B .E. ELECTRICAL ENGINEERING
THIRD YEAR FIRST SEMESTER EXAMINATION 2024

INTRODUCTION TO STATISTICAL AND PROBABILISTIC METHODS

Time: Three hours **Full Marks 100**
(√50 marks for each part)

Part-I

Use a separate Answer-Script for each part

Q1a) Explain the concept of conditional probability. State Bayes Theorem. (CO1)

Q1b) State and explain the notion of Random Variable. (CO1) 4+4

Q2a) A Gaussian voltage random variable X has a mean value $\mu_x=0$ and variance $\sigma_x^2=9$. The voltage X is applied to a square-law, full-wave diode detector with a transfer characteristics $Y=5X^2$. Find the mean value of the output voltage Y. Derive all the steps. (CO2) 7

Q2b) Prove that the Exponential Distribution does have a memory-less property. With the help of a suitable example explain the significance of this property. (CO2) 7

OR

Q2a) Let X be a continuous R.V. having exponential density function with parameter $\lambda=1$. Compute $P(X\geq7)$. Also determine the corresponding Markov, Chebyshev and Cernoff bounds. (CO2) 8

Q2b) Derive the moment generating function / the characteristic function of a Gamma distributed R.V. Hence, obtain the mean and variance of the distribution. (CO2) 6

Q3a) Enumerate the properties of a joint cumulative distribution function. (CO3) 6

Q3b) Consider the following function

$$G_{X,Y}(x,y)=u(x)u(y)[1-e^{-(x+y)}]$$

Determine whether the function is a valid joint probability distribution function or not. Justify your answer through proper derivations. (CO3) 8

OR

Q3a) Two statistically independent random variables X and Y have respective density functions

$$f_X(x) = 5u(x)\exp(-5x)$$
$$f_Y(y) = 2u(y)\exp(-2y)$$

Find the density function of the sum W=X+Y. (CO3) 4

Q3b) Let X and Y be statistically independent random variables with

$\mu_X = \frac{3}{4},\ E\left[X^2\right]=4,\ \mu_Y = 1,\ E\left[Y^2\right]=5.$ For a random variable W=X-2Y+1, find

a) R_{XY}, b) R_{XW}, c) R_{YW} and d) C_{XY}. Are X and Y uncorrelated? (CO3) 6

Q3c) Define correlation of two random variables. Explain the effect of statistical independence of the random variables on their correlation. (CO3) 4

Q4) Write short notes on any two (CO4) 7+7

i) Concept of Random Process as an extension of Random Variable
ii) White Noise
iii) Power Spectral Density of a Random Process

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SUBJECT: INTRODUCTION TO STATISTICAL AND PROBABILISTIC METHODS

Time: Three Hours

Full Marks: 100
(50 Marks for each part)

Use separate Answer-Script for each part
Two marks for neat and well-organized answers

Part-II

Answer any three questions
(The symbols used have their usual meanings)

To answer the following questions, you can use (with linear interpolation and extrapolation if necessary) the following standard normal distribution table where ever required:

z:	0.38	0.39	0.40	0.41	0.42	0.43	0.44	……..
P (z):	0.6480	0.6517	0.6554	0.6591	0.6628	0.7019	0.7054	……..
z:	1.95	1.96	1.97	1.98	1.99			
P (z):	0.9744	0.9750	0.9756	0.9761	0.9767			

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1. (a) With necessary derivation and clarification, comment on the distribution of sample mean and sample variance when the sample is randomly chosen from a normal population having mean μ and variance σ^2. 9

(b) The amount of water used per day by a person residing in a city has a mean value of 145 litres with a standard deviation of 60 litres. If a random sample of 25 persons is taken, determine the approximate probability that the average amount of water consumed per day by the members of the group exceeds 150 litres. Clearly mention the assumptions you have to take to solve the problem. 7

2. (a) A Signal having value μ is transmitted from station A and the value received at station B is normally distributed with mean μ and variance 4. If the successive values received are as follows: 10

5, 8.5, 12, 15, 7, 9, 7.5, 6.5, and 10.5

Determine the sample size so that an interval of length 0.05 is obtained which we can assert, with 95% confidence, contains μ. Deduce the expression for confidence interval you might have used.

(b) If X_i, i=1, 2, ..., n constitute a random sample from a normal population with mean μ and variance σ^2 where both μ and σ^2 are unknown, then derive the expression for a $100(1-\alpha)$ % confidence interval for μ. 6

3. (a) A public health official claims that the mean home water used is normally distributed having mean, $\mu=350$ gallons a day with $\sigma=20$ gallons. To verify this claim, a study of 20 randomly selected homes was performed with the results that the average daily water use of these 20 homes were as follows: 10

340, 356, 332, 362, 318, 344, 386, 402, 322, 360, 362, 354, 340, 372, 338, 375, 364, 355, 324, and 370.

Verify whether the data contradicts the officials claim with a test of 5% significance level. Derive the expression that you have used for the test.

(b) With necessary derivation, discuss how type-II error is involved for calculating the mean of a normal population. 6

4. Derive the expressions for the Least Square Estimators of the Regression Parameters of a simple linear regression model. Also, determine the expected values and variance of those parameters. 16

5. Write short notes on the followings: 6+6+4

(i) Test of the following hypothesis:
H_0: $\mu = \mu_0$ Vs. H_1: $\mu \neq \mu_0$ for a normal population with mean μ and variance σ^2, when both the μ and σ^2 are unknown.

(ii) Chi-square distribution

(iii) Continuity Correction