BACHELOR OF ARCHITECTURE EXAMINATION, 2024

(1st Year, 2nd Semester)

MATEMATICS – II

Time : Three Hours | Full Marks : 100

Use separate Answer scripts for each part.

Symbols/Notations have their usual meaning.

PART – I (50 Marks)

Answer ***any five*** questions.

1. a) Find the angle between the lines whose direction cosines are proportional to 1, 2, 1 and 2, –3, 6.

   b) A, B, C are three points on the axis of x, y and z respectively at distances a, b, c from the origin O. Find the coordinates of the point which is equidistant from A, B, C and O.

   5+5

2. a) Find the equation of the plane through the point (1, 2, 3) and parallel to the plane $4x+5y-3z=7$.

   b) Find the equation of the plane through the point (2, 1, 0) and normal to the planes $2x-y-z=5$ and $x+2y-3z=5$.

   4+6

3. a) Show that the line given by the equations $3x-4y+z+1=0$, $x-2y+z+2=0$ is equally inclined to the axes.

   b) Find in symmetrical form the euaqations of the line of intersection of the planes $2x-y-1=0$ and $2y-z+1=0$.

   5+5

4. Prove that the lines

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \quad \text{and} \quad \frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$$

   are coplanar and find the equation of the plane containing them.

   10

5. Find the magnitude and the equation of the line of shortest distance between the lines

$$\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7} \quad \text{and} \quad \frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}.$$

   10

[Turn over

6. a) Find the equation of the sphere which passes through the points (0, 0, 0), (a, 0, 0), (0, b, 0) and (0, 0, c).

   b) A sphere of constant radius k passes through the origin and meets the axes at A, B, C. Prove that the centroid of the triangle ABC lies on the sphere $9\left(x^2+y^2+z^2\right)=4k^2$.

4+6

7. Prove that the circles

$$x^2+y^2+z^2-2x+3y+4z-5=0,\ \ 5y+6z+1=0,$$

and $x^2+y^2+z^2-3x-4y+5z-6=0,\ \ x+2y-7z=0,$

lie on the same sphere and find its equations.

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BACHELOR OF ARCHITECTURE FIRST YEAR SECOND SEMESTER EXAMINATION - 2024

Ref. No. : Ex/Arch/MATH/T/124/2024

Subject : MATHEMATICS - II

Time : Three Hours **Full Marks : 100**

Use Separate Answer Scripts for each Part.

Part - II (50 Marks)

Answer any five questions.

1. (a). What do you mean by inverse of a matrix? Does the inverse of a skew symmetric matrix of odd order exist? Justify. If $A = \begin{pmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix} = f(x)$, then show that $A^{-1} = f(-x)$. $1+1+3$

(b). (i). Let A and B be two square matrices of the same order such that $AB = A$ and $BA = B$. Show that $A^2 + B^2 = A + B$.

(ii). What do you mean by a nilpotent matrix? Is $\begin{pmatrix} ab & b^2 \\ -a^2 & -ab \end{pmatrix}$ is a nilpotent matrix? Justify. $2+(2+1)$

2. (a). Express $A = \begin{pmatrix} 2 & 5 & -3 \\ 7 & -1 & 1 \\ -1 & 3 & 4 \end{pmatrix}$ as a sum of a symmetric and a skew symmetric matrix. Is this representation unique? Justify. $3+2$

(b). Define singular matrix. Give an example of a nonzero singular matrix. If the matrix $A = \begin{pmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ \lambda & -3 & 0 \end{pmatrix}$ is a singular matrix then find the value of λ. $1+2+2$

3. (a). (i). Show that $\Delta = \begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix}$ is a perfect square.

(*ii*). Let A be an $n \times n$ nilpotent matrix of index 2 and I_n be the identity matrix of order n. Show that both $I_n - A$ and $I_n + A$ are non-singular matrices. $3+2$

(*b*). (*i*). Let A and B be two square matrices of the same order. Is $det(A+B) = det(A) + det(B)$? Justify.

(*ii*). Let A and B be two orthogonal matrices of the same order and $det(A) + det(B) = 0$. Show that $A + B$ is a singular matrix. $2+3$

4. (*a*). Prove that the radius of curvature at any point of the cycloid given by $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$ is $4a\cos\frac{\theta}{2}$. 4

(*b*). Find the equations of tangent and normal, the lengths of tangent and subtangent, the lengths of normal and subnormal for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $(\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}})$. 6

5. (*a*). Determine the values of b and c given that the line $3x + y - 8 = 0$ is tangent to the curve $y = x^2 + bx + c$ at the point $(1, 3)$. 5

(*b*). Find all the asymptotes of the curve $y^3 + x^2y + 2xy^2 - y + 1 = 0$. 5

6. (*a*). Find the points on the parabola $y^2 = 8x$ at which the radius of curvature is $\frac{125}{16}$. 5

(*b*). Find the radius of curvature of the curve defined by the equation $y^2 = \frac{4a^2(2a-x)}{x}$ at the points where the curve intersects the x-axis. 5

7. (*a*). If the normal to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ makes an angle ϕ with the x-axis, show that its equation is $y\cos\phi - x\sin\phi = a\cos 2\phi$. 6

(*b*). The curves $y = x^2$ and $y^2 = x$ pass through the point $(1, 1)$. Find their angle of intersection at this point. 4 4