Ref. No.: **Ex/EE/PE/B/T/414A/2024**

B.E. ELECTRICAL ENGINEERING FOURTH YEAR FIRST SEMESTER – 2024

DIGITAL CONTROL TECHNIQUES

Time : 3 hours

Full Marks : 100
(50 marks for each part)

Use separate answer-scripts for each parts.

Part-I

Answer ANY THREE questions. Two marks reserved for neatness.

Answer all parts of a question in the sequential order.

1. a) Explain with neat sketches, the various dynamic characteristics observed in the Sample and Hold operations during the A/D conversion of a time-varying analog signal.

b) Derive the transfer function of a Zero Order Hold.

[10+6=16]

2.a) Explain how the absolute stability of a discrete-time closed-loop system can be tested.

b) Consider the discrete-time system described by the following model:

$$y(k) - 0.6y(k-1) - 0.81y(k-2) + 0.67y(k-3) - 0.12y(k-4) = x(k)$$

Where x(k) is the input and y(k) is the output of the system. Determine the stability of the system employing Jury's Stability test and comment on the result.

[4+12=16]

3a) Derive the steady-state errors of Type-0, Type-1 and Type-2 LTI discrete-time control systems in response to standard test signals.

b) What are the considerations for the selection of suitable sampling frequency for a Digital Control System implementation?

[12+4=16]

4. A closed-loop system has the characteristic equation:

$$1 + GH(z) = 1 + K\frac{0.368(z+0.717)}{(z-1)(z-0.368)} = 0.$$

Where K is the variable gain of the controller. Draw the root locus of the system. Hence comment on the stability of the given plant from the root locus.

[16]

5. Derive the P-I-D controller equation for Velocity Algorithm of Direct Digital Control employing suitable numerical methods for calculating the integral and derivative terms. Hence, draw the flow chart of the same.

[10+6=16]

[Turn over

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B. E. ELECTRICAL ENGINEERING 4TH YEAR 1ST SEMESTER EXAMINATION, 2024

Subject: DIGITAL CONTROL TECHNIQUES **Time: Three Hours** **Full Marks: 100**

Part II (50 marks)

Question No. | ***Question 1*** is compulsory | Marks
Answer ***Any Two*** questions from the rest (2×20)

Q1 *Answer any* ***TWO*** *Questions* (2 × 5=10)

(a) Obtain the state-variable model of the system described by the difference equation

$$y(k+2) = u(k) + 1.7y(k+1) - 0.72y(k)$$ 5

where, $u(k)$ is the input and $y(k)$ is the output of the system.

(b) Obtain the pulse transfer function for the system described by the state equations

$$x(k+1) = \begin{bmatrix} 1.35 & 0.55 \\ -0.45 & 0.35 \end{bmatrix} x(k) + \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} u(k)$$ 5

$$y(k) = [1 \quad -1]x(k)$$

(c) What is Similarity Transformation? Show that the Pulse Transfer Function is invariant under Similarity Transformation. 2+3

(d) What is Deadbeat Response? How it can be achieved for discrete-time systems? 2+3

Q2 (a) Consider the discrete-time system defined by

$$\frac{Y(z)}{U(z)} = \frac{b_0 z^n + b_1 z^{n-1} + \cdots + b_n}{z^n + a_1 z^{n-1} + \cdots + a_n}$$

Show that a state-space representation of this system may be given by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ \vdots \\ x_{n-1}(k+1) \\ x_n(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_{n-1}(k) \\ x_n(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u(k)$$ 10

$$y(k) = [(b_n - a_n b_0) \quad (b_{n-1} - a_{n-1} b_0) \cdots (b_1 - a_1 b_0)] \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_{n-1}(k) \\ x_n(k) \end{bmatrix} + b_0 u(k)$$

(b) Consider a system given by the pulse transfer function

$$G(z) = \frac{0.17z + 0.04}{z^2 - 1.1z + 0.24}$$

(i) Obtain the state and output equations of the system. 4

(ii) Solve the state equations by methods of recursion and derive the state and output values for the first 5 instants of sampling. Consider the input to be unit step. 6

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Q3 (a) State and derive the Necessary condition for arbitrary pole-placement via state-feedback. 8

(b) For the sampled-data system as shown in the Figure Q3(b), let $T = 0.1$ sec and

$$G_p(s) = \frac{10}{s(s+1)}$$

(i) Obtain the discrete-time state-space model of the system. 6

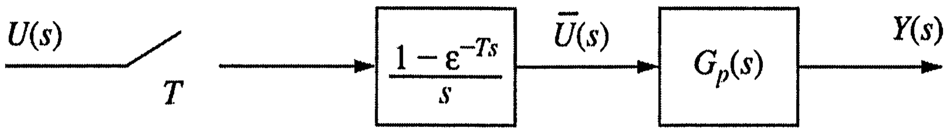

Figure Q3(b)

(ii) Determine the values of state feedback gain such that the overall system has a characteristic equation 6

$$z^2 - 1.78\,z + 0.82 = 0$$

Q4 (a) (i) Define complete state controllability and observability for a discrete-time system. 2+2

(ii) State and discuss the *Principle of Duality* for discrete-time systems. 4

(b) Consider a double integrator system given by

$$\ddot{y} = u,$$

with, y and u being output and input, respectively.

(i) Obtain a continuous-time state-space representation of the system. 4

(ii) Then determine the discrete-time equivalent state-space model of the system. 4

(iii) Derive the pulse transfer function for the discrete-time system. 4

Q5 (a) What is a State Observer? 2

(b) Draw the block diagram for observer-based state-feedback control system. 4

From the block diagram derive the necessary equations to describe the dynamics of a Full-Order State-observer. 4

(c) Consider a system given by the state equations of the form

$$\boldsymbol{x}(k+1) = \boldsymbol{G}\,\boldsymbol{x}(k) + \boldsymbol{H}\,u(k)$$

$$y(k) = \boldsymbol{C}\,\boldsymbol{x}(k)$$

with, $\boldsymbol{G} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$, $\boldsymbol{H} = \begin{bmatrix} T^2/2 \\ T \end{bmatrix}$, $\boldsymbol{C} = [1 \quad 0]$ and T being the sampling period.

(i) Design a full-order state observer such that the observer error vector exhibits deadbeat response. 6

(ii) Verify, with the obtained observer feedback gain matrix, that the observer error dynamics follow a deadbeat response. 4