Performance Evaluation of Engineering Materials in Optimized Design

Thesis submitted by

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CERTIFICATE FROM THE SUPERVISOR/S

This is to certify that the thesis entitled "**Performance Evaluation of Engineering Materials in Optimized Design**" submitted by Shri **Debasis Das**, who got his name registered on **01.04.2016** for the award of Ph. D. (Engineering) degree of Jadavpur University is absolutely based upon his own work under the supervision of **Dr. Somnath Bhattacharya** and that neither his thesis nor any part of the thesis has been submitted for any degree/diploma or any other academic award anywhere before.

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Abstract

Engineering design is the judicious trade-off among shape, materials, and manufacturing that requires a wide range of decisions. Decision-making in engineering design allocates all the resources optimally while fulfilling the design objectives within economic constraints, quality constraints, safety constraints, environmental constraints etcetera under uncertainty. Material selection is a fundamental step in the mechanical design that has to meet all the functional requirements of the component. This thesis provides a material selection framework and two new methods for performance evaluation of the alternatives. One of them is newly developed method and all the approaches with overcoming the previous are outlined as:

- Design is the formulation of information or data (quantitative and qualitative) where some degree of risk and uncertainty always exist. Therefore, the problem should be precisely structured according to the decision requirements. A framework is proposed based on decision-based design (DBD) framework where the alternatives are generated by Suh's design axioms (SDA) and evaluated by AHP (analytical hierarchy process) that assigns the weightage to the criteria and MAUT (multi attribute utility theory) that assigns the utility value to the alternatives. The entire approach is termed as **normative-prescriptive approach (NPA)**.
- In the above-mentioned approach MAUT considers the observed utility. An alternative has also unobserved utility which is stochastic in nature that should be considered. The conditional logit (CLGT) from the domain of discrete choice theory is introduced in place of MAUT that assigns the utility value to the alternatives in terms of choice probability to address the choice under risk and uncertainty. The entire approach is termed as **discrete choice analysis (DCA)**.
- Above-mentioned conditional logit (CLGT) gives good result under uncertainty phenomena. Decision making is the process to choose an appropriate alternative

based on the belief of the decision maker. Belief is the function of knowledge and confidence. A designer rather enjoys a spatial relationship evidence that can give confidence under uncertainty. A new spatial approach is developed to make the belief as a justified belief where the choice parameter is considered as a Manhattan norm (Taxicab geometry) in turn which is a function of the Euclidean norm and cosine similarity to raise a preeminent alternative under the MADM (multi attribute decision-making) framework. The entire approach is termed as **nearest neighbour search (NNS)**.

In this thesis, four case studies such as: two stages spur gear reduction unit; cryogenic storage tank; flywheel; and spar of human powered aircraft are considered. The findings are compared with the available alternative evaluation methods and the previous woks on material selection approaches. Hopefully, the results are consistent with the results that were raised by previous literature. The sensitivity analysis (based on changing the criteria weightages and normalizing process) is also conducted for the new methods and compared and results show that NNS is more sensitive than DCA.

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Finally, I want to dedicate this thesis to my parents and uncle who have done so much to raise me up, to educate me, to guide me under all circumstances. This work is also the outcome of the blessing and support of my sisters and brothers, my all family members, class mates and colleagues.

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Chapter 1

Research introduction

1.1 Introduction

Human civilization has changed rapidly in the last thousand years. This is not the age of one material. With the advancement of new technologies, varieties of materials are being invented such as, ceramic and composite materials. Today the number of engineering materials is around 50000.

In fact, human evolution took a new dimension when tree dwellers started to live in the cave. It was Stone Age; people began breaking stones to create sharp edge and used it as a weapon or to cut wood and meat. Gradually they started building roads; through these roads they scattered around the world and established the Bronze Age.

By around 3000 BC bronze was discovered by melting copper and tin together and stone tools and weapons were replaced by bronze. Towns grew into cities with market places, courts and temple. People were able to train large animals, such as horse, donkeys and camels to carry goods over long distance with in short time. At this age the most revolutionary invention was a wheel. By horse drawn chariots bronze aged people entered the Iron Age.

Iron is one of the most common elements on the earth, but it is rarely ever found in metallic form. Around 1200 BC, people learned to extract large quantities of iron by smelting iron ore in the furnace. In 1620 AD cast iron technology established the dominance of metals in engineering.

In the present age we are in a very complex situation. From the product design point of view, two most important criteria for a successful product are to meet all the functional requirements and to be economically competitive. In many manufacturing operations the cost of the materials may amount to more than 50 percent of the total cost (Figure 1.1). Thus, an improperly chosen material may increase the cost of production or affect the service performance. The sole attention of this research is to evaluate the performance of the engineering materials during material selection in engineering design to ensure an optimal design. Optimal design is a feasible solution that optimizes the overall objective function or maximizes the utility.

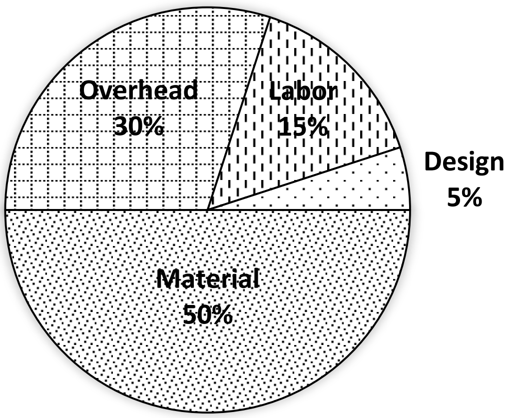


Figure 1.1 Manufacturing cost [source: Ullman, 2003]

1.2 Research problem

Design is a sequential decision-making process of shape, materials, and manufacturing. The performance of a material describes how a part constructed from the given material behaves under certain loading conditions or design requirements. Material properties are an indication of the performance of the material in use and thus link structure to material performance. Material selection in design is the process of alternative generation followed by alternative selection through performance evaluation of the alternative that requires data. Data are the information about an idea in the real world. As a whole design is an articulation of the information where some degree of risk and uncertainty always exist. Material selection is an integral part of the engineering design process and rather uncertainty takes place in the formulation of design and thereby alternatives generation in material selection. Therefore, a strategic decision under risk and uncertainty is required to balance between functional requirements and cost those depend on physical construction and properties of the constituent materials of the product (Pfeifer, 2009).

Material selection involves using a material database to select the best material for a product. By following materials selection techniques, the most appropriate material is selected from all known materials in order to satisfy product requirements and goals. The material selection method that is most widely used in practice is the method developed by Ashby (1999). This process begins with a database containing all known materials. Screening and ranking techniques reduce the number of feasible material based on product geometry and loading conditions. The resulting subset of feasible materials is further reduced by conducting research on these materials. At this stage in the material selection process, engineering expertise plays a role in eliminating materials that would be poor choices in the overall product design by including information about economics (cost and availability) and manufacturing needs. After the prime candidates have been selected, local load conditions combined with design requirements lead to the final material choice.

From material selection, decision makers generally choose a preeminent material from the finite set of alternatives. There are numerous decision matrix techniques have been developed in which the Pugh method and Pahl and Beitz method are very popular due to its simplicity as the alternative evaluation takes place at the conceptual stage of the design process (Pahl & Beitz, 1988; Otto & Wood, 2001). The above-mentioned methods can be regarded as multiple criteria decision making and typically studied in the domain of engineering design. At the same time, there is another domain termed as MCDA (Multiple Criteria Decision Analysis) and can be classified as Multi-Objective Decision Making (MODM) where design space is continuous and Multi-Attribute Decision

Making (MADM) where design space is discrete. MADM processes are becoming popular due to its user-friendly nature to select the best material from that finite number of alternatives where the performances of the alternatives are expressed in terms of multiple attributes or criteria shown in Figure 1.2.

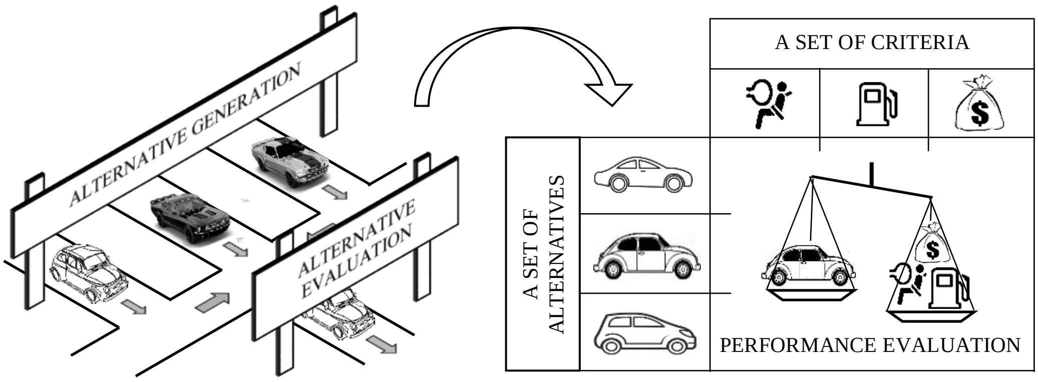


Figure 1.2 Graphical multi-attributed decision-making framework

MADM approaches are more likely to be modelled with uncertain values for the attributes (Jahan & Edwards, 2015). However, the existing methods are deemed to be inadequate. A method alone is not only sufficient to choose a preeminent alternative under uncertainty during design optimization. The chosen solution should be in agreement with the preferences and beliefs of the decision maker. It is important that how the methods are implemented in decision-making. There is some lack of rational approaches in the previous works of this field. Rationality has different meanings in different perspectives. To ensure rationality in choice is to analyse the problem in a systematic way. From the material selection point of view in engineering design, the rationality is the balancing between the customer's requirements and design requirements by,

- selecting a single best or optimal outcome that optimizes the design or maximizes the overall utility;
- and ddressing the risk and uncertainty with mathematically sound approaches.

1.3 Aims and objectives

The existence of a designer depends on the satisfaction of a customer and a designer and a customer both are in the same situation of 'which alternative' is to choose from a set of alternatives under uncertainty. The aim of this paper is to investigate the 'which' in a rational decision-making framework (Figure 1.4) which is germane to the material selection in design under risk and uncertainty to ensure an optimal design.

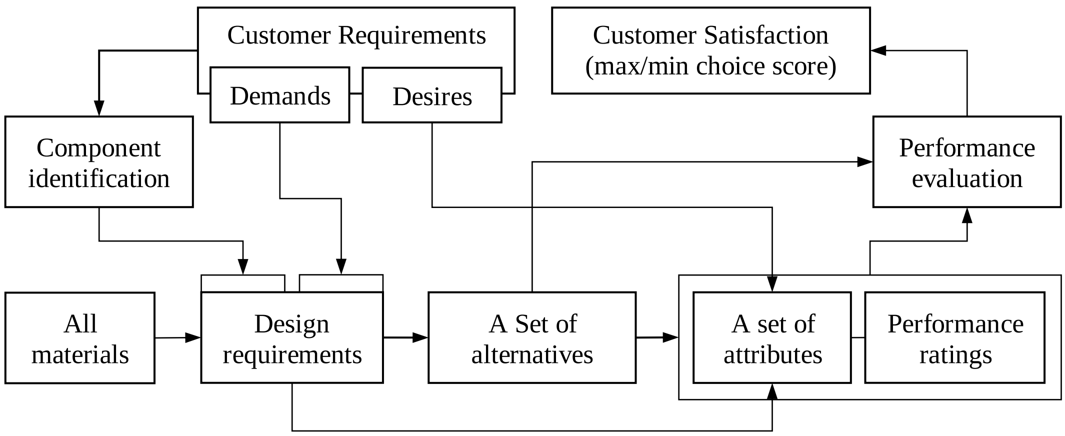


Figure 1.3 Rational decision-making framework of material selection

The specific objectives of this research that will frame the aim shown in Figure 1.4 and can be stated as:

- *Understanding the scope* is to review the literature thoroughly and summarize the current knowledge regarding MCDA process which have been applied in material selection and to identify the major shortcomings in the existing MCDA process specifically in MADM process;
- *Development of the notion* is to suggest the ways to improve the conventional decision-making process used in material selection by overcoming the shortcomings in the existing methods;

- *Implementation of the notion* is to apply the suggested ways in some practical cases and investigate the suitability of the suggested ways.

