Bachelors of Computer Science and Engineering 2024

(2nd Year, 2nd Semester)

Mathematics IV

Time : Three hours **Full Marks : 100**

USE SEPARATE ANSWER SCRIPTS FOR GROUP A AND GROUP B

Group A

FULL MARKS: 50

Answer question 1 any SIX from the rest:

1. Prove that $\alpha = \frac{1+i}{\sqrt{2}}$ is an algebraic number. Is α an algebraic integer? 2

2. A box contains 6 red, 8 green, 10 blue, 12 yellow and 15 white balls. What is the minimum number of balls we have to choose randomly from the box to ensure that we get at least two balls of same color? 8

3. Let F be the collection of all functions $f : \{1,2,3\} \longrightarrow \{1,2,3\}$. Define a binary relation $\sim$ on F by $f \sim g$ if and only if $f(3) = g(3)$.

   (a) Show that $\sim$ is an equivalence relation.

   (b) Find the number of equivalence classes defined by $\sim$.

   (c) Find the number of elements in each equivalent class.

4. Let $S = \{x \in \mathbb{R} \mid -1 < x < 1\}$ and a function $f : \mathbb{R} \longrightarrow S$ be defined by $f(x) = \frac{x}{1+|x|}$. Show that f is a bijection and find f^{-1}.

5. Let X and Y be two non-empty sets. Show that $|X| \leq |Y|$ if and only if there exists a map from Y onto X. 8

[Turn over

6. Show that the subset X of $\mathbb{R}^3$ defined by

$$X = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid 0 \leq x_i < 1 \text{ for } i = 1, 2, 3\}$$

has cardinal number c. 8

7. Let $(L, \wedge, \vee)$ be a lattice. Show that $x \wedge y = x \Longleftrightarrow x \vee y = y$. 8

8. Define a limit point of a subset of $\mathbb{R}$, the set of real numbers. Prove that every bounded infinite subset of $\mathbb{R}$ has a limit point. 8

9. Using truth table, prove the following:

"If p implies q and q implies r, then p implies r." 8

Ex./CSE/BS/Math/T/221/2024

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USE SEPARATE ANSWER SCRIPTS FOR GROUP A AND GROUP B

Group B

Answer any FIVE questions 5×10

1. (i) Let A and B be two independent events such that (a) $P(A) < P(B)$, (b) $P(AB) = \frac{6}{25}$ and (c) $P(A|B) + P(B|A) = 1$. Evaluate $P(A)$ and $P(B)$.

   (ii) State Tchebycheff's inequality. Hence show that in 2000 throws with a fair coin the probability that the number of heads lies between 900 and 1100 is at least $\frac{19}{20}$.

   (iii) An integer is chosen at random from the set $\{1, 2, 3, \ldots, 200\}$. Find the probability that the integer is divisible by (a) 4 or 6, (b) 2 and 3, (c) 2 or 3 or 5. $3 + 4 + 3$

2. (i) A point P is chosen at random on a circle of radius a and a point A is fixed on the circle. Show that the probability that the chord AP will exceed the length of the side of an equilateral triangle inscribed in the circle is $\frac{1}{3}$.

   (ii) Three concentric circles of radii $\frac{1}{\sqrt{3}}$, 1 and $\sqrt{3}$ feet are drawn on a target board. If a shot falls within the innermost circle, 3 points are scored; if it falls within the next two rings, then scores are respectively 2 and 1, and the score is zero if the shot falls outside the outermost circle. If the probability density of the distance of the hit from the centre of the target is $\frac{2}{\pi} \frac{1}{1+r^2}$, then find the probability distribution of the score. $4 + 6$

3. (i) Let the range of a random variable X be $\{1, 2, 3, \ldots, n\}$. If $P(X = i)$ is proportional to $\frac{1}{i(i+1)}$ for all $i = 1, 2, \ldots, n$, then determine $P(m < X \leq r)$ where $m, r \leq n$.

   (ii) Derive the mean and standard deviation of a Poisson distribution with parameter λ. $5 + 5$

4. (i) The probability density function of a two-dimensional random variable (X, Y) is given by

$$f(x,y) = \begin{cases} 3x^2y + 3y^2x, & 0 < x < 1, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the marginal density functions and $P\left(\frac{1}{3} < Y \leq \frac{2}{3} \middle| \frac{1}{2} < X \leq \frac{3}{4}\right)$.

(ii) Two points P and Q are chosen at random on a line segment of length a. Find the probability that the distance between P and Q is less than b $(< a)$. 5+5

5.

$$\mathbf{P} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left\| \begin{array}{ccccccc} \frac{1}{7} & \frac{1}{7} & 0 & \frac{2}{7} & \frac{1}{7} & \frac{1}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{2}{5} & 0 & 0 & \frac{3}{5} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right\| \end{array}$$

Let $\mathbf{P}$ be the one-step transition matrix of a Markov chain with state space $\mathcal{S} = \{0, 1, 2, 3, 4, 5, 6\}$.

(i) Find all the closed and irreducible recurrent classes and the transient class of this Markov chain.

(ii) Fix any one such recurrent class C, find the absorption probabilities $\rho_C(x)$, $\forall\, x \in \mathcal{S}$. 6+4

6. Arrival of machinists at a tool crib are considered to be Poisson distributed with an average rate of 7 per hour. The service time at the tool crib is exponentially distributed with mean of 4 minutes.

(i) What is the probability that a machinist arriving at a tool crib will have to wait?

(ii) What is the average number of machinists at the tool crib queue?

(iii) The company made a policy decision that it will install a second crib if a machinist has to wait at least 5 minutes before being served. What should be additional flow of machinist to the tool crib to justify a second tool crib? 3+2+5