control system has forward-reverse type of thrusters with a dead zone and a proportional plus derivative controller. With the help of a phase plane plot, investigate the stability of the system assuming standard notations for the parameters and variables.	**CO4**	**[10]**

4.	a) An on-off type controller is used for level control of a water tank. Develop expressions for the time response of the above system considering a first order plant without any delay.	**CO3**	**[7]**
	b) Derive approximate expressions for on-time, off-time, duty cycle and maximum temperature.	**CO4**	**[8]**

OR

5. a) The output of an ideal (without hysteresis) bi-directional relay element is as follows: **CO3** **[7]**

$$y(\omega t) = \begin{cases} +1; & \forall 0 < \omega t < \pi \\ -1; & \forall \pi < \omega t \le 2\pi \end{cases}.$$

Find the describing function for the above relay element.

b) A closed loop unity feedback control system has the relay element described in 5(a) above, along with a linear process with transfer function **CO4** **[8]**

$G(s) = \dfrac{1}{s(s+1)^2}$ in the forward path. Investigate the stability of the closed loop nonlinear control system by using the describing function obtained in 5(a) and comment about the presence of limit cycle, if any, on the basis of the above analysis.

Ref. No.: Ex/EE/PE/B/T/421A/2024

Part II

Answer any one question from Question nos. 6 and 7
AND
Any one question from Question nos. 8 and 9
AND
Question no. 10 which is compulsory

6. a) Distinguish between the terms 'Structured uncertainty' and 'Unstructured uncertainty'. **CO2** **[4]**

   b) Analyze the robust stability of the system with the characteristic polynomial **CO4** **[12]**
   $p_5 s^5 + p_4 s^4 + p_3 s^3 + p_2 s^2 + p_1 s + p_0 = 0$,
   where
   $p_5 \in [1,1], p_4 \in [2,3],\ p_3 \in [5,11], p_2 \in [6,8],\ p_1 \in [4,7]$ and $p_0 \in [1,5]$.

OR

7. a) Distinguish between the terms 'Stability Robustness' and 'Performance Robustness'. **CO2** **[4]**

   b) A process plant given by $G_1(s) = \dfrac{100}{(s+1)(0.01s+1)}$ is modeled by using the transfer function $G_2(s) = \dfrac{100}{s+1}$. **CO4** **[12]**

   Analyze the suitability of using the above model for the given process plant on the basis of (i) the open loop unit step responses, (ii) the frequency responses (Bode plots) of the plant and its model.

Ref. No.: Ex/EE/PE/B/T/421A/2024

8. a) What is an observer? Draw the structure of a full-order observer and explain its uses. **CO1** **[4]**

b) Design a reduced order observer for observing the second and third state variables of the following continuous time system so that the observer poles are located at $-16.15 \pm j16.5$ **CO5** **[12]**

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -2.7 & -1.7 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u; \; y = [1 \quad 0 \quad 0]\mathbf{x}.$$

OR

9. a) Explain the physical significance of the term 'quadratic performance index'. **CO1** **[2]**

b) With the help of an example, explain what is meant by "terminal control problem". **CO1** **[2]**

c) Design an optimal controller for the following linear quadratic regulator by finding **CO5** **[12]**

(i) the Ricatti matrix P, and

(ii) the optimal control law.

The plant is described by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u; y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

and has the performance index

$$J = \int_0^{\infty} \left[\mathbf{x}^T \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\mathbf{x} + u^2 \right] dt.$$

10 a) Develop the concept of system sensitivity S_{α}^{M} for a unity feedback system with forward path transfer function given by $\frac{1}{s+\alpha}$, where $M(s)$ is the closed loop transfer function of the system. **CO3** **[6]**

b) Given a transfer function

$$G(s) = \frac{12}{(s+1)(s+2)^2}.$$

Determine $\|G\|_2$. **CO4** **[6]**

c) For the system with transfer function $G(s) = \frac{0.5s+1}{0.2s+1}$, determine $\|G\|_{\infty}$. **CO4** **[6]**