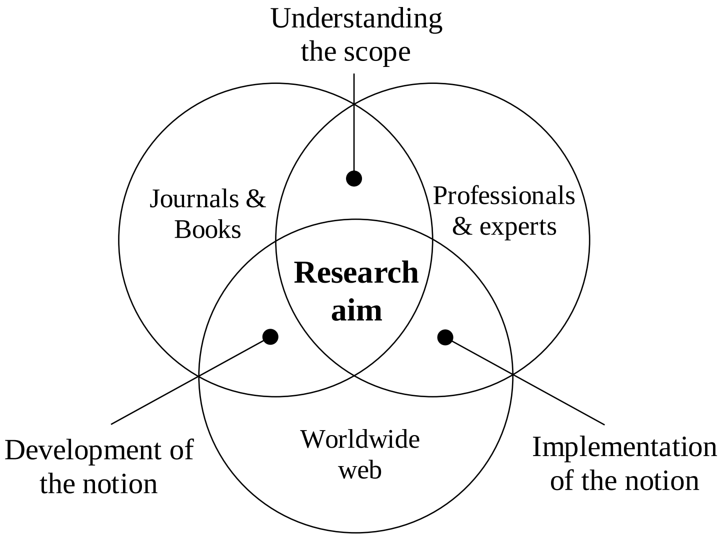


Figure 1.4 Aims and objectives of the research

1.4 Thesis structure

In this section, the entire thesis regarding overviews of the all chapters are portrayed in words as well as shown in Figure 1.5.

Chapter 1 provides background information of this research. It explains why this research was undertaken and how this research on material selection is significant in manufacturing industry. Research aims and objectives and material selection framework adopted are highlighted.

Chapter 2 provides a theoretical foundation of engineering design process and ingredients associated in the design process by reviewing literature and previous research. It provides information and argument for the importance of precise ranking and confidence under uncertainty in material selection that reveals Chapter 3.

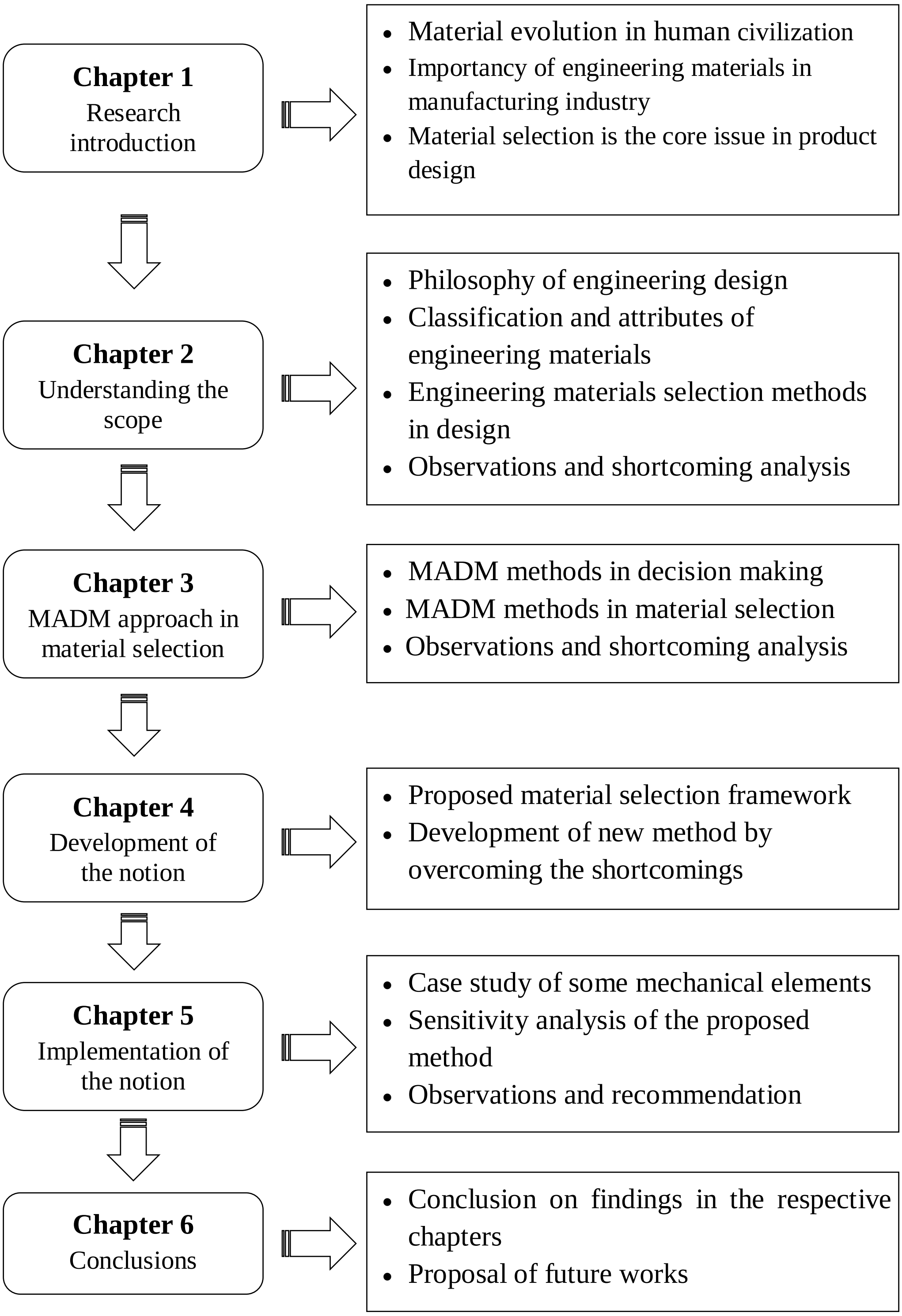


Figure 1.5 Thesis structure and overview

Chapter 3 focuses on the broad discussion on multi-criteria decision analysis (MCDA) from the domain of decision making theory that overcomes the shortcoming of lack of precise ranking of the alternatives arose in Chapter 2 but the confidence under uncertainty is still exist.

Chapter 4 is devoted exclusively to the development of the multi criteria decision model for material selection. This chapter provides a material selection framework and two new methods for performance evaluation of the alternatives. One of them is newly developed method and tried to overcome the shortcoming that was still exist in Chapter 3.

Chapter 5 implements the developed methods in various case studies. Consistency in ranking is checked by comparison with other MADM methods and sensitivity analysis is also conducted among the developed methods in order to demonstrate the shortcoming of the existing methods and the benefit of the proposed methods.

Chapter 6 summarises the research findings and states the conclusions. Conditional statements are made with respect to the application of the conceptual model in engineering design. Limitations of the research and the possibilities of further research are made at the end of the chapter.

References contain an extensive reference list summarising the literature reviewed in this research, including books, journal papers, reports, and conference papers around the world.

Chapter 2

Understanding the scope

2.1 Introduction

The design process begins with a need which is based on customers' and markets' demands and ends to a finished product. A successful product enjoys good profits, good market share, and good customer satisfaction. The importance of materials selection in design has increased in recent years. The selection of the proper material is a key step in the design process. In most of the manufacturing unit, materials costs comprise 50% or more of the manufacturing cost. At the same time, material science worldwide has created a variety of new materials and focused attention on the competition between broad classes of materials like metals, polymers, ceramics, composites and other advanced materials. Thus, the range of materials available to the engineer is much larger than ever before. This presents the opportunity for innovation in design by utilizing these materials in products that provide greater performance at lower cost. To achieve this requires a more rational process for materials selection.

2.2 Engineering design

In philosophically, engineering design is the set of creative activities to meet the customer's demands and desires. Creative means capable of creation, inventive, imaginative, showing imagination in addition to habitual skill and knowledge. Creativity does not occur but need to be explored with the emphasis on ingenuity and functionality including the extensive sections of knowledge of various fields. In psychology, creativity occurs as a result of a natural tension between intellectual and intuitive mental modes (Hubka & Eder, 1996). The intellectual mode (systematic, methodical, analytical) can recognize that a problem exists and can analyse its nature. In the intuitive mode (erratic, inconstant, non-calculable) a sense of dissatisfaction can then arise, which triggers and motivates the mental interaction with the intellect to attempt to solve the problem. More specifically, according to The ABET (Accreditation Board for Engineering and Technology), engineering design is the process of devising a system, component, or process to meet the desired needs. Engineering design is a sequential decision-making process, in which the basic sciences, mathematics, and engineering sciences are applied to convert the available resources to meet a stated objective. Among the fundamental elements of the design process is the establishment of objectives and criteria, synthesis, analysis, construction, testing, and evaluation.

The entire design process can be sliced in four phases as (Pahl & Beitz, 1988; Cross, 2005):

- ***Clarification of the task*****:** In this phase, all information (qualitative and quantitative) are collected from the customer requirements domain and translated into the design requirements (specification).
- ***Conceptual design*****:** The goal in this phase is to validate the need, establishment of function structures with generating the number of concepts and evaluate the concepts on the basis of technical criteria though rough economic criteria should also be considered. It may be that several concept variants look equally promising, but the final decision depends on the designer’s sense of creativity.
- ***Embodiment design*****:** During this phase, the concept takes the form in terms of product or system in accordance with technical (function, strength, spatial compatibility etc.) and economic feasibility. An important task in embodiment design is to quantify the parameters so as to establish the optimal solution.
- ***Detail design*****:** In this phase the design is formalized with complete engineering description of a tested and producible product. The arrangement, form,

dimensions tolerances and surface properties of all the individual parts are finally laid down and the technical and economic feasibility again re-checked and all the drawings and other production documents are prepared.

As a whole, the engineering design is the judicious trade-off among shape, materials, and manufacturing that requires a wide range of decisions (Figure 2.1). Decision-making in engineering design allocates all the resources optimally while fulfilling the design objectives within economic constraints, quality constraints, safety constraints, environmental constraints etcetera under uncertainty (Ullman, 2003; Hatamura, 2006).

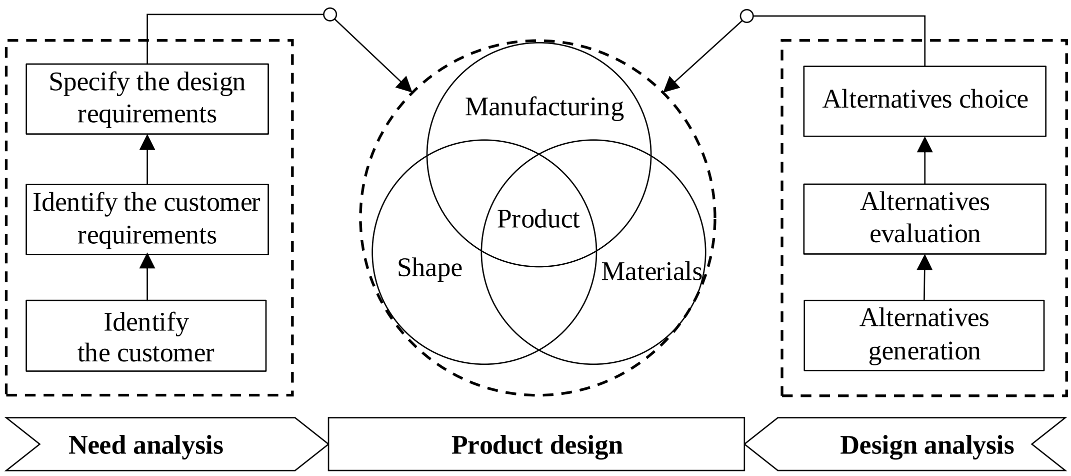


Figure 2.1. A concise framework of product design

2.3 Material selection in design

One aspect of optimized product design is that of selecting the materials that best meet the needs of the design by maximizing its performance and minimizing its cost. The choice of the best material among a host of alternative materials might greatly impact the eventual success or failure of a product in the market place. An improper choice can adversely affect productivity and profitability. Like any other decision-making process, material selection can be characterized as the outcomes of (Dieter, 1983):

- ***Component identification and performance specification*:** To recognize the component and lay down the possible performance specification in terms of demands and desires from the customer requirements.
- ***Identify the design and material requirements*:** To determine the conditions of service and environment that the product must withstand and to translate them into critical material properties or attributes.
- ***Screening of the candidate materials*:** To compare the needed properties with large materials property data base to select a few promising materials.
- ***Evaluation of the candidate materials*:** To conduct evaluations to determine whether a material does satisfy all the design requirements of the product element under consideration. When there is more than one option of materials, evaluations provide information that will help identify the material with properties that best meet the design requirements, at the lowest cost.

Material selection takes place at the earlier stage of the design process. The above-mentioned material selection steps are shown in Figure 2.2.

