Ref.No. : *Ex/Arch/Math/T/114/2024(S)*

Bachelor of Architecture Examinations, 2024

(First Year, First Semester, Supplementary Exam, 2024)

MATHEMATICS I

Time: Three hours | Full Marks:100

(50 marks for each part)

Use separate answer script for each part

Symbols/Notations have their usual meanings

Part - I (50 Marks)

Answer ***any five*** from the ***seven*** questions.

1. Evaluate the integrals $\int f(x)dx$, where (a) $f(x) = \frac{x-3}{2x^2-x-3}$, (b) $f(x) = \frac{x^2}{(x+2)(x+1)(x-3)}$. [5 + 5]

2. Determine (a) $\lim_{x\to 1}\left(\frac{x}{x-1} - \frac{1}{\log(x)}\right)$, (b) $\lim_{x\to\infty} x^{\frac{1}{x}}$. [5 + 5]

3. Find the local maxima and minima of the following functions if possible. Also determine their nature. (a) $f(x) = \cos^2(\pi x)$ in $[0, 1]$, (b) $f(x) = x^3 - 2x^2$ in $[-1, 1]$. [5 + 5]

4. Find the n-th derivative of the functions: (a) $f(x) = e^{-x} - \sin(x)$, (b) $f(x) = \frac{1}{x}$. [5 + 5]

5. (a) Define Maclaurin's series.
   (b) Using it show that e^x can be expressed as
   $$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ..., \text{ for all } x \in \mathbb{R}.$$
   [3 + 7]

6. (a) Define Rolle's theorem and verify it for the function $f(x) = x^2 - 5x + 10$ on $[2,\ 3]$.
   (b) Define Lagrange's Mean Value theorem and verify it for the function $f(x) = x^3 - 3x + 1$ on $[1,\ 3]$ [5 + 5]

7. (a) State Leibnitz's Theorem for successive differentiation.
   (b) Using it, show that if $y = x^{n-1}\log(x)$, then $y_n = \frac{(n-1)!}{x}$. [3 + 7]

Ref. No.: Ex/Arch/Math/T/114/2024(S)

BACHELOR OF ENGINEERING IN ARCHITECTURE ENGINEERING
SUPPLEMENTARY EXAMINATION, 2024

(1st Year, 1st Semester)

Mathematics-I

Time:Three hours | Full Marks: 100

(50 marks for each Part)

(Symbols and notations have their usual meanings)

Use a separate Answer-Script for each Part

PART-II (50 Marks)

Answer *Q. No.1* and any *three* from the rest

Answer the following questions:

1. Evaluate $\int_0^1 x^3 (1-x^2)^{5/2}\,dx$. **5**

2. Examine the convergence of the following integrals:

   a) $\int_2^\infty \frac{dx}{\log x}$ b) $\int_0^1 \frac{1}{(x+1)(x+2)\sqrt{x(1-x)}}$ c) $\int_0^{\frac{\pi}{2}} \frac{x^m}{Sinx^n}\,dx$ when n <1+m **15**

3. a) Find the volume of the solid obtained by revolving the cardioide $r = a(1+\cos\theta)$ about x-axis.

   b) Evaluate $\iint_D e^{\frac{y-x}{y+x}}\,dxdy$, where D is the region bounded by the triangle with vertices (0, 0), (1, 0), (0, 1). **8+7**

4. a) Calculate the value of $\int_2^4 \frac{x}{x-1}\,dx$ using Simpson's $\frac{1}{3}$ rule by taking eight intervals.

   b) Let f:[-3, 3]→ R be define by $f(x) = \begin{cases} 2x\sin\frac{\pi}{x} - \pi\cos\frac{\pi}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$.
   Examine whether f is Riemann integral in [-3, 3] and hence find $\int_{-3}^{3} f\,dx$.

   c) Compute the length of one arch of the cycloid
   $x = a(\theta - \sin\theta), y = a(1-\cos\theta)$. **5+5+5**

5. a) Changing the order of integration, evaluate $\int_0^\infty \int_0^\infty e^{-xy} \sin nx\,dxdy$.
   Hence deduce that $\int_0^\infty \frac{\sin nx}{x}\,dx = \frac{\pi}{2}$.

   b) Find the area of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

   c) Evaluate $\int_0^\infty e^{-ax^2}\,dx$. (a>0) **7+5+3**

---------------------------------xxx---------------------------------