

MODELLING AND CONTROL OF SECOND ORDER PLUS DEAD TIME (SOPDT) SYSTEMS

ABSTRACT

A. Objectives:

The objectives of the proposed research work is executed in the following steps:

1. Review of the work:

Many industrial processes, like those in chemical engineering and manufacturing, involve complex dynamics with time delays. The SOPDT (Second-Order Plus Dead Time) model simplifies these processes for analysis and control. Parameters are estimated using methods like step response analysis, integral techniques, and the method of moments, but these approaches can be sensitive to noise and precise inflection point identification. Dead time is critical, often approximated with techniques like Pade's approximation. While some methods use closed-loop modeling, they can complicate systems and limit adaptability compared to open-loop approaches.

Fuzzy Logic is effective for PID tuning using expert rule tables, while methods like Genetic Algorithms (GAs) also achieve optimal parameters. However, fuzzy logic often involves trial-and-error for scaling factors, making it less efficient. GAs are promising but depend on a well-chosen cost function for effective optimization.

Major heads of study and proposed research may be outlined as follows:

- To develop mathematical models that represent the behaviour of SOPDT systems - one for underdamped systems and the other for critically/overdamped systems.
- To test the modelling approaches for noisy systems as commonly encountered in real-world situations.
- To conduct a case study on the modelling of a fast-steering mirror assembly along with the associated drive electronics viz. a system typically modelled as a SOPDT system.
- To compare the developed modelling methods with standard methods existing in the literature.

- To test two control schemes, viz. fuzzy control and GA-based PID control on a simulation platform for typical SOPDT systems.
- To analyze the performance of the proposed controllers, taking into account standard performance indices, in the time domain.
- To evaluate the performance of the controllers in the presence of measurement noise and disturbances.
- To investigate the efficacy of the proposed modelling and control strategies on a Temperature Controller Kit (FEEDBACK Trainer 37-1000), a typical SOPDT system.

2. Modelling of SOPDT systems

2.1 Methodology developed for Underdamped SOPDT systems

This modelling methodology proposes the usage of constrained convex optimization in improving the quality of the parameter estimates of a typical process plant, (the majority of which are identified as second-order plus dead time (SOPDT) systems) from its time response data by incorporating system-specific constraints that are not considered in standard estimation methods. Traditional methods for parameter estimation in SOPDT systems have often relied on heuristic approaches or simplified assumptions, leading to suboptimal results. In this work, a novel iterative algorithm based on gradients is developed for parameter estimation of SOPDT systems from step responses using Newton's Quadratic Model and Sequential Quadratic Programming which provide a rigorous mathematical framework for parameter estimation. By incorporating system constraints, such as bounds on the parameters or stability requirements, it is ensured that the obtained parameter estimates adhere to physical and practical limitations. The proposed approach is demonstrated using simulations and on a real-time system (second-order LP Butterworth filter with an input delay block) as shown in Fig. 1., and the results show that it is effective not only in accurately estimating the parameters of the underdamped SOPDT systems but also works efficiently for parameter estimation of SOPDT systems in the presence of measurement noise. The efficacy of the proposed algorithm is verified by comparing it with similar published methods. The proposed methodology is applied to a third-order dead time system with a complex conjugate dominant pole pair, and a fourth-order system, with two complex conjugate pole pairs: one dominant and the other away from origin to find out whether the proposed method can model such a system by second-order dynamics. The simulation results suggest that the proposed methodology successfully models complex third and fourth-order systems with second-order dynamics.



Figure 1 experimental set-up for modelling second-order LP Butterworth filter with an input delay block

2.2 Methodology developed for overdamped/critically damped SOPDT systems

The proposed least-square-based iterative parameter estimation algorithm estimates the parameters of the canonical transfer function of the plant by creating a correlation that precisely connects parameters of the transfer function with the three specific time instants of the step response. A three-point measurement of the transient response, viz. t_5 , t_{10} and t_{90} is acquired (as shown in Figure 2) for estimation of *the parameters* where t_{90} is the time when step response attains 90% of its final value, t_{10} is the time when step response attains 10% of its final value and t_5 is the time when step response attains 5% of its final value.