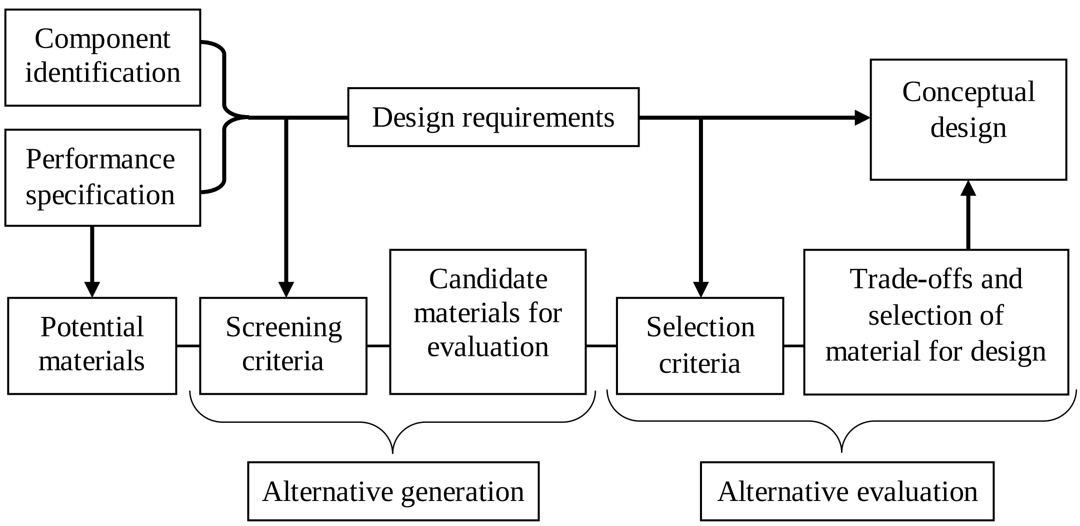


Figure 2.2 Material selection framework of a product

2.3.1 Alternatives: Engineering materials

The engineering materials can be classified as three categories: metals, polymers, and ceramics. In these modern days many traditional engineering materials is being replaced by new advanced material as shown in Figure 2.3, in order to meet the weight reduction and performance enhancement such as composite materials taking the vital role in spite of having a high cost (Gupta, 2015; Kutz, 2002). Composites are so versatile that they are now being used even to build large-scale structures. A 460-ft (140-m) bridge that carries four lanes of traffic through San Diego, California, has recently been constructed from composites and is estimated to be one fifth as light as an equivalent metal bridge.

2.3.1.1 Ferrous alloys

Ferrous alloys are iron based alloys that has extensive use in wide range of industries because of its flexibility to meet strength, toughness, and impact of diverse industrial applications. This flexibility depends on the heat treatment procedures, which modifies the final micro-structure. Examples of ferrous alloys include carbon steels, alloy steels, stainless steels, tool steels, cast iron, and cast steel. Carbon steels are iron-carbon alloys contained the carbon up to 2.06 %, up to 1.65 % Mn. 0.5 % Silicon, Sulphur and Phosphorus and 0.40 % Copper. In carbon steels, Carbon content addresses the strength and ductility. As a consequence, these steels are relatively soft and weak. Mechanical properties for specific uses can be remarkably improved in plain carbon steels (i.e. pure metals) when alloyed with one or more elements and these steels are known as alloy steels. Alloy steels are more expensive to produce due to higher cost of alloying elements and processing and also, they are more difficult to fabricate and machine. Different alloying elements and their notable effects are given in Table 2.1. Stainless steels are steels possessing high corrosion resistance due to the presence of substantial amount of chromium. Chromium forms a thin film of chromium oxide on the steel surface. Most of the stainless steels contain 12% - 18% of chromium. Cast iron is a type of iron that contains more than 1.7% carbon, usually in between 2% and 4.5% C. Cast iron is the cheaper metallurgical material available to the engineer. Apart from its low cost, other commendable properties of cast iron including good rigidity, high compressive strength, excellent fluidity so that it makes good casting impressions, and Good machinability. Various ferrous materials are given in Table 2.2.

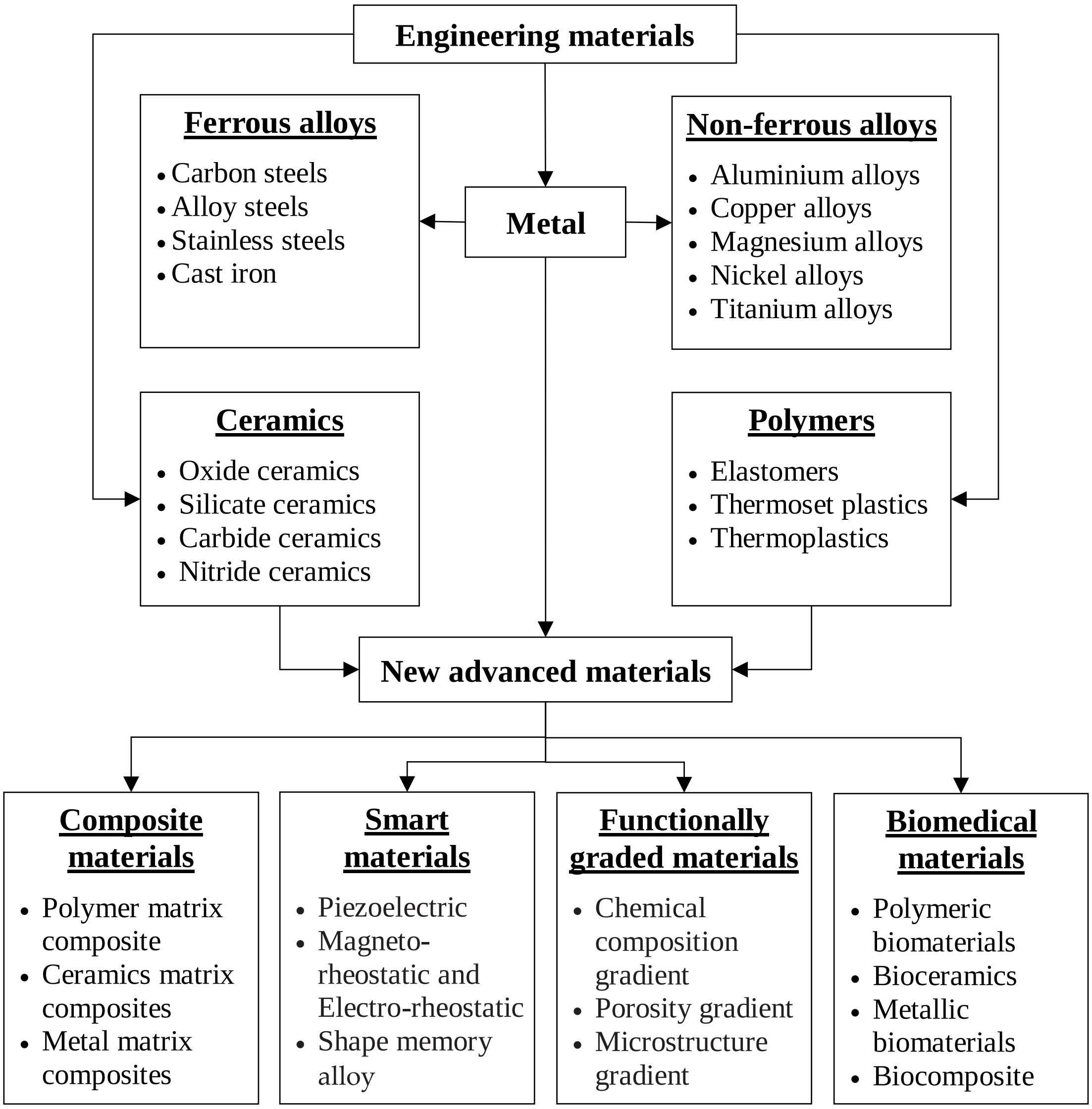


Figure 2.3 Engineering materials classification

Table 2.1 Alloying elements in carbon steel and its effect.

Alloying element	Imparted properties	Alloying element	Imparted properties
Tungsten (W)	Imparts red hardness	Manganese (Mn)	Imparts wear resistance
Vanadium (V)	Induces fie-grain distribution	Titanium (Ti)	Increases abrasion resistance
Chromium (Cr)	Improves resistance to corrosion and oxidation	Nickel (Ni)	Improves tensile strength and toughness
Molybdenum (Mo)	Increases hardenability	Phosphorous (P)	Enhances machinability

Table 2.2 Steel and cast-iron classification

Variant		Properties	Application
Plain carbon and low alloy steels	**Low carbon steels** (up to 0.30% C)	Good formability and weldability, low strength, low cost.	Deep drawing parts, chain, pipe, wire, nails, some machine parts.
	Medium carbon steels (0.30 to 0.60% C)	Good toughness and ductility, relatively good strength, may be hardened by quenching	Shafts, axles, screws, cylinders, crankshafts, heat treated machine parts.
	High carbon steels (0.60 to 1.00% C)	High strength, hardness and wear resistance, moderate ductility.	Rolling mills, rope wire, screw drivers, hammers, wrenches, band saws.
	Ultra-high carbon steels (1.25 to 2.06% C)	Very high strength, hardness and wear resistance, poor weldability low ductility.	Punches, shear blades, springs, milling cutters, knives, razors.
Stainless steels (high alloy steels)	**Austenitic stainless steels** (200 and 300 series)	Highest corrosion resistance, weldability and ductility.	Chemical equipment, food equipment, kitchen sinks, medical devices, heat exchangers, parts of furnaces and ovens.

Variant		Properties	Application
	Ferritic stainless steels (400 series)	Low cost and best machinability, these steels are ferromagnetic. Ductility and formability of ferritic steels are low. Corrosion resistance and weldability are moderate. Resistance to the stress corrosion cracking is high.	Decorative and architectural parts, automotive trims and exhausting systems, computer floppy disc hubs, hot water tanks.
	Martensitic stainless steels **(**400 and 500 series)	Poor weldability and ductility. Corrosion resistance of these steels is moderate.	Turbine blades, knife blades, surgical instruments, shafts, pins, springs.
	Austenitic-ferritic (Duplex) stainless steels (contain increased amount of chromium (18% -28%))	High resistance to the stress corrosion cracking and to chloride ions attack. These steels are weldable and formable and possess high strength.	Desalination equipment, marine equipment, petrochemical plants, heat exchangers.
	Precipitation hardening stainless steels	High strength, good weldability and fair corrosion resistance. They are magnetic.	Pump shafts and valves, turbine blades, paper industry equipment, aerospace equipment.
Cast irons	**White cast irons**	Hard and brittle highly wear resistant cast irons consisting of pearlite and cementite.	Brake shoes, shot blasting nozzles, mill liners, crushers, pump impellers and other abrasion resistant parts.
	Grey cast irons	High compressive strength, fatigue resistance and wear resistance. Presence of graphite in grey cast irons impart them very good vibration damping capacity.	Gears, flywheels, water pipes, engine cylinders, brake discs, gears.

Variant		Properties	Application
	Malleable cast irons	Good ductility and machinability. Ferritic malleable cast irons are more ductile and less strong and hard, than pearlitic malleable cast irons.	Parts of power train of vehicles, bearing caps, steering gear housings, agricultural equipment, railroad equipment.
	Ductile cast irons	High ductility, good fatigue strength, wear resistance, shock resistance and high modulus of elasticity.	Automotive engine crankshafts, heavy duty gears, military and railroad vehicles.

2.3.1.2 Non-ferrous alloys

Non-ferrous alloys have been recognized from many industries for their undoubted advantages. Since non-ferrous are much lighter than ferrous ones, they are mainly preferred in configurations where strength is needed, but weight is a factor (e.g. the aerospace industry). As their name reveals they don't contain iron, which guarantees higher resistance to rust and corrosion. And last, but not least non-ferrous metals are not-magnetic and therefore best choice for electronics and wiring. Some important non-ferrous alloys are given in Table 2.3.

