Ref.No. : $Ex/Arch/Math/T/114/2024$

Bachelor of Architecture Examinations, 2024

(First Year, First Semester)

MATHEMATICS I

Time: Three hours | Full Marks:100

(50 marks for each part)

Use separate answer script for each part

Symbols/Notations have their usual meanings

Part - I (50 Marks)

Answer ***Question No. 1*** and ***any seven*** from the rest.

1. State and prove Lagrange's Mean Value Theorem. Give the geometrical interpretation of this theorem. [6 + 2]

2. If $y = \sin(\log(x))$, then show that $x^2y_2 + xy_1 + y = 0$, where $y_1, \ y_2$ are first and second derivatives of y with respect to x. [6]

3. Evaluate $\int f(x)dx$, where $f(x) = \frac{x^2}{(x^2-1)(3x-2)}$. [6]

4. Prove that $\int \dfrac{x^2}{(x-a)(x-b)}dx = x + \dfrac{1}{a-b}a^2 \ln(|x-a|) - b^2 \ln(|x-b|)$. [6]

5. If $y = e^{tan^{-1}x}$ then show that $(1 + x^2)y_2 + (2x - 1)y_1 = 0$ and [6]

$$(1 + x^2)y_{n+2} + (2nx + 2x - 1)y_{n+1} + n(n + 1)y_n = 0.$$

6. Find the value of $\lim_{x\to 0}(\frac{\sin(x)+x}{x\sin(x)})$. [6]

7. Determine the points of maxima or minima of the function $f(x) = x^4$ in the interval $[-1, 1]$. Also, find its type. [6]

8. Use the relation $f(x) = f(0) + xf'(\theta x), \ 0 < \theta < 1$, to prove that

$$\frac{2x}{1-x^2} > \ln(\frac{1+x}{1-x}) > 2x$$

when $0 < x < 1$. [6]

9. If $f(x) = e^x$ and $g(x) = e^{-x}$ are continuous in $[a, b]$ and $f'(x)$, $g'(x)$ exist in (a, b), then prove that c is the arithmetic mean between a and b. [6]

10. Find the Maclaurin series of the function $f(x)$, where $f(x) = e^{-x}$. [6]

PART-II (50 Marks)

Answer*Q. No.1* and any ***three*** from the rest

Answer the following questions:

1. Evaluate $\int_0^{\pi/2} \sqrt{\cot x}\, dx$. **5**

2. Examine the convergence of the following integrals:

   a) $\int_0^1 \frac{\sin\frac{1}{x}}{\sqrt{x}}\, dx$ b) $\int_0^1 \frac{\sin x+\cos x}{(1-x^3)^{\frac{1}{5}}}\, dx$ c) $\int_3^{\infty} \frac{1}{\sqrt{x(x-1)(x-2)}}\, dx$ **15**

3. a) From the definition of integral calculus evaluate $\int_a^b e^x dx$.

   b) Using a double integral, prove that $B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$; $a,b > 0$.

   c) State condition for convergence of beta function.

   6+7+2

4. a) Evaluate $\iint_R \frac{\sqrt{a^2b^2-b^2x^2-a^2y^2}}{\sqrt{a^2b^2+b^2x^2+a^2y^2}}\, dxdy$, where R is the region bounded by the first quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

   b) Find the surface area of the solid obtained by revolving one arch of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1-\cos\theta)$ about x-axis.

   8+7

5. a) Calculate the value of $\int_0^1 \frac{x}{x+1} dx$using Simpson's $\frac{1}{3}$ rule by taking six intervals.

   b) Find the area of the cardioid $r = a(1+\cos\theta)$.

   c) Show that $\lim_{n\to\infty} \left|\frac{1}{1+n}+\frac{1}{2+n}+\cdots+\frac{1}{n+n}\right| = ln2$.

   5+6+4