Ref. No.: Ex/CSE/PC/B/T/325/2024

B.E. COMPUTER SCIENCE AND ENGINEERING THIRD YEAR SECOND SEMESTER - 2024
Subject: DESIGN AND ANALYSIS OF ALGORITHMS

Time: 3 hours   Full Marks: 100

Answer question no. **1 or 2** from Group A and question no. **3 or 4** from Group B and question no. **5 or 6** from Group C and question no. **7 or 8** from Group D

Group A

(Answer question no. 1 or 2 from this group)

[CO1]

1. a) $\Omega(n^{10}) = \Omega(n^3) = \Omega(n^2)$. Is this correct? Justify.
   b) Use recursion tree method for solving the following recurrence:
   $T(n) = 2T(n/2) + n\,log(n)$, T(1)= 1.
   c) Compare the running time of the BFS algorithm when (1) adjacency list is used for graph implementation and (ii) when adjacency matrix is used for graph implementation.
   d) Show that $\log(n!) = O(n \cdot \log(n))$
   e) Using limit method, prove that $e^n = \Omega(n2^n)$ 5 x 5 = 25 marks

2. a) Explain Big-oh, Omega and theta notation with the help of graphical representation.
   b) Using limit method prove that $log\ n = O(n)$
   c) Show that if c is a positive real number, then $g(n) = 1 + c + c^2 + \ldots + c^n$ is $\Theta(1)$ if $c<1$
   d) Prove that Prim's algorithm is correct although it is greedy algorithm. Explain why it is a greedy algorithm.   5 x 3 + 10= 25 marks

Group B

[CO2]

(Answer question no. 3 or 4 from this group)

3. a) Write an efficient "divide and conquer" algorithm for multiplying two n-digit numbers. Compute the asymptotic order of the worst case running time for this algorithm.

   b) Use Master theorem to find the value of the following recurrence:
   $T(n) = 3T(n/2) + O(n^2)$
   c) State Master theorem and prove it.   10 + 5+10= 25 marks

4. a) Apply divide and conquer approach to compute $y = x^n$ (n is an integer) and develop an algorithm. Analyze the running time of the algorithm in detail.
   b) Mention the three basic steps of *divide and conquer algorithm design* technique. Does the *Quicksort* algorithm follow all these three steps? - Explain
   c) Write the *Mergesort* algorithm and analyze the running time of the algorithm in detail.
   d) Compare *Quicksort* algorithm and *Mergesort* algorithm on the basis of their merits and demerits.   10 + 5 x 3 = 25 marks

[Turn over

Group C
[CO3 and CO4]
(Answer question no. 5 or 6 from this group)

5\. a) Consider the following graph search algorithm and analyze its running time in detail.

Graph-search(G) \\G=(V, E) and

\\G is implemented using adjacency matrix

\\Assumptions: do not allow any node twice

Step1. Mark a node as S and Explored= {S}, Unexplored= ( ).

Step2. Search for an edge E= (u, v) where u∈ Explored, v ∈ Unexplored

if $\exists E$, mark v and Explored= Explored ∪ v

Step3: Stop where |explored|= |v|

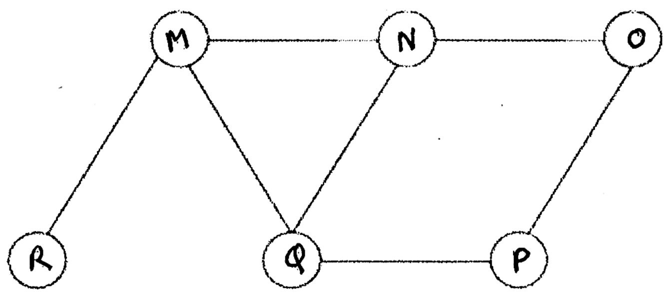
b) Describe the greedy algorithm for coin changing problem. Explain why and when this greedy algorithm may fail.

c) Consider the following 5 data points and create a weighted graph of 5 vertices where each vertex corresponds to a data point and the weight of an edge between any two vertices is the reciprocal of the Euclidean distance between the corresponding vertices. Now apply MST-based clustering algorithm to find two clusters. Apply the Prim's algorithm to find the minimum spanning tree(MST).

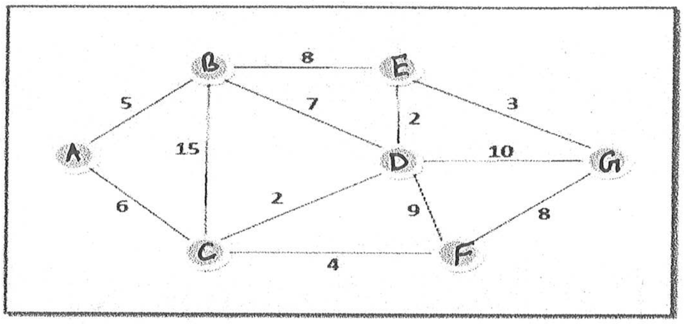
Data points: (1, 1), (1, 2), (2, 1.5), (5,2), (6, 2.5)

d) Compute the running time of the part of the Prim's algorithm that finds the lightest edge crossing a cut and adding the last end point of the edge to the explored set of vertices. Consider priority queue-based implementation. 5+5+10+5=25 marks

6\. a) How is the following graph represented using the adjacency list method? If the *Breadth First Search algorithm* has been implemented using the queue data structure, find one possible order of visiting the nodes of the following graph. Show the step by step dry-run of the algorithm on this graph.



b) Stating from A, apply the Prim's algorithm for finding the minimum spanning tree (MST) for the following weighted graph. Show the step by step dry run of the algorithm. Mention the order of visiting the edges forming the MST and draw the obtained minimum spanning tree. Is there a unique MST in this case? Justify.



10 +15 = **25 marks**

Group D

(Answer question no. 7 or 8 from this group) **[CO5 and CO6]**

7. a) A thief enters a house for robbing it. He can carry a maximal weight of 5 kg into his bag. There are 4 items in the house with the following weights and values. To maximize the values in the bag, what items should thief take if he either takes the item completely or leaves it completely? Show the dynamic programming solution to the problem.

Item	Weight (kg)	Value ($)
Mirror	2	3
Silver nugget	3	4
Painting	4	5
Vase	5	6

b) Is P = NP? Justify your answer. Why is it a million dollar question?

c) Does the halting problem belongs to NP? - Explain.

d) Consider an input to the randomized Quicksort algorithm is a set of numbers, $Z_{ij} = \{z_i, z_{i+1}, .., z_j\}$, find the probability that z_i *is compared with* z_j. Give justifications in favour of your answer.

10+3 x 5 = **25 marks**

8. a) What are the important features of dynamic programming approach?

b) What is the running time of the Brute-force solution to the 0-1knapsack problem? Why is the dynamic programming-based 0-1 Knapsack Algorithm a pseudo-polynomial time algorithm? - Justify.

c) If any of the NP complete problem can be solved in polynomial time, then P = NP. Justify this statement.

d) State the SAT problem. What is the time complexity of the brute force algorithm for the SAT problem?- Explain.

e) Consider that *Partition(A, p, r)* is the partition subroutine used in the traditional Quicksort algorithm. Now write the version of the Randomized quicksort by doing changes to *Partition(A, p, r)* subroutine. You need not expand the body of the *Partition(A, p, r)* subroutine.

5 x 5 = **25 marks**