Table 2.3 Non-ferrous alloys classification

Variant		Properties	Application
Aluminium alloys	**Wrought aluminium alloys**	Ductile and moderately strong	Deep drawn parts, sheets, foil, tubes, wire, extruded parts, pressure vessels,

Variant		Properties	Application
	Cast aluminium alloys	Low melting point, good fluidity of most of the alloys, capability to control grain structure, good surface finish, low solubility of gases (except Hydrogen), ability to be strengthened by heat treatment.	cylinder heads for automotive and aircraft engines, pistons for diesel engines, exhausting system parts.
Copper alloys	**Wrought Brasses**	Less ductile and they have good formability only in hot state and poor formability in cold state.	Architectural and decorative applications, automotive radiators and tanks, screw parts, heat exchangers tubes, marine hardware.
	Wrought Bronzes	Excellent formability in cold state and good corrosion resistance.	Sleeve bushings, springs, clutch discs, fittings, wear resistant parts, electrical contacts, sliding bearings, screw products, valve parts, pressure vessels, pump body castings, impellers for chemical plants.
	Cast Brasses	Machinability of the Cast Brasses (particularly alloyed with lead) is very good.	Heating and cooling equipment, electrical equipment, parts of valves, pump impellers, pipe fittings, small gears, gas line fittings.
	Cast Bronzes	Very high mechanical strength combined with good ductility and good corrosion resistance.	Flanges, gears, shafts, cams, heavy load bearings, bushings, screw down nuts, gun mounts, hydraulic cylinder parts, pump components, impellers,

Variant		Properties	Application
Magnesium alloys	**Wrought magnesium alloys**	Low mechanical strength combined with excellent weldability and corrosion resistance.	Parts of power train of vehicles, bearing caps, steering gear housings.
	Cast magnesium alloys	Good mechanical strength combined with excellent ductility and impact toughness.	Automotive wheels and structures, components of electric instruments and motors, plastic moulds, aircraft engines, airframe structures.
Nickel alloys	**Commercially pure nickel alloys** (contains at least 99% of nickel (Ni))	Good corrosion resistance in oxidizing media and excellent corrosion resistance in alkaline solutions, non-oxidizing acids and halogen gases, high thermal and electrical conductivity.	Food processing equipment, chemicals containers, caustic handling equipment, electrical and electronic parts, anodes for electroplating, heat exchangers, fluorescent lamps, decorative and protective coating.
	Nickel-copper alloys (nickel base alloys containing 29-33% of copper (Cu))	High corrosion resistance in acids and alkalis, high mechanical strength combined with good ductility and low coefficient of thermal expansion, machinability of the alloys is poor.	Chemical processing equipment, valve stems, springs, pumps, shafts, fittings, heat exchangers, screw machine products, marine equipment.
	Non-heat-treatable nickel-chromium-iron alloys (Nickel base alloys, containing 15-22% of chromium (Cr) and up to 50% of iron (Fe) as the major alloying elements.)	Very good mechanical strength and high resistance to Creep combined with excellent corrosion resistance in chlorine-ion solutions, good machinability, weldability and workability	gas turbine parts, evaporator tubes, chemical processing equipment, distillation columns, combustion systems, heat exchangers, steam generator tubing of nuclear power plants.

Variant		Properties	Application
	Heat-treatable nickel-chromium-iron alloys (Nickel base alloys, containing 15-22% of chromium (Cr) and up to 33% of iron (Fe) as the major alloying elements.)	High mechanical strength and high resistance to Creep at temperatures up to 1500°F (815°C) combined with good Corrosion and oxidation resistance.	Turbines components (blades, rings, combustors, flame holders, discs), parts of nuclear steam generators, die casting cores, hot working tools, exhaust valves in internal combustion engines.
Titanium alloys There are two crystallographic forms of titanium: **α-titanium**, in which atoms are arranged in Hexagonal Closest Packing (HCP) crystal lattice; **β-titanium**, in which atoms are arranged in Cubic Body Centered (BCC)crystal lattice;	**Commercially pure and low alloyed titanium alloys** (Consist of grains of α-phase and dispersed spheroid particles of β-phase. Small amounts of iron (Fe), present in the alloys, stabilize β-phase.)	Low mechanical strength, high ductility and formability and good corrosion resistance.	Airframes, plate-type heat exchangers, condenser and evaporator tubing, aircraft engine parts, surgical implants, gas compressors.
	Titanium alpha and near-alpha alloys (consist entirely of α-phase. They contain aluminium (Al) as the major alloying element, stabilizing α-phase)	Good fracture toughness and creep resistance combined with moderate mechanical strength, which is retained at increased temperatures up to 1100 ºF (600ºC).	High pressure cryogenic vessels, aircraft engine compressor blades, missile fuel tanks and structural parts.
	Titanium alpha-beta alloys (containing 4-6% of β-phase stabilizers: molybdenum (Mo), vanadium (V), tungsten (W), tantalum (Ta), silicon (Si))	High tensile strength and fatigue strength, good hot formability and creep resistance up to 570ºF - 800ºF (300ºC - 425ºC).	Pressure vessels, blades and discs of aircraft turbines, aircraft hydraulic tubing, rocket motor cases, cryogenic parts, marine components.

Variant		Properties	Application
	Titanium beta alloys (contains β-phase stabilizers: molybdenum (Mo), vanadium (V), tungsten (W), tantalum (Ta), silicon (Si))	Heat-treatable to very high strength and have good hot formability, ductility and fatigue strength of the alloys in heat-treated conditions are low.	Aerospace components, high-strength fasteners, torsion bars, high-strength aircraft sheets, burn-resistant aircraft engine parts.

2.3.1.3 Ceramics

Ceramics are non-metallic, inorganic, amorphous solids and are mostly metallic oxides. They have poor tensile strength and are brittle. They can be either crystalline or non-crystalline. Many ceramics are workable in extremely low (cryogenic) temperature range, while many others are able to sustain high temperatures. There are various classification systems of ceramic materials, which may be attributed to one of two principal categories: application base system or composition base system. According to composition base system, these are oxide ceramics, silicate ceramics, carbide ceramics, and nitride ceramics given in Table 2.4.

Table 2.4 Ceramics classification

Variant		Properties	Application
Oxide ceramics	**Alumina ceramics** (the most important, widely used and cost-effective oxide ceramic material. The technical alumina ceramics contain at least 80% of aluminium oxide (Al_2O_3))	High mechanical strength and hardness, high wear resistance, high stiffness, excellent insulating properties, low coefficient of thermal expansion, good fracture toughness, good thermal conductivity, Good biocompatibility.	Insulators, capacitors, resistors, furnace tubes, sealing refractory parts, foundry shapes, ballistic armors, laboratory equipment, bio-ceramic parts for orthopaedic and dental surgery, bearings.

Variant		Properties	Application
	Magnesia Ceramics (ceramic materials consisting of at least 90% of Magnesium Oxide (MgO))	High thermal stability, high resistance to molten metals (iron, steel, aluminium), good corrosion resistance even at high temperatures, good insulating properties, Good thermal conductivity, infrared transparency	High temperature crucibles, thermocouple tubes, heating elements, foam ceramic filters for molten metal, insulators, steel making refractories,
	Zirconia Ceramics (ceramic materials consisting of at least 90% of Zirconium Dioxide (ZrO_2))	Low thermal conductivity, high fracture toughness, very high flexural strength and hardness, high chemical resistance, good wear resistance.	Cutting tools, balls and seats for ball valves, thread and wire guides, pump seals, impellers and shaft guides, engine parts
Silicate ceramics	**Technical Porcelain** (consists of silica (SiO_2) and alumina (Al_2O_3))	High mechanical strength, excellent dielectric properties, high chemical resistance.	Technical porcelain is generally used in electrical engineering as a good insulator.
	Magnesium Silicates (consist of silica (SiO_2), magnesia (MgO) and some alumina (Al_2O_3))	High mechanical strength, good dielectric properties, very low loss factor.	Sockets, control housings, insulating beads, low-voltage power fuses and base plates, parts of water heaters, pipes of heating element, spark protectors and catalyst carriers in automobiles.
	Mullite Ceramics (consist of mullite ($3Al_2O_3*2SiO_2$), alumina (Al_2O_3) and glass (SiO_2))	High strength, high thermal shock resistance, relatively low thermal expansion, good creep resistance.	High temperature parts, kiln furniture, slide gates, ladles for molten metal, protection tubes for thermocouples, glass industry refractories.
Carbide ceramics	**Boron carbide (B_4C)**	excellent wear and abrasion resistance, high	Used as an abrasive in polishing and lapping

Variant		Properties	Application
		hardness, high elastic modulus, and low density.	applications, nozzles for slurry pumping, grit blasting and in water jet cutters, absorbent for neutron radiation arising in nuclear power plants, ballistic armour
	Silicon carbide (SiC)	Excellent thermal shock resistance, good wear resistance, low coefficient of thermal expansion, high thermal conductivity, Semi-conducting properties.	Melting crucibles, kiln furniture, heat exchangers, burner nozzles, wear plates, heating elements, milling balls, wear plates.
	Tungsten carbide	High mechanical strength, high melting point, high thermal conductivity, excellent hardness	Cutting tool, rock drill bits, neutron reflector, surgical instruments, jewellery.
Nitride ceramics	**Silicon Nitride (Si_3N_4)** (produced from a mixture of fine sub-micron powder of Silicon Nitride and binders (oxides))	High fracture toughness, high mechanical strength and hardness even at high temperatures, good creep resistance, high thermal shock resistance, high wear resistance, low coefficient of thermal expansion.	Tools, turbine blades, bearing balls and rollers, kiln furniture, valves and turbocharger rotors for engines,
	Aluminium Nitride (AlN) (produced by sintering of Aluminium Nitride powder with or without binders)	Very high thermal conductivity, thermal expansion similar to silicon, good dielectric properties, good corrosion resistance, stability in semiconductor processing atmospheres.	Substrates for semiconductors, housing and heat sinks, power transistors bases, IC packages, microwave device packages.

Variant		Properties	Application
	Silicon Aluminium Oxynitride (SIALON) (produced from a mixture of Silicon Nitride powder and Aluminium Oxide powder)	Low wettability by non-ferrous molten metals, high toughness.	Metallurgical applications, cutting tools.

2.3.1.4 Polymers

A compound consisting of long-chain molecules, each molecule made up of repeating units connected together. The word polymer is derived from the Greek words poly, meaning many, and meros (reduced to mer), meaning part. As engineering materials, it is appropriate to divide them into the following three categories: Thermoplastic polymers, Thermosetting polymers, and Elastomers shown in Table 2.5.

Table 2.5 Polymers classification

Variant		Properties	Application
Thermoplastics	**Low Density Polyethylene (LDPE)**	Good impact strength, good chemical resistance, good flexibility, poor UV resistance, good hot formability.	Packaging films (general purpose, shrink, lamination), containers, cable insulation, chemically resistant linings.
	High Density Polyethylene (HDPE)	Good strength, good impact strength, good chemical resistance, good stiffness, poor UV resistance.	Packaging films, heavy duty shrink film, pipes, containers, bags, blown bottles.

Variant		Properties	Application
	Polyvinyl Chloride (PVC)	Rigid or flexible (depending on formulation), high density, environmentally durable.	Drainage pipes, water service pipes, bottles, window frames, wire and cable insulation, resilient floors, automotive interiors.
	Polyethylene Terephthalate (PET)	High strength, high rigidity, excellent processing properties, good weather resistance.	Window wiper holders, exterior mirror housing, electrical plugs and sockets, under bonnet parts, wire and cable insulation, insulation tapes.
	Polytetrafluoroethylene (PTFE)	Excellent anti-friction properties, excellent chemical resistance, excellent high temperature resistance, very good low temperature toughness, very good weather resistance, high dielectric strength (up to 100 kV/mm).	Bearings pump parts, non-sticking coating, tubes for chemicals, thread seal tape, high and low temperature electrical and medical applications.
Thermosetting polymers	**Epoxy (EP)**	High strength, excellent corrosion resistance, excellent dimension stability, excellent toughness, good dielectrical properties, good adhesion.	Electrical moulding, electrical circuits (reinforced with glass fibre), protective coating, pipe fittings, adhesive, rocket motors components (reinforced with glass-filament-wound).
	Phenolics (PF)	High hardness, brittle, excellent thermal stability, very good dielectric properties.	Wiring devices, lamps holders, switchboards, bottle caps, automotive parts, plugs and switches, motor housings,

Variant		Properties	Application
	Unsaturated Polyester (UP)	Excellent rigidity, high strength, high creep strength, excellent dielectrical properties, good chemical resistance.	Fiberglass boats, building panels, fans, fences, helmets, compressor housing, auto body components.
	Alkyds (AMC)	Good rigidity, high toughness, good colour stability, good dimensional stability, good dielectric properties.	Circuit breakers, switch gears, coloured parts.
Elastomers	**Polyisoprene (natural rubber)**	Excellent abrasion resistance, Excellent tear strength, excellent resilience, excellent low temperature flexibility, excellent dielectric strength.	Automobile tires, gaskets, hoses.
	Butyl (Isobutene-Isoprene)	Very low permeability to air, excellent resistance to acids and alkali, excellent heat resistance, poor resistance to solvents, fuel, oil.	Inner lining of automobile tires, steam hoses and diaphragms.
	Neoprene (Chloroprene)	Excellent abrasion resistance, good resistance to oil, fuel and petroleum-based solvents, excellent resistance to ozone, very good resistance to sunlight.	Oil and crude oil hoses, gaskets, diaphragms, lining of chemical vessels.

Variant		Properties	Application
	Ethylene-Propylene (EPDM)	Excellent resistance to sunlight and heat, excellent resistance to water and steam, excellent low temperature flexibility, good dielectric strength, good abrasion resistance.	Electrical insulation, shoe soles, hoses, conveyor belts.

2.3.1.5 New and advanced materials

New and advanced materials do not emphasis on the materials other than traditional materials like metallic alloys or ceramics. It is based on increasing knowledge of the microscopic properties of matter and on mastering industrial reproduction processes of these microscopic properties, enabling different materials to be combined to make new alloys or composites and to customize their properties. The concept of 'new and advanced materials' used here thus refers to substances possessing compositions, microstructures, properties, performances, or application potentials derived from the industrial reproduction of their microscopic properties. Every traditional material can become "new" through the adoption of advanced shaping and manufacturing techniques and processes permitting the control of its microscopic structure. Some of the advanced materials are described in Table 2.6.

