

**B.E COMPUTER SCIENCE AND ENGINEERING 3<sup>rd</sup> YEAR 1<sup>st</sup> SEMESTER EXAMINATION 2024****Formal Languages and Automata Theory**

Time: 3 hours

Full Marks: 100

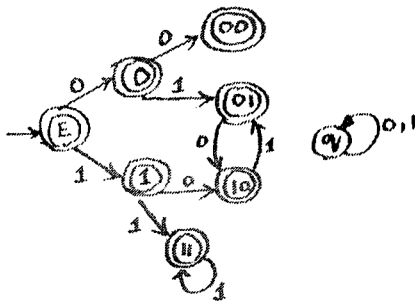
Answer from Groups A, B, C, D

**Group A**

1(i) Which one of the following regular expressions is NOT equivalent to the regular expression  $(a+b+c)^*$ ?

- (A)  $(a^* + b^* + c^*)^*$       (B)  $(a^*b^*c^*)^*$       (C)  $((ab)^* + c^*)^*$       (D)  $(a^*b^* + c^*)^*$

ii) Consider the set of strings on  $\{0,1\}$  in which, every substring of 3 symbols has at most two zeros. For example, 001110 and 011001 are in the language, but 100010 is not. All strings of length less than 3 are also in the language. A partially completed DFA that accepts this language is shown below.



The missing arcs in the DFA are

(A)

	00	01	10	11	q
00	1	0			
01				1	
10	0				
11			0		

(B)

	00	01	10	11	q
00		0			1
01		1			
10				0	
11		0			

(C)

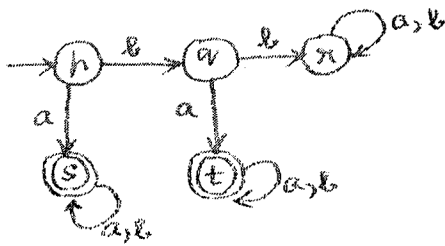
	00	01	10	11	q
00		1			0
01		1			
10			0		
11		0			

(D)

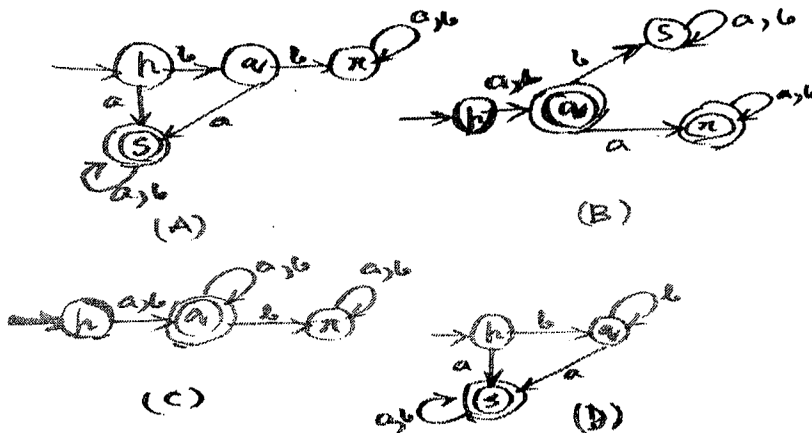
	00	01	10	11	q
00		1			0
01				1	
10	0				
11			0		

iii) A deterministic finite automation (DFA) M on alphabet  $\Sigma = \{a,b\}$  is given below

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Which of the following finite state machines is a valid minimal DFA which accepts the same language as does M?



iv) Let  $L = \{0^k \mid k = a + \text{ve constant}, k > 0\}$ . That is, if  $n=3$  then  $L = \{0^3, 0^6, 0^9, \dots\}$ . What is the minimum number of states needed in a DFA to recognize  $L$ ?

- (A)  $k+1$  (B)  $n+1$  (C)  $2^{n+1}$  (D)  $2^{k+1}$

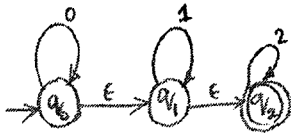
v) Let  $w$  be any string of length  $n$  in  $\{0,1\}^*$ . Let  $L$  be the set of all substrings of  $w$ . What is the minimum number of states in a non-deterministic finite automaton that accepts  $L$ ?

- (A)  $n-1$  (B)  $n$  (C)  $n+1$  (D)  $2n-1$

vi) Consider the languages  $L_1 = \emptyset$  and  $L_2 = \{a\}$ . Which one of the following represents  $L_1 L_2^* \cup L_1^*$ ?

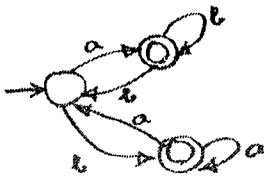
- (A)  $\{\epsilon\}$  (B)  $\emptyset$  (C)  $a^*$  (D)  $\{\epsilon, a\}$

vii) What are the final states of the DFA generated from the following NFA?



- (A)  $[q_0, q_1], q_2$  (B)  $q_0, [q_1, q_2]$   
 (C)  $[q_0, q_1], [q_0, q_2], []$  (D)  $q_0, q_1, q_2$

viii) Which one of the following regular expressions correctly represents the language of the finite automaton given below?



- (A)  $ab^*bab^*+ba^*aba^*$  (B)  $(ab^*b)^*ab^*+(ba^*a)^*ba^*$   
 (C)  $(ab^*b+ba^*a)^*(a^*+b^*)$  (D)  $(ba^*a+ab^*b)^*(ab^*+ba^*)$

ix) Consider the following context free grammars:

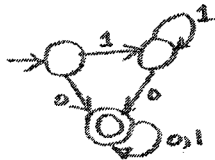
**G1:**  $S \rightarrow aS \mid B, B \rightarrow b \mid bB$

**G2:**  $S \rightarrow aA \mid bB, A \rightarrow aA \mid B \mid \epsilon, B \rightarrow bB \mid \epsilon$

Which one of the following pairs of languages is generated by **G1** and **G2** respectively?

- (A)  $\{a^m b^n \mid m > 0 \text{ or } n > 0\}$  and  $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$   
 (B)  $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$  and  $\{a^m b^n \mid m > 0 \text{ or } n \geq 0\}$   
 (C)  $\{a^m b^n \mid m \geq 0 \text{ or } n > 0\}$  and  $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$   
 (D)  $\{a^m b^n \mid m \geq 0 \text{ and } n > 0\}$  and  $\{a^m b^n \mid m > 0 \text{ or } n > 0\}$

x) Consider the DFA A given below.



Which of the following are **FALSE**?

- (1) Complement of  $L(A)$  is context - free.  
 (2)  $L(A) = (11^*0+0)(0+1)^*0^*1^*$   
 (3) For the language accepted by A, A is the minimal DFA  
 (4) A accepts all strings over  $\{0,1\}$  of length at least 2

- (A) 1 and 3 only (B) 2 and 4 only (C) 2 and 3 only (D) 3 and 4 only

xi) Let  $N_f$  = the classes of languages accepted by Non Deterministic Finite Automata (NFA)

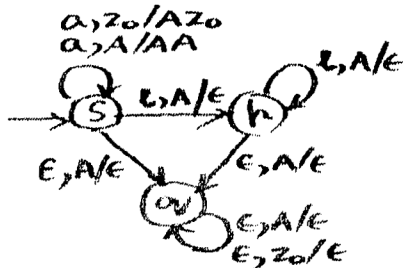
$D_f$  = the classes of languages accepted by Deterministic Finite Automata (DFA)

$N_p$  = the classes of languages accepted by Non deterministic Push Down Automata (NPDA)

$D_p$  = the classes of languages accepted by Deterministic Push Down Automata (DPDA)  
Which one of the following is TRUE?

- (A)  $D_f \subset N_f$  and  $D_p \subset N_p$  (B)  $D_f = N_f$  and  $D_p \subset N_p$   
(C)  $D_f = N_f$  and  $D_p = N_p$  (D)  $D_f \subset N_f$  and  $D_p = N_p$

xii) Consider the pushdown automaton (PDA) P below with  $\Sigma = \{a, b\}$ ,  $T = \{Z_0, A\}$ ,  $Q = \{s, p, q\}$ ,  $s =$  start state. The PDA accepts by the empty stack.



Which one of the following options correctly describes the language accepted by P ?

- (A)  $\{a^m b^n \mid 1 \leq m \text{ and } n < m\}$  (B)  $\{a^m b^n \mid 0 \leq n \leq m\}$   
(C)  $\{a^m b^n \mid 0 \leq m \text{ and } 0 \leq n\}$  (D)  $\{a^m \mid 0 \leq m\} \cup \{b^n \mid 0 \leq n\}$

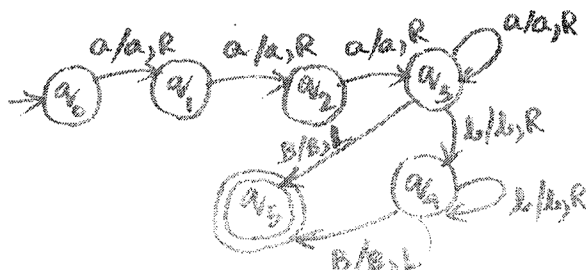
xiii) Let  $p$  be a problem that belongs to the class NP. Then which one of the following is TRUE?

- (A) There is no polynomial time algorithm for  $p$ .  
(B) If  $p$  can be solved deterministically in polynomial time, then  $P = NP$ .  
(C) If  $p$  is NP-hard, then it is NP-complete.  
(D)  $p$  may be undecidable.

xiv) Which of the following statements is False?

- (A) Halting problem for Turing Machines is Undecidable  
(B) Determining whether a context free grammar is ambiguous is Undecidable  
(C) Given two arbitrary Context free grammars  $G_1, G_2$ , it is undecidable whether  $L(G_1) = L(G_2)$   
(D) Given two regular grammars  $G_1, G_2$  and it is undecidable whether  $L(G_1) = L(G_2)$

xv) The transition diagram of a single tape Turing machine  $M$  with  $\Sigma = \{a, b\}$ ,  $T = \{a, b, B\}$ ,  $B =$ blank symbol is shown below:



Which of the following is TRUE?

- (A)  $M$  accepts the set of all strings on  $\{a, b\}$   
(B)  $M$  accepts the language  $L = aaa^*b^*$   
(C)  $M$  accepts the language  $L = aaa(a+b)^*$   
(D)  $M$  accepts the language  $L = aaaa^*b^*$

Group B

Answer any three

- 2(a) Give the transition diagram of a Deterministic Finite Automaton (DFA) that accepts all binary strings in which both the number of 0's and the number of 1's are even.  
 (b) How would you decide whether two regular languages are equivalent?

(8+4)

- 3(a) State the Pumping Lemma for regular languages.  
 (b) Prove that  $L = \{0^n 1^{n+1} | n > 0\}$  is not regular

(5+7)

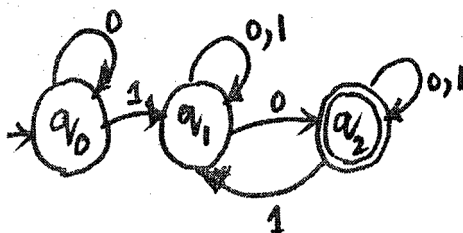
- 4(a) Consider the following state table of a Deterministic Finite Automaton (DFA). Form the table of distinguishability for this automaton and give the transition diagram of the minimum state equivalent DFA.

	0	1
q0	q1	q3
q1	q2	q4
q2	q1	q4
q3	q2	q4
q4	q4	q4

- (b) Prove that no other equivalent DFA with lesser number of states than the one obtained by your method can exist.

(5+7)

- 5(a) Consider the following Non Deterministic Finite Automaton (NFA). Give the transition diagram of an equivalent DFA.



- (b) Let  $L = \{abab, baba\}$  be a regular language on  $\Sigma = \{a, b\}$  and  $h$  a homomorphism defined as  $h(0) = ab$ ,  $h(1) = \epsilon$ . Find  $h^{-1}(L)$ .  
 (c) Prove that regular languages are closed under difference.

4+4+4

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Group C

Answer any two

- 6(a) State the Pumping Lemma for context free languages.  
 (b) Let  $L = \{ww | w \text{ in } \{0,1\}^*\}$ .  $0^n 1^n 0^n 1^n$  is a string of L. Prove that L is not a context free language. (4+6)
- 7(a) Let G be a CFG in Chomsky normal form and w be the yield of a parse tree formed with G. Prove that  $|w| \leq 2^{n-1}$  where n is the length of the longest path in the parse tree for w.  
 (b) Consider the language  $L = \{0^i 1^j 2^k | i, j, k \geq 0 \text{ and } i=j \text{ or } j=k\}$ . Give a Context free grammar for the language L. (7+3)
- 8) Prove that context free languages are not closed under complementation.

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Group D

Answer any two

- 9(a) State Halting problem for Turing machines.  
 (b) Assuming undecidability of Halting problem for Turing machines, prove undecidability of Blank tape halting problem. (2+5)
- 10) Define P, NP, NP hard and NP complete classes of problems. (1+(2x3))
- 11) Give the transition diagram of a Turing machine M such that  $L(M) = \{a^n b^{n+1} | n > 0\}$

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