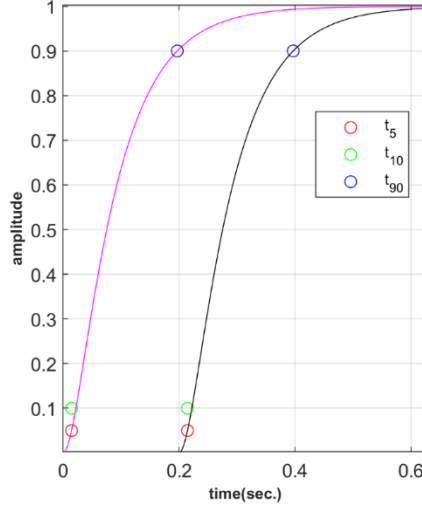


Figure 2 System response at three-time instants
 (Color magenta $t_d = 0$ sec, $\xi = 1.2$, $\omega_n = 12$ rad/sec
 Color black $t_d = 0.2$ sec, $\xi = 1.2$, $\omega_n = 12$ rad/sec

After a thorough investigation, it is found that for a given *damping ratio* ξ , the ratio $T_R = \frac{t_{90}-t_5}{t_{10}-t_5}$, a performance index defined for this purpose, remains the same for different values of ω_n (*undamped natural frequency*). The inclusion of t_5 in calculating T_R ensures the elimination of the dead time. Moreover, for a fixed value of ω_n , T_R increases monotonically as ξ is increased. This points to the fact that a correlation between ξ and T_R may be formulated that is independent of ω_n . Further, the investigations revealed that for a fixed value of ξ , a linear relationship exists between ω_n and $f_r = \frac{1}{t_{90}-t_{10}}$, where f_r is the reciprocal of the rise time t_r . Thus, for a given value of ξ , a correlation may also be formulated between ω_n and f_r . The quantitative indices for measuring the accuracy of the closeness of response of the estimated SOPDT system with the actual system, viz. VAF (variance accounted for) and IAE (integral absolute error), illustrate comparable results with the existing estimation schemes in literature. The illustrative examples also clearly indicate that the proposed method is more robust against measurement noise compared to the other two existing standard methods in literature.

2.3 Methodology developed for modelling a real-time SOPDT system -a dual axis fast steering mirror assembly

This work introduces a systematic black-box approach to model a dual-axis tip-tilt fast steering mirror assembly, featuring integrated piezo actuators and pick-off sensors. The method employs a two-fold paradigm utilizing time response (TR) or frequency response (FR) data,

contingent on the system's damping characteristics. It estimates the cross-coupling coefficient between the two axes by transforming the time response signal into the frequency domain using the FFT of the output on one axis due to a sinusoidal input on the other axis. The TR method employs step response data, collected via a digital storage oscilloscope, to formulate a mathematical model. Meanwhile, the FR method employs sinusoidal excitation to derive the frequency response, combined with a least square estimator. The experimental set-up is shown in Figure 3.

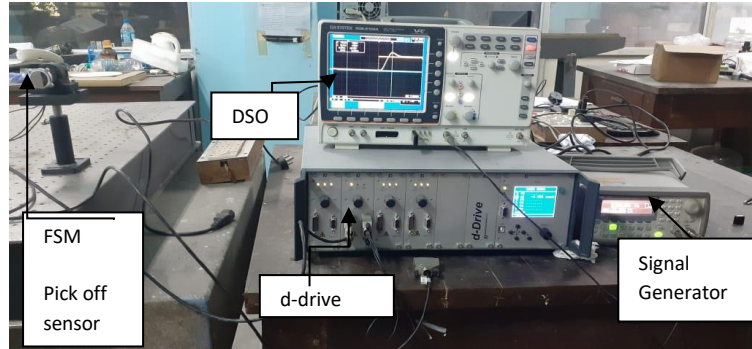


Figure 3 Experimental Set-up of dual axis FSM assembly

Operating in open-loop mode, the algorithm obviates the need for controller insight and works uniformly across all damping cases. Validation depends on data type: TR modelling is verified in the frequency domain and FR modelling in the time domain. Model accuracy is assessed using variance accounted for (VAF) and compared against conventional data-driven methods. To capture the effect of cross-coupling between the two axes the state space model of the FSM is formulated

3. Control of SOPDT system

3.1 Fuzzy Logic Control

In designing a fuzzy logic controller (FLC), a PI (Proportional-Integral) model is typically chosen as the foundation. Following this choice, a rule base is formulated to define the controller's response under different input conditions. This rule base serves as the guiding framework for the controller's decision-making process. Once the FLC structure is established, it is implemented in MATLAB for simulation and testing purposes. One common challenge in FLC design is tuning the scaling factor, which directly influences the controller's performance. Traditionally, this tuning process involves trial and error, which can be time-consuming and less efficient. To address this challenge, a self-tuning mechanism is introduced using a fuzzy rule base. This mechanism dynamically adjusts the output scaling factor based on the system's behavior and input conditions. By leveraging fuzzy logic for tuning, the controller becomes more adaptive and responsive.

Subsequently, the performance of the self-tuned FLC is compared with that of a conventional Ziegler-Nichols PI controller. Both controllers undergo testing under various scenarios, including those involving measurement noise and external disturbances. Performance evaluation is conducted in the time domain, assessing each controller's ability to track setpoints and reject disturbances.

Performance indices like IAE, ITAE, and ISE measure a controller's error and disturbance handling. Results show self-tuned fuzzy logic controllers outperform Ziegler-Nichols PI controllers, highlighting the advantages of adaptive fuzzy tuning over traditional fixed methods.

3.2 Genetic Algorithm-based PID controller

In the optimization of PID controller parameters for a SOPDT system, MATLAB's Genetic Algorithm toolbox was employed to translate the problem into GA chromosomes. These chromosomes represent sets of real-number values for PID gains, within predefined bounds derived from system characteristics, such as delay, and established control methods Ziegler-Nichols rule. Careful choice of bounds is essential for successful convergence, as arbitrary bounds may impede finding optimal solutions. The lower bound chosen for $[K_p, K_i, K_d]$ are $[0,0,0]$ and the upper bound as $[200, 200, 200]$. The choice of an objective function, also known as a fitness function or cost function, is a crucial decision in the design and success of a Genetic Algorithm (GA). Here, the fitness function f is chosen as the maximum of IAE (Integral of Absolute Error), ITAE (Integral of Time-weighted Absolute Error), and ISE (Integral of Squared Error) as shown in (1) to increase the scalability of GA algorithm.

$$f = \max([IAE, ISE, ITAE]) \quad (1)$$

Following this, a self-adaptive GA was designed incorporating a worst-case cost function to guide the optimization process. This adaptive mechanism dynamically adjusts GA parameters during optimization to enhance performance.

Subsequently, the performance of the self-adaptive GA-tuned PID controller was compared against that of a Ziegler-Nichols PI controller. Both controllers underwent time domain testing, including scenarios involving measurement noise and external disturbances. Ultimately, the self-adaptive GA-tuned PID controller consistently demonstrated superior performance compared to the Ziegler-Nichols PI controller. This underscores the effectiveness of GA-based optimization techniques tailored to the system's characteristics in achieving enhanced control performance.

3.3 Practical Case Study

The thesis investigates the efficacy of the proposed modelling and control strategies on a Temperature Controller Kit (FEEDBACK Trainer 37-1000), a typical SOPDT system. The proposed controllers are plugged in with the plant model and their performances are evaluated and compared with controller implementations for the same trainer kit. To obtain the process characteristics, open loop tests were performed, as shown in Figure 5. A 4V (peak to peak) square wave signal of frequency 0.1 Hz with offset signal -2V was applied externally to the Trainer 37-100 by a signal generator (AFG-2005) and it was observed at Channel 2 of the DSO (GDS-1054B).

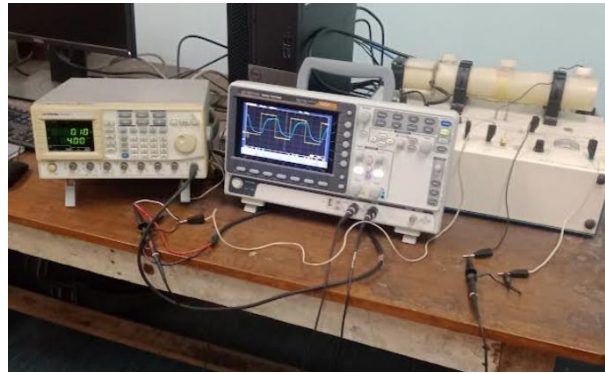


Figure 5 Experimental set-up

The .csv format of the acquired signals is read in MATLAB to determine if the inflection point exists. The identification of inflection points in the unit step response of a system serves as a valuable indicator for understanding its dynamic behavior. When an inflection point is present, it signifies characteristics akin to a second-order system. The sluggish response of the trainer without any oscillation suggests that its dynamics can be characterized as Second-Order Plus Dead Time (SOPDT) with overdamped/critically damped ($1 \leq \xi \leq 2$) behavior. Next applying modelling methodology of overdamped system, the mathematical model of the Trainer is identified. Once the model is at hand the fuzzy logic control and GA-tuned PID control is used in MATLAB Simulink environment to compare these control actions with the conventional ZN tuned PI control. The performance analysis of the proposed controllers, considering standard performance indices, is done in the time domain. The performance of the controllers is also evaluated in the presence of measurement noise and disturbances, to test the robustness. It is found that the self-tuned fuzzy logic PI controller outperforms self-adaptive GA and Ziegler Nichols-PI controller. Fuzzy Logic Controllers (FLCs) are more resilient to noise due to their ability to convert crisp input values into fuzzy sets, accommodating

variability and imprecision. Additionally, FLCs operate on rule-based systems and utilize linguistic terms and membership functions, which inherently accommodate variations and uncertainties enabling smoother responses to noise-induced fluctuations in input signals.