Table 2.6 New advanced materials classification

Variant		Properties	Application
Composite materials (a material composed of two or more distinct phas es: matrix	**Polymer Matrix Composites (PMC)** (consisting of a polymer (resin) matrix combined with a fibrous reinforcing dispersed phase)	High tensile strength, high stiffness, high Fracture Toughness, good abrasion resistance, Good puncture resistance, good corrosion resistance, low cost.	Boat bodies, canoes, kayaks, automotive parts, radio-controlled vehicles, sport goods, brake and clutch linings.

Variant		Properties	Application
phase and dispersed phase)	**Ceramics Matrix Composites (CMC)** (consisting of a ceramic matrix combined with a ceramic (oxides, carbides) dispersed phase)	High mechanical strength even at high temperatures, high thermal shock resistance, high stiffness, high toughness, high thermal stability, low density, high corrosion resistance even at high temperatures.	Combustion liners of gas turbine engines, heat exchangers, rocket propulsion components, high performance braking systems, turbojet engine components.
	Metal Matrix Composites (MMC) (consisting of a metallic matrix combined with a ceramic (oxides, carbides) or metallic (lead, tungsten, molybdenum) dispersed phase)	Low density, high stiffness (modulus of elasticity), high wear resistance, good strength even at elevated temperatures, good creep resistance.	Automotive parts (pistons, pushrods, brake components), bicycles, golf clubs, gearboxes, transmissions, compressors and engine.
Smart materials	**Piezoelectric materials**	When a piezoelectric material is deformed, it gives off a small but measurable electrical discharge. Alternately, when an electrical current is passed through a piezoelectric material, it experiences a significant increase in size.	Piezoelectric materials are most widely used as sensors in different environments; they are often used to measure fluid compositions, fluid density, fluid viscosity, and the force of an impact.
	Electro-Rheostatic(ER) and Magneto-Rheostatic (MR) materials (these materials are fluids, which can experience a dramatic change in their viscosity)	These fluids can change from a thick fluid to nearly a solid substance within the span of a millisecond when exposed to a magnet or electric field. The effect can be completely reserved just as quickly when the field is removed.	MR fluids are used in car shocks, damping washing machine vibration, prosthetic limbs, exercise equipment, and surface polishing of machine parts. ER fluids are developed for use in clutches and valves as well as engine mounts designed to reduce noise and vibration in vehicles.

Variant		Properties	Application
	Shape memory alloy (NiTi, CuZnAl, CuAlNi)	Two very unique properties, namely, pseudo elasticity and the shape memory effect i.e., when deformed returned to its pre-deformed state when heated.	Automotive thermostats, Aircraft flap/slat adjusters, electrical circuit breakers, robot actuators, vibration dampers, keyhole surgery instruments, autofocus (AF) actuator for a smart phone.
Functionally graded materials	**Chemical Composition Gradient FGM**	The properties in FGMs are not uniform across the entire material and depend on the spatial position of the material in the bulk structure of the material. Functionally graded materials are designed with varying properties that include changing chemical properties, changing mechanical, magnetic, thermal, and electrical properties.	rocket engine components, spacecraft truss structure, heat exchange panels, leading edge of missiles, diesel engine pistons, spark plugs, flywheels, inner wall of nuclear reactors,
	Porosity Gradient FGM		Human body parts including the bones and the teeth, skeletal replacement implants,
	Microstructure Gradient FGM		Body of bullet-proof vehicles, bullet-proof vests, armour plates.
Biomedical materials	**Polymeric biomaterials**	Synthetic Flexibility, controllable mechanical properties, biomolecule compatibility, highly processable, many commercial vendors available	Blood vessel prosthesis, artificial heart, catheter, tissue engineering, drug delivery.
	Bioceramics	High compression strength, wear & corrosion resistance, can be highly polished, bioactive/inert, low	Pacemakers, kidney dialysis machines, and respirators, dental and bone implants, artificial total hip, knee,

Variant		Properties	Application
		strength in tension, low fracture toughness.	shoulder, elbow, wrist, Intramedullary nails.
	Metallic biomaterials	High strength, fatigue resistance, wear resistance, easy fabrication, easy to sterilize, shape memory.	Dental implant, joint replacement (hip, knee), bone plate, heart valve.
	Biocomposites	High strength, wear resistance, corrosion resistance.	Total hip replacement, spine instrumentation, bone plates, tendon/ligament, intramedullary nails.

2.3.1.6 Designation of carbon and alloy steels

American Iron and Steel Institute (AISI) together with Society of Automotive Engineers (SAE) have established four-digit (with additional letter prefixes) designation system:

AISI 1XXX

First digit 1 indicates carbon steel (2-9 are used for alloy steels);

Second digit indicates modification of the steel.

0 - Plain carbon, non-modified; 1 – Resulfurized; 2 - Resulfurized and rephosphorized; 5 - Non-resulfurized, Mn over 1.0%

Last two digits indicate carbon concentration in 0.01%.

Example: **AISI 1040** means non-modified carbon steel, containing 0.40% of carbon.

AISI/SAE classification divide alloy steels onto groups according to the major alloying elements:

- Low alloy steels (alloying elements ≤ 8%);
- High alloy steels (alloying elements > 8%).

According to the four-digit classification SAE/AISI system for low alloy steels:

First digit indicates the class of the alloy steel:

2- Nickel steels; 3- Nickel-chromium steels; 4- Molybdenum steels; 5- Chromium steels; 6- Chromium-vanadium steels; 7- Tungsten-chromium steels; 9- Silicon-manganese steels.

Second digit indicates concentration of the major element in percent (1 means 1%).

Last two digits indicate carbon concentration in 0.01%.

Example: **AISI 4130** means alloy molybdenum steel, containing 1% of molybdenum and 0.30% of carbon.

2.3.2 Attribute: Engineering material properties

A designer or an engineer has to take into account a large number of factors when selecting materials. These factors range from mechanical and electrical properties to corrosion resistance and surface finish. In mechanical design it is the mechanical properties which are of greatest importance. There are a wide range of material properties which can be considered in mechanical design some of which are density, strength, elasticity, creep, toughness, hardness etc. which are also shown in Table 2.7. Appropriate combinations of these properties will dictate the suitability of a material for a specific application.

Table 2.7 Design-limiting material properties

Class	Property	Definition	Units
General	Cost		\$/kg
Physical	Density	Mass per unit volume	kg/m^3
	Molecular weight	Mass per mole of a substance.	Kg/mole
	Permeability	Amount of gas or liquid that can pass through a material.	m^2
Mechanical	Elastic moduli (Young's, shear, bulk)	Ratio of stress to strain during elastic (nonpermanent) deformation.	GPa, MPa

Class	Property	Definition	Units
	Yield strength	Stress at the onset of plastic deformation during tensile loading	MPa
	Ultimate strength	Maximum stress that can be supported before fracture during tensile loading.	MPa
	Shear strength	Maximum stress that can be supported before fracture during shear loading.	MPa
	Poisson's ratio	Ratio of lateral strain to the linear strain	--
	Toughness	Total energy required to cause fracture.	kJ/ m^2
	Fracture toughness	Maximum stress that can be applied before catastrophic crack propagation.	MPa $m^{1/2}$
	Hardness	Amount of deformation induced in the surface as the result of an applied indentation load.	BHN
	Fatigue strength	Relationship between applied cyclic stresses and the number of cycles to failure.	MPa
	Creep resistance	Resistance to deformation while stressed at elevated temperatures.	°K
Thermal	Thermal conductivity	Rate at which heat flows through a material.	W/mK
	Thermal diffusivity	Rate of heat transfer of a material from the hot side to the cold side.	m^2/s
	Specific heat	Amount of heat required to raise a substance's temperature 1°C.	J/kg-K
	Melting point	Temperature at which a material changes from solid to liquid.	K
	Glass transition temperature	Temperature of the transition from solid to very viscous liquid.	K
	Coefficient of thermal expansion	Thermal strain per degree of temperature change.	$°K^{-1}$

Class	Property	Definition	Units
	Thermal shock resistance	Change in electrical resistance per degree of temperature change.	W/m
Electrical	Conductivity	Ability of electrons to flow through a material.	S/m
	Dielectric strength	Maximum voltage gradient that a material can withstand before breakdown.	V/m
	Dielectric constant (permittivity)	Ability of a material to resist the formation of an electric field within it.	F/m
	Magnetic permeability	Change in magnetic induction in response to a magnetizing force.	H/m or kg-m/s^2
Optical	Refractive index	Ratio of the velocity of light in a vacuum to its velocity in a material.	--
	Transmission	Amount of light of a specific wavelength that can pass through a material.	
Wear	Archard wear constant	Rate of wear due to frictional force	m^2/N
Corrosion/ Oxidation	Corrosion rate	The speed at which any given metal deteriorates in a specific environment.	mm/year
	Parabolic rate constant	Rate of diffusion of reactants (metal cations and O_2 anions) through the oxide layer	m^2/s

2.3.3 Alternative generation

Design methods all aim to lead a designer to one or more good solutions to a design problem. There is no one way method to generate the alternatives. Significantly, from the Suh's design axioms (Suh, 1990) alternatives can be generated by mapping the functional requirements with the design parameters (Dieter, 1983). Ashby (1999) formalized the Suh's design axioms in noetic way and proposed a chart known as Ashby's Chart to generate the potential materials under specific condition. It can be regarded as a *satisficing* (Simon, 1988) model in the field of material selection in engineering design where he showed the performance of the machine elements is the function of the material properties. The chart is a log-log plot whose abscissa and ordinate are that material properties, but we are left in this chart with many alternatives or a set of potential alternatives. Due to the lack of information, uncertainty takes place in the design-formulation and thereby alternative generation in material selection.

2.3.3.1 Suh's design axioms

Suh (19**) defined the engineering design process as a constant interplay between 'what' (customers' perspective) we want to achieve and 'how' (designers' perspective) we want to achieve it. The former objectives are always stated in the functional domain, while the latter (the physical solution) is always generated in the physical domain. Suh has proposed two conceptually simple design axioms.

Axiom 1: *The independence axiom*

Maintain the independence of functional requirements (FRs)

Axiom 2: *The information axiom*

Minimize the information content

Axiomatic Design operates with a model of the design process that uses state spaces to describe different steps in generating design concepts.

- Consumer Requirements (CRs): Variables that characterize the design in the consumer domain. CRs are the customer needs and wants that the completed design must fulfil.
- Functional Requirements (FRs): Variables that characterize the design in the functional space. These are the variables that describe the intended behaviour of the device.

- Design Parameters (DPs): Variables that describe the design in the physical solution space. DPs are the physical characteristics of a particular design that has been specified through the design process.

The design parameters (DPs) depict a physical embodiment of a feasible design that will fulfil the FRs. As Figure 2.4 illustrates, the design process consists of mapping the FRs of the functional domain to the DPs of the physical domain to create a product, process, system, or organization that satisfies the perceived societal need. Note that this mapping process is not unique. Therefore, more than one design may result from the generation of the DPs that satisfy the FRs. However, the design axioms provide the principles that the mapping techniques must satisfy to produce a good design, and they offer a basis for comparing and selecting designs.

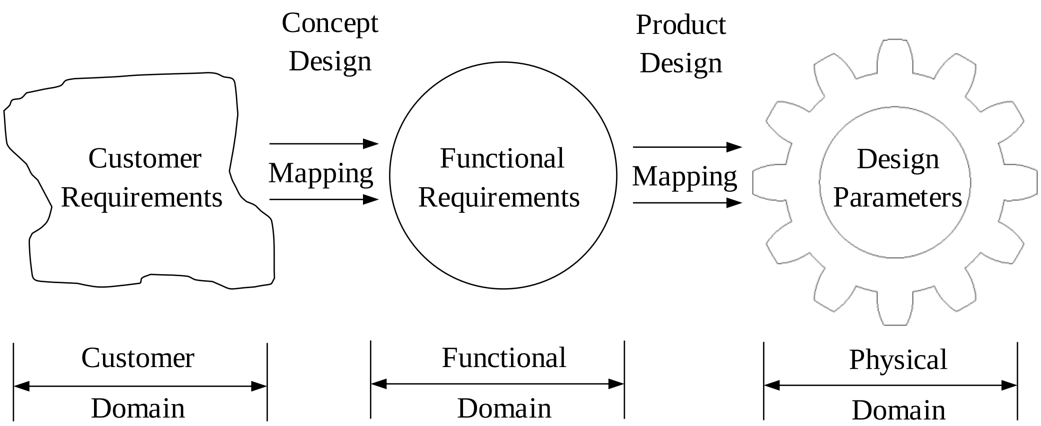


Figure 2.4 Axiomatic Design perspective

It can be concluded that the functional requirements (FRs) is the function of design parameters (DPs). In typical mechanical design, the design DPs are the function of material (attributes) and shape (geometry) and can be represented as,

$$FR = f\{\text{material}(\text{attribute}), \text{shape}(\text{geometry})\} \tag{2.1}$$

In engineering design, material attributes and shape are interlinked and well trade off between material and shape generates the wide range of alternatives (in terms of material or in terms of geometry) to fulfil the required functional requirement.

2.3.3.2 Ashby's chart

Any engineering component has one or more functions: to support a load, or to store energy etc. In designing the component, the designer has an objective: to make it as cheap as possible, perhaps, or as light, or as safe, or perhaps some combination of these. This must be achieved subject to constraints: that certain dimensions are fixed, that the component must carry the given load or to store energy without failure, and many more. Ashby (1999) translated the design requirements for a component into,

- the function of the component,
- the objectives the designer has selected to optimize its performance, and
- the constraints it must meet.

Structural elements are components which perform a physical function: they carry loads, transmit heat, store energy and so on; in short, they satisfy functional requirements. The functional requirements are specified by the design: a tie must carry a specified tensile load; a spring must provide a given restoring force or store a given energy, a heat exchanger must transmit heat with a given heat flux, and so on. The design of a structural element is specified by three things as

$$p = f\left[\begin{pmatrix}\text{Functional}\\ \text{requirements}, F\end{pmatrix}, \begin{pmatrix}\text{Geometric}\\ \text{parameters}, G\end{pmatrix}, \begin{pmatrix}\text{Material}\\ \text{properties}, M\end{pmatrix}\right] \tag{2.2}$$

$$\text{or } p = f\,(F\,.G, M)$$

where, p describes some aspect of the performance of the component: such as mass, or volume, or cost, or life. f means 'a function of'. Optimum design is the selection of the material and geometry which maximize or minimize p. The three groups of parameters in equation (3.1) are said to be separable when the equation can be written

$$p = f_1(F) f_2(G) f_3(M) \tag{2.3}$$

where, f_1, f_2 and f_3 are separate functions which are simply multiplied together. The factor $f_3(M)$ is known as 'material index'. Each combination of function, objective and constraint leads to a material index and is a combination of material properties which characterizes the performance of a material in a given application. For example, a light stiff beam (Figure 2.5) can be characterized as,

Function: Beam

Objective: Minimize the mass

Constraints: Specified stiffness and Specified length

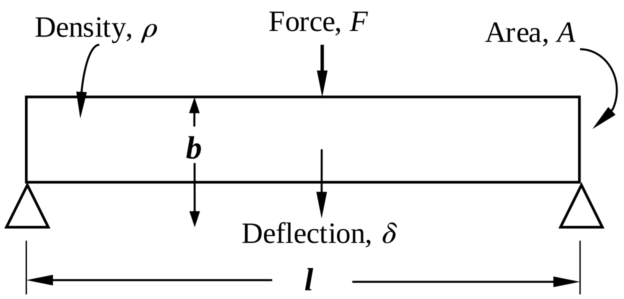


Figure 2.5 A beam of square cross-section, loaded in bending

The equation of objective function is given by,

$$m = \left(\frac{12S}{C_1 l}\right)^{1/2} l^3 \frac{1}{\left(\frac{E^{1/2}}{\rho}\right)} \tag{2.4}$$

Where, *E* is Young's modulus, C_1 is a constant which depends on the distribution of load and *I* is the second moment of the area of the section, *S* is the stiffness and *l* is the length. Comparing the equation (3.7) with (3.1), the performance *m* can be characterized as functional requirements, geometry and material. The best materials for a light, and stiff beam are those with large values of the material index, $M = \frac{E^{1/2}}{\rho}$. When width is specified and height is free, then $I = \frac{A^3}{12b^2}$. which gives $M = \frac{E^{1/3}}{\rho}$. When height is specified and width is free, then $I = \frac{Ab^2}{12}$, which gives, $M = \frac{E}{\rho}$.

Figure 2.6 shows, the modulus *E*, plotted against density *p*, on log scales. The material indices $\frac{E}{\rho}$, $\frac{E^{1/2}}{\rho}$ and $\frac{E^{1/3}}{\rho}$ can be plotted onto the figure. For the condition, $\frac{E}{\rho} = C$, taking logs both side which gives,

$$\log E = \log \rho + \log C \tag{2.5}$$

which is a set of parallel straight lines of slope 1 on a plot of $\log E$ against $\log \rho$; each line corresponds to a value of the constant *C*.

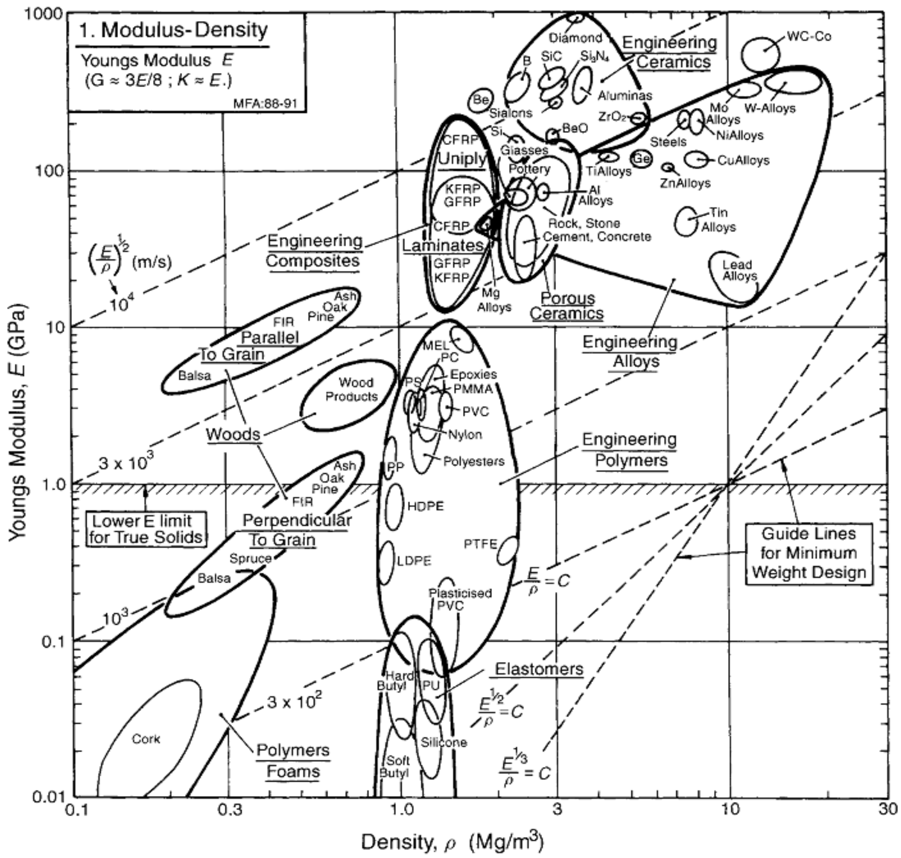


Figure 2.6 Ashby's materials chart (Young modulus Vs Density).

These lines are referred as selection guide lines. They give the slope of the family of parallel lines belonging to that index. It is now easy to read off the subset materials which optimally maximize performance for each loading geometry. All the materials which lie on a line of constant E/ρ perform equally well as a light, stiff beam but these are not the interest of optimization concept; those above the line are qualified candidate materials, those below, are disqualified materials for the particular design requirements. There are many charts like this according to the design requirements those are applied to generate the alternatives.

2.3.4 Alternatives evaluation

When a range of alternative designs has been created, the designer is then faced with the problem of selecting the best one. At various points in the design process there may also be decisions of choice to be made between alternative sub-solutions or alternative features that might be incorporated into a final design. Choosing between alternatives is therefore a common feature of design activity. The goal of any materials selection endeavour is to choose the preeminent material for a given application that requires a finite set of alternatives, a set of well-defined objectives (criteria) and performance ratings of i^{th} alternative with j^{th} criterion to form a decision matrix. Due to the importance of reliability in product design, for contemporary materials selection systems, the suitability of candidate materials is evaluated against multiple criteria rather than considering a single factor. An evaluation involves a comparison of the alternatives with an imaginary ideal solution, a 'rating' or degree of approximation to that ideal or the comparison may take place among alternatives. There are numerous decision matrix techniques have been developed in which the Pugh method and Pahl and Beitz method are very popular due to its simplicity as the alternative evaluation takes place at the conceptual stage of the design process. At the same time multiple attribute decision-making (MADM) methods is becoming very popular to select the most appropriate material for high technology components used in biomedical, aerospace and nuclear industries.

2.3.4.1 Decision-matrix method (Pugh's method)

The Pugh method is the simplest among the above-mentioned methods, with all criteria qualitatively evaluated with equal weight (Otto & Wood, 2001). Selection is based on qualitative comparison to a datum or reference alternative. The Pugh method is useful early in design, since it requires the least amount of detailed information. It is also useful in redesign, because the current material design serves automatically as the datum. The sequential steps are followed as:

Step 1: Alternatives are listed as column headers in the decision matrix. The first column lists criteria. One alternative is selected to be the datum, perhaps the alternative that is best known and understood, or the alternative that is intuitively considered to be the best.

Step 2: Each blank cell is associated with a row (criterion) and a column (alternative). Each alternative is compared with datum or reference alternative in respect of each criterion and performance are rated better (+), worse (-), or the same (S) and each cell in the decision matrix is filled with appropriate symbol.

Step 3: The next step is to create three more rows at the bottom of the matrix with the row headers +, −, and S. For each alternative except the datum, sum the number of +, −, and S responses in the appropriate blank. Analyse the results. For each alternative, consider whether some modification might mitigate its weak points relative to the datum. If so, modify the alternative, enter it as a new column on the decision matrix, and evaluate it relative to the datum. Retain the unmodified alternative.

For example, let us consider a set of ferrous gear materials under the specific condition those are listed in Table 2.8 following the Step 1. Generating these alternatives (gear materials) will be further discussed in Chapter 4 in details. The overall ratings of the materials are tabulated in Table 2.9 following the Step 2 and Step 3 where ductile cast iron (Grade 80-55-06) is the best material. At the same time molybdenum steel (AISI 4130) and grey cast iron (ASTM class 60) can also be considered as a good gear material.

Table 2.8 Performance ratings of the gear materials

Criteria	AISI 4130	AISI 1040	AISI 304	AISI 405	ASTM class 60	Grade 60-40-18	Grade 65-45-12	Grade 80-55-06
Ultimate tensile strength in MPa	560	515	515	448	431	416	464	559
Brinell hardness number	156	149	147	150	285	167	167	192
Density in g/cc	7.85	7.84	7.80	7.80	7.30	7.10	7.10	7.10
Cost in US $/kg	2.00	1.30	2.85	3.50	3.30	4.00	4.00	4.00

Table 2.9 Pugh ratings of the gear materials

Criteria	AISI 4130	AISI 1040	AISI 304	AISI 405	ASTM class 60	Grade 60-40-18	Grade 65-45-12	Grade 80-55-06
Ultimate tensile strength in MPa	S	−	−	− −	− −	− −	− −	S
Brinell hardness number	S	−	−	−	+ +	−	−	+ +
Density in g/cc	S	S	S	S	+	+	+	+
Cost in US $/kg	S	+	−	− −	− −	− −	− −	− −
$\sum +$	0+	1+	0+	0+	3+	1+	1+	3+
$\sum -$	0 −	2 −	3 −	5 −	4 −	5 −	5 −	2 −
$\sum$	0	1 −	3 −	5 −	1 −	4 −	4 −	1+
Rank	2	4	5	8	3	6	7	1

In using the Decision Matrix there are two possible types of comparisons. The first type is *absolute* in that each alternative concept is directly (i.e., absolutely) compared with some target set by a criterion. The second type of comparison is *relative* in that alternative concepts are compared with each other using measures defined by the criteria. In choosing to use a datum the comparison is relative. However, many people use the method for absolute comparisons. Absolute comparisons are possible only when there is a target. Relative comparisons can be made only when there is more than one option.

2.3.4.2 Weighted objectives method (Pahl and Beitz method)

In order to make any kind of evaluation it is necessary to have a set of criteria, and these must be based on the design objectives. These objectives should have been established at an early point in the design process. The objectives will include technical and economic factors, user requirements, safety requirements, and so on. The Pahl and Beitz method allows quantitative evaluation of alternatives using weighted criteria (Pahl & Beitz, 1988). Because of the level of detail associated with its application and the objectivity of numerical approaches, it is particularly well suited for configuration design, using a large number of criteria that require input from technically diverse groups or individuals. The sequential steps are followed as:

Step 1: Identifying evaluation Criteria

The first step is to identify a set of criteria based on technical requirements, economic factors, and other general constraints. A range of objectives should satisfy the following conditions:

- *Relevancy*: The objectives must cover the decision-relevant requirements and general constraints as completely as possible, so that no essential criteria are ignored.
- *Independency*: The individual objectives on which the evaluation must be based should be as independent of one another as possible-that is, provisions to increase the value of one variant with respect to one objective must not influence its values with respect to the other objectives.
- *Measurability*: The properties of the system to be evaluated must, if possible, be expressed in concrete quantitative or at least qualitative (verbal) terms

Step 2: Weighting evaluation criteria

The criteria are weighted numerically such that the sum of all weighting factors equals 1. An objective tree is a useful construction for assigning weighting factors (Figure 2.7). The first level represents a simple statement describing the part or component being designed and has an entry value of 1; at each lower level the subobjectives are then given weights relative to each other but which also total 1.0. However, their 'true' weights are calculated as a fraction of the 'true' weight of the objective above them.

Each box in the tree is labelled with the objective's name and given two values: its local value relative to its neighbours at the same level, and its global value relative to the overall objective. Thus, in the example, strength and hardness are given local values relative to each other of 0.55:0.45; but their global values can only total 0.80 (the global value of demand objectives) and are therefore calculated as $0.55 \times 0.80 = 0.44$ and $0.45 \times 0.80 = 0.36$.

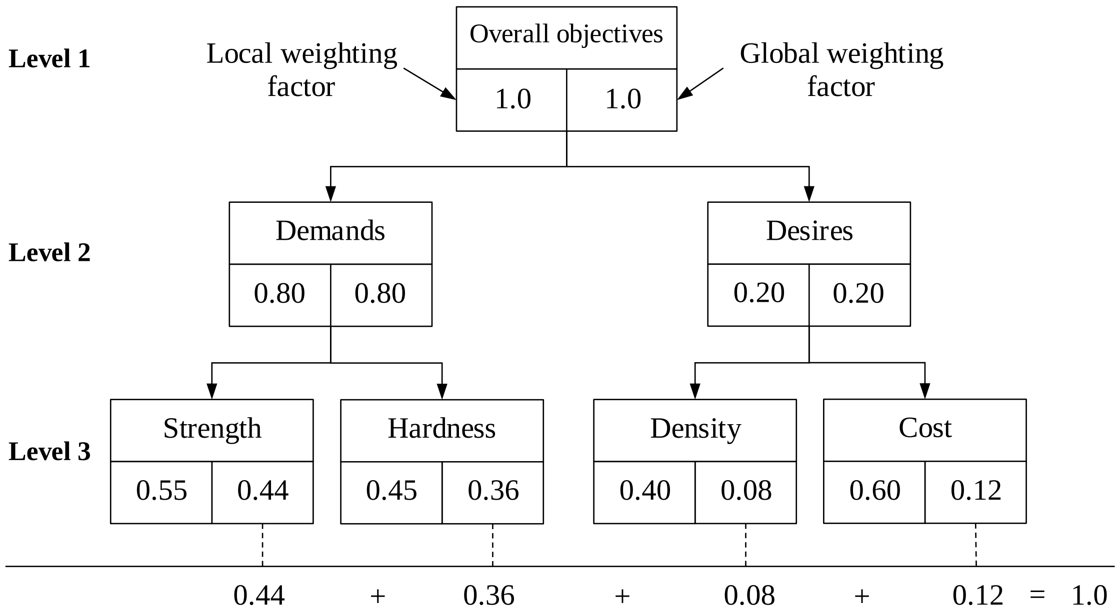


Figure 2.7 Objectives tree for assigning relative weights to sub-objectives

Step 3: Assessing Values

The next step is the assessment of values and hence the actual evaluation. These values derive from a consideration of the relative scale where the performances of the alternatives are expressed by the points. Pahl and Beitz (1988) suggested either 11-points scale or 5-points scale as shown in Table 2.10.

Step 4: Determining overall value

The sub-values for every variant having been determined, the overall value must now be calculated.

A set of alternatives, $A = \{a_i | i = 1,2, \cdots, m\}$

A set of criteria or attributes for, $E = \{e_j | j = 1,2, \cdots, n\}$

Weightage to the attributes, $W = \{w_j | j = 1,2, \cdots, n\}$

Performance ratings, $D = \{d_{ij} | i = 1,2, \cdots, m; j = 1,2, \cdots, n\}$

Scaled value of performance ratings, $V = \{v_{ij} | i = 1,2, \cdots, m; j = 1,2, \cdots, n\}$

Overall weighted value of an alternative is given by,

$$\mathrm{OWV}(a_i) = \sum_{j=1}^{n} w_j \cdot v_{ij} \qquad i = 1,2, \cdots m \tag{2.7}$$

The preeminent alternative is maximum of $\mathrm{OWV}(a_i)$. The example of gear materials is considered here and data is tabulated in Table 2.11. The same data will be considered and analysed throughout the thesis for better understanding of the various methods.

Table 2.10 Assessing value scale with magnitude of gear material attributes

Value scale (v_i)				Gear material attribute magnitude			
11-points scale		5-points scale		σ^{ut} MPa	BHN	ρ g/cc	c \$/kg
Points	Meaning	Points	Meaning				
0	Absolutely useless solution	0	Unsatisfactory	370	100	8.00	5.00
1	Very inadequate solution			390	120	7.90	4.60
2	Weak solution	1	Just tolerable	410	140	7.80	4.20
3	Tolerable solution			430	160	7.70	3.80
4	Adequate solution	2	Adequate	450	180	7.60	3.40
5	Satisfactory solution			470	200	7.50	3.00
6	Good solution with few drawback	3	Good	490	220	7.40	2.60
7	Good solution			510	240	7.30	2.20
8	Very good solution	4	Very good (ideal)	530	260	7.20	1.80
9	Solution exceed the requirements			550	280	7.10	1.40
10	Ideal solution			570	300	7.00	1.00
σ^{ut}- Ultimate tensile strength, BHN - Brinell hardness number, ρ - Density, c - Cost							

Table 2.11 Overall value matrix for gear materials

Alternatives	$\sigma^{ut}(w_1 = 0.44)$			$\text{BHN}(w_2 = 0.36)$			$\rho(w_3 = 0.08)$			$c(w_4 = 0.12)$			OWV	Rank
	d_{i1}	v_{i1}	$w_1 v_{i1}$	d_{i2}	v_{i2}	$w_2 v_{i2}$	d_{i3}	v_{i3}	$w_3 v_{i3}$	d_{i4}	v_{i4}	$w_4 v_{i4}$		
AISI 1040	515	7	3.08	149	2	0.72	7.84	2	0.16	1.30	9	1.08	5.04	4
AISI 4130	560	9	3.96	156	3	1.08	7.85	2	0.16	2.00	8	0.96	6.16	2
AISI 304	515	7	3.08	147	2	0.72	7.80	2	0.16	2.85	5	0.60	4.56	5
AISI 405	448	4	1.76	150	3	1.08	7.80	2	0.16	3.50	4	0.48	3.48	7
ASTM class 60	431	3	1.32	285	9	3.24	7.30	7	0.56	3.30	4	0.48	5.6	3
Grade 60-40-18	416	2	0.88	167	3	1.08	7.10	9	0.72	4.00	3	0.36	3.04	8
Grade 65-45-12	464	5	2.20	167	3	1.08	7.10	9	0.72	4.00	3	0.36	4.36	6
Grade 80-55-06	559	9	3.96	192	5	1.80	7.10	9	0.72	4.00	3	0.36	6.84	1

2.3.4.3 observations

Based on above mentioned literature survey the following points have been noted with reference to important research paper of contemporary relevance,

- In case of Pugh method of alternative evaluation, there is a lack of precise ranking among the alternatives. The approach is very straightforward that an alternative is compared with a base alternative in respect of each criterion and performance are rated as the sum of collection of total number of better (+), worse (−), and same (0). Some researchers modified this rating by assigning numbers on the basis of max-min approach (best of the worse). This concept is taken from the MADM methods.
- In case of Pahl and Beitz method, the performance ratings of an alternative is converted into the scaled values on the basis of a common ordinal-cardinal scale. It can be considered as a normalizing process. There is also a lack precise scaling. Suppose, in Table 2.12, 140 BHN is 'weak solution (2)' and 160 BHN is 'tolerable

solution (3)' then the scaled value for the material AISI 405 having BHN 150 will be a fraction but, the ordinal-cardinal scale does not allow the fractional value.

However, besides the above-mentioned points, there is some lack of joy of mathematical interpretation which motivate us towards the multiple attribute decision making (MADM) methods. For last two decades the MADM methods have been popular and widely used in field of material selection those are portrayed in the next chapter.

Chapter 3

MADM approach in material selection

3.1 Introduction

When there is a set of alternatives, tangible or not, subject to restrictions and limitations, and when it is necessary to perform a selection and ranking, i.e. when there is a complex choice, it is convenient to solve the problem by applying a set of sequential procedures in MADM framework. MADM methods are generally sub- domain of multiple-criteria decision analysis (MCDA) where the alternatives are described in terms of evaluative criteria. MCDA methods can be categorized as:

- Multi-attributed decision making (MADM) where the decision space is explicitly known and discrete.
- Multi-objective decision making (MODM) where the decision space is continuous.

MADM processes are becoming popular due to its user-friendly nature to select the best material from that finite number of alternatives. MADM approaches are more likely to be modeled with uncertain values for the attributes (Shahinur et al., 2017; Wallenius et al., 2008). The most rational approach to select the best alternative is about its utility. In MODM there is usually no attempt to capture the alternatives utilities. Some of the popular MADM models in material selection point of view are:

- *Scoring* model that selects an alternative which deserves the maximum score, such as, MAUT (Hatush & Skitmore, 1998; Yuan-pei et al., 2010), SAW (Kahraman et al., 2008), FUZZY logic (Girubha & Vinodh, 2012; Chan et al., 2008) and AHP (Mujgan et al., 2004; Chan, 2003).
- *Compromising* model that skims off an alternative which is closest to the ideal solution, such as, TOPSIS (Gupta, 2011; Thakker et al., 2008) and VIKOR (Opricovic & Tzeng, 2004; Bahraminasab & Jahan, 2011).
- *Outranking* model that arranges a set of performance relations among alternatives to acquire information on the best alternative, such as, ELECTRE (Shanian, 2008) and PROMETHEE (Peng & Xiao, 2013).

Material selection is an integral part of the engineering design process and rather uncertainty takes place in material attributes and design formulation. Engineering design is a decision-making process where the decision should be formulized under risk and uncertainty. More specifically the above-mentioned scoring models under MADM in the domain of risk and uncertainty are dichotomized as (Howard, 1992):

- *Descriptive* model that deals with human behaviour of real-life choice under certain heuristics (availability, representativeness, and anchoring and adjustment). One of the most prominent descriptive models under uncertainty is the Prospect Theory model of Kahneman and Tversky (1979).
- *Normative* model that is built on some basic assumptions (cancellation, transitivity, dominance, and invariance) and focuses a rational choice. Most inspirational normative models are expected utility model of von Neumann and Morgenstern (1947) and subjective expected utility model of Savage (1954).
- *Prescriptive* model is the assemblage of theoretical and operational assumption that helps the people to make better decision. Many prescriptive applications of expected utility theory have been carried out to capture the risk and uncertainty

especially for problems that have multiple attributes e.g. MAUT (Keeney and Raiffa (1976)) and AHP (Satty, 1980).

3.2 Multi attribute decision making framework

The decision-making process starts by defining the different objectives, the various alternatives that are often present in a given scenario either in one or in different projects, and criteria. These alternatives must be extensively examined from the technical, social, economic, and environmental point of view. The decision maker is the person who, with the knowledge, information, and solutions provided by a model, must make a decision, helped by the ability that most models have in analysing various scenarios, circumstances and situations, as well as changing conditions. The basic steps in all MADM approaches are generally considered as:

- *Decision matrix* is the discrete decision space where a finite set of alternatives is expressed by its performance ratings in multiple attribute or criteria.
- *Weightage* or priority is assigned to each criterion according to the design requirements to satisfy the customer’s demands and desires. AHP (analytical hierarchy process) (Escober & Moreno-Jimenez, 2000; Mujgan et al., 2004; Hu et al., 2014), fuzzy AHP (Chan et al., 2008), DL (digital logic), MDL (modified digital logic) (Dehghan-Manshadi et al., 2007), and entropy are some of the popular methods for priority distribution (Jahan, 2012).
- *Normalization* is the process by which the performance ratings measured in different unit are converted into a common unit. Vector normalization based on Euclidean distance, standardization based on standard deviation, and feature scaling based on the difference between the best and worst value are widely used (Opricovic & Gwo-Hshiung, 2004; Girubha & Vinodh, 2012; Rao & Patel, 2010).
- *Overall performance analysis* that assigns the overall performance rating to each alternative and the alternatives are ranked accordingly. According to the nature of

the mathematical models used in the analysis, the MADA methods are named (Mousavi-Nasab & Sotoudeh-Anvari, 2017; Peng & Xiao, 2013; Shanian, 2008).

- *Sensitivity analysis* examines the overall ordering of the options for another choices of preferences or weights and the advantage and disadvantages of selected options and compare pairs of options.

Compiling the above mentioned first four steps, various MADM models are formed. Some of the popular MADM methods are discussed to the next consecutive sections. In MADM methods, a set of alternatives (A) with performance ratings (D) is decided according to the design requirements. Performance is the measure of effectiveness in the form of attributes (E). Attributes are set according to the demands and desires of the design requirements and can be split up in benefit and cost attributes. The basic inputs in all MADM methods are:

A set of alternatives, $A = \{a_i | i = 1,2, \cdots, m\}$

A set of criteria or attributes for, $E = \{e_j | j = 1,2, \cdots, n\}$

Weightage to the attributes, $W = \{w_j | j = 1,2, \cdots, n\}$

Performance ratings, $D = \{d_{ij} | i = 1,2, \cdots, m; j = 1,2, \cdots, n\}$

3.2.1 AHP (Analytical Hierarchy Process)

The Analytic Hierarchy Process (AHP) is a multiple criteria decision-making procedure developed by Saaty (1980). It is based on well-defined mathematics of consistent matrices and their associated right eigenvector's ability to generate true or approximate weights, and philosophy which compares criteria, or alternatives with respect to a criterion, in a natural, pair-wise mode. To do so, the AHP uses a fundamental scale of absolute numbers shown in Table 3.1. It converts individual preferences into ratio scale weights. This method is also applicable to evaluate the alternatives. Negative priorities can be derived from positive dominance comparisons and from ratings just as positive priorities are, except that the sense in which the question is asked in making the comparisons is opposite to that used to derive positive numbers.

Table 3.1 Satty's fundamental scale

Intensity of Importance	Definition	Intensity of Importance	Definition
1	Equal importance	7	Demonstrated importance
3	Weak importance of one over another	9	Absolute importance
5	Essential or strong importance	2,4,6,8	Intermediate values between the two adjacent judgements

Steps are followed to develop the weights for the criteria:

Step 1: Pair-wise comparison matrix for the criteria

$$B = [b_{ij}], \quad \text{where, } b_{ij} = \frac{w_i}{w_j} \qquad i, j = 1,2, \dots, m \qquad (3.1)$$

where, b_{ij} indicates how much the ith objective is more important than jth objective and B is said to be random reciprocal pairwise matrix if $b_{ji} = {}^{1}/_{b_{ij}}$ and $b_{ii} = 1$ (Vargas, 1982).

Step 2: For a given pairwise reciprocal matrix $B = [b_{ij}]$, with, $b_{ij} > 0$, the priorities vector obtained according to row geometric mean without normalization is given by (Crawford & Williams, 1985),

$$\bar{w}_i = \sqrt[m]{\prod_{j=1}^{m} b_{ij}} \qquad i = 1,2, \dots, m \qquad (3.2)$$

Step 3: Normalized priority vectors is given by

$$w_i = \frac{\bar{w}_i}{\sum_{j=1}^{m} \bar{w}_j} \qquad i = 1,2, \dots, m \qquad (3.3)$$

Step 4: It is necessary to check the consistent of matrix *B*. The priorities vector obtained according to eigenvector method is given by,

$$Bw = \lambda_{\max} w \tag{3.4}$$

where, $\lambda_{\max}$ is the largest eigenvalue of *B* and $Bw = b_{ij} w_j = v_j$. Therefore,

$$\lambda_{\max} = \frac{1}{m} \sum_{i=1}^{m} \frac{v_i}{w_i} \tag{3.5}$$

Consistency Ratio (*CR*) is given by,

$$CR = \frac{CI}{RI} \tag{3.6}$$

where, consistency index, $CI = \frac{\lambda_{\max} - m}{m-1}$. Random Inconsistency Index, *RI* is derived as an average of a large sample of consistency indices of a matrix of order *m* with random reciprocal entries. A table of *RI* was first constructed at Oak Ridge National Laboratory shown in Table 3.2 (Munier, 2011).

Table 3.2 Random index table

m	2	3	4	5	6	7	8	9	10
RI	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.51

If $CI/RI \leq 0.10$, the degree of consistency is satisfactory.
If $CI/RI \geq 0.10$, the matrix is inconsistent.
The priorities vector also known as criteria weights. Let us consider a same example of gear materials in Section 2.3.4.2 with same attributes: ultimate tensile strength, Brinell hardness number, density, and cost. The criteria weights are tabulated in Table 3.3 following the expressions (3.1) to (3.3) which gives,

$$w_{j=1,2,\cdots,m} = (0.47,0.33,0.08,0.12)$$

From expressions (3.4) and (3.5), $\lambda_{\max} = 4.12$ for $m = 4$. From expression (3.6), $CR = 0.044 \ll 0.1$, i.e. the degree of consistency is satisfactory.

Table 3.3 Pair-wise comparison Matrix of attributes

Material attribute	e_1	e_2	e_3	e_4	$\bar{w}_i$	$\boldsymbol{w_i}$	v_i
Ultimate tensile strength (e_1)	1	2	4	4	2.37	**0.47**	1.93
Brinell hardness number (e_2)	1/2	1	4	4	1.68	**0.33**	1.36
Density (e_3)	1/4	1/4	1	1/2	0.42	**0.08**	0.34
Cost in (e_4)	1/4	1/4	2	1	0.59	**0.12**	0.48

3.2.2 MAUT (Multi Attribute Utility Theory)

MAUT (Multi Attribute Utility Theory) is based on utility theory; it has had considerable success especially in the United States. It is an additive method consisting in multiplying the score for each alternative and for a criterion, by the weight assigned to that criterion. Further, it proceeds with the summation of values found; the selected alternative is the one that gets the highest value from this summation. The purpose in developing MAUT was to take into account uncertainty caused by lack of precise information or data. Steps are followed as:

Step 1: To find the utility value of j^{th} attribute for i^{th} alternative.

$$u_{ij} = \frac{d_{ij} - d_{ij}^{-}}{d_{ij}^{+} - d_{ij}^{-}} \tag{3.7}$$

where, d_{ij}^{+} is the best value of d_{ij} and d_{ij}^{-} is the worst value of d_{ij} (for benefit criteria, higher value is the best and for the cost criteria, lower value is the best).

Step 2: To find the overall utility value of the respective alternative and is given by,

$$U(a_i) = \sum_{j=1}^{n} w_j(e_j) \cdot u_{ij}(d_{ij}) \qquad i = 1,2,\cdots,m \tag{3.8}$$

where, w_j is the weightage to the attributes such that, $\sum_{j=1}^{n} w_j = 1$. Let us solve the same problem of performance evaluation of the gear materials following the expressions (3.7) and (3.8) and the results are tabulated in Table 3.4.

Table 3.4 Overall performance of the gear materials following MAUT

Alternative (a_i)		e_1 (max) ($w_1 = 0.47$)		e_2 (max) ($w_2 = 0.33$)		e_3 (min) ($w_3 = 0.08$)		e_4 (min) ($w_4 = 0.12$)		$U(a_i)$	Rank
		d_{i1}	u_{i1}	d_{i2}	u_{i2}	d_{i3}	u_{i3}	d_{i4}	u_{i4}		
a_1	AISI 1040	515	0.687	149	0.014	7.84	0.013	1.30	1.000	0.449	4
a_2	AISI 4130	560	1	156	0.065	7.85	0.000	2.00	0.741	0.580	2
a_3	AISI 304	515	0.687	147	0	7.80	0.067	2.85	0.426	0.379	5
a_4	AISI 405	448	0.222	150	0.021	7.80	0.067	3.50	0.185	0.139	7
a_5	ASTM class 60	431	0.104	285	1	7.30	0.733	3.30	0.259	0.469	3
a_6	Grade 60-40-18	416	0	167	0.145	7.10	1.000	4.00	0.000	0.128	8
a_7	Grade 65-45-12	464	0.333	167	0.145	7.10	1.000	4.00	0.000	0.284	6
a_8	Grade 80-55-06	559	0.993	192	0.326	7.10	1.000	4.00	0.000	0.654	1

3.2.3 TOPSIS (Technique for Order Preferences by Similarity to an Ideal Solution)

TOPSIS (Technique for Preference by Similarity to Ideal Solution) was developed by Hwang & Yoon (1981) to determine the best alternative based on the concepts of the compromise solution. The compromise solution can be regarded as choosing the solution with the shortest Euclidean distance from the ideal solution and the farthest Euclidean distance from the negative ideal solution shown in Figure 3.1(a). Generally, the steps are followed in TOPSIS as;

Step 1: To develop the normalized decision matrix (performance ratings).
Vector normalization is used to eliminate the units of criterion functions, so that all the criteria are dimensionless.

$$r_{ij} = \frac{d_{ij}}{\sqrt{\sum_{i=1}^{m} {d_{ij}}^2}} \qquad j = 1,2,\cdots,n \tag{3.9}$$

Step 2: To prepare the weighted normalized decision matrix.

$$x_{ij} = r_{ij} \cdot w_j = \frac{d_{ij}}{\sqrt{\sum_{i=1}^{m} {d_{ij}}^2}} \cdot w_j \qquad j = 1,2,\cdots,n \tag{3.10}$$

Step 3: To find out the ideal and the negative-ideal solution.

$$y_j^+ = \{(\max x_{ij}) | j = 1,2,\cdots,n-r; (\min x_{ij}) | j = n-r+1,\cdots,n\} \tag{3.11}$$

$$y_j^+ = \{(\min x_{ij}) | j = 1,2,\cdots,n-r; (\max x_{ij}) | j = n-r+1,\cdots,n\} \tag{3.12}$$

Step 4: To calculate the separation measure using the Euclidean distance.

$$S_i^+ = \sqrt{\sum_{j=1}^{n} (x_{ij} - y_j^+)^2} \qquad i = 1,2,\ldots,m \tag{3.13}$$

$$S_i^- = \sqrt{\sum_{j=1}^{n} (x_{ij} - y_j^-)^2} \qquad i = 1,2,\ldots,m \tag{3.14}$$

Step 5: To determine the relative closeness to the ideal solution.

$$M_i = \frac{S_i^-}{S_i^+ + S_i^-} \qquad 0 < M_i < 1, \quad i = 1,2,\ldots,m \tag{3.15}$$

The same problem discussed in Section 2.3.4.1 of performance evaluation of the gear materials is solved following the expressions (3.9) to (3.15) and the results are tabulated in Table 3.5. The ideal point and negative ideal point are given by the following expressions (3.11) and (3.12),

$$y_j^+ = 0.189, 0.183, 0.027, 0.017$$

$$y_j^- = 0.141, 0.094, 0.030, 0.052$$

Table 3.5 Overall performance of the gear materials following TOPSIS

(a_i)	e_1 (max) ($w_1 = 0.47$)		e_2 (max) ($w_2 = 0.33$)		e_3 (min) ($w_3 = 0.08$)		e_4 (min) ($w_4 = 0.12$)		S_i^+	S_i^-	M_i	Rank
	d_{i1}	x_{i1}	d_{i2}	x_{i2}	d_{i3}	x_{i3}	d_{i4}	x_{i4}				
a_1	515	0.174	149	0.096	7.84	0.030	1.30	0.017	0.089	0.048	0.353	4
a_2	560	0.189	156	0.100	7.85	0.030	2.00	0.026	0.083	0.055	0.398	3
a_3	515	0.174	147	0.094	7.80	0.029	2.85	0.037	0.092	0.036	0.283	5
a_4	448	0.152	150	0.096	7.80	0.029	3.50	0.046	0.099	0.013	0.113	8
a_5	431	0.146	285	0.183	7.30	0.028	3.30	0.043	0.050	0.089	0.640	1
a_6	416	0.141	167	0.107	7.10	0.027	4.00	0.052	0.097	0.014	0.123	7
a_7	464	0.157	167	0.107	7.10	0.027	4.00	0.052	0.090	0.021	0.189	6
a_8	559	0.189	192	0.123	7.10	0.027	4.00	0.052	0.069	0.056	0.448	2

3.2.4 VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje)

VIKOR method was developed for multicriteria optimization of complex systems. It determines the compromise ranking list, the compromise solution, and the weight stability intervals for preference stability of the compromise solution obtained with the initial (given) weights. This method focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria. It introduces the multicriteