Bachelor in Computer Science and Engineering
2^{nd} Year, 2^{nd} Semester Exam 2024
Graph Theory and Combinatorics

Full Marks : 100 Time : 3 Hrs

Write answers to the point. Make and state all the assumptions(wherever made). ALL PARTS OF A QUESTION SHOULD BE ANSWERED TOGETHER

Section A Answer all questions [$8 \times 5 = 40$]

(1) What is the generating function for the sequence 2,2,2,2,2?

(2) There are 4 roads from A and B and 6 roads from B and C. Show that the number of ways to go

(a) from A to C is 24

(b) from A to C and back to A is 576

(c) from A to C and back to A without using a road more than once is 360

(3) How many triangles can be made by joining 10 points in a plane, given that 5 are in one line.

(4) Use characteristic root method to solve the recurrence relation: $a_n - 8a_{n-1} + 214a_{n-2} - 18a_{n-3} = 0$

(5) For each of the following, draw an Eulerian graph that satisfies the conditions, or prove that no such graph exists.

(a) An even number of vertices, an even number of edges.

(b) An odd number of vertices, an even number of edges.

(6) Can three houses be connected to three utilities, which are on opposite sides of a street without any connections crossing each other. Justify.

(7) If a simple regular graph has n vertices and 24 edges, find all possible values of n.

(8) Define with example: connected and disconnected graphs, Walk, Path, Circuit, Component of a graph

Section B Answer any Five(5) Questions. [$5 \times 5 = 25$]

You can keep your answer in the "Combinatorial Form"

(1) Use the principle of inclusion and exclusion to find the number of solutions of the equation $x_1 + x_2 + x_3 = 18$, where x_1, x_2, x_3 are non negative integers such that $x_1 \leq 5, x_2 \leq 6, x_3 \leq 18$.

(2) Find the solution for the linear non-homogeneous recurrence relation $a_n - 2a_{n-1} = 6n$ for all values of $n \geq 2$, the initial condition being $a_1 = 2$.

(3) Find the sequence corresponding to the following generating function: $g(x) = \frac{5(x^5-1)}{x-1}$

(4) Show that if 10 colors are used to paint 201 houses, at least 21 houses will have the same color.

(5) Prove by induction that every third element in a Fibonacci sequence is an even number.

(6) Prove that the number of circular permutations of n objects is $(n-1)!$

Section C Answer any Five(5) Questions [$5 \times 5 = 25$]

[Turn over

(1) Let $G(V, E)$ be a graph with 20 vertices numbered 1 to 20. Two vertices v_i, v_j are adjacent only if $|i - j| = 8$ or $|i - j| = 10$. Find the number of connected components.

(2) A graph contains 2 disjoint Hamiltonian circuits. What is the minimum degree of any vertex of this graph?

(3) Given a simple undirected graph with N vertices such that every possible pair of of vertices is connected with an edge. How many DFS and BFS traversals are possible for such a graph?

(4) Find the chromatic polynomial equation of a tree with n vertices.

(5) Show that the Petersen graph does not have a Hamilton circuit, but the subgraph obtained by deleting a vertex v has a Hamilton circuit.

(6) Let $G(V, E)$ be a simple graph with 25 vertices and 6 connected components. Find the

i. the minimum number of edges that $G(V, E)$ can have

ii. the maximum number of edges that $G(V, E)$ can have

Section D Answer any one question [10]

(a) The degree sequence for a graph $G(V, E)$ given as $Deg(G) = (4, 4, 3, 2, 1, 1, 1, 1, 1)$ is claimed to be a tree. Prove by drawing the tree or explain why the tree cannot exist.

(b) Let $G(V, E)$ be a tree T, which is a complete n-ary tree with m vertices of which k are non-leaves and l are leaves. Prove that $m = nk + 1$ and $l = (n - 1)k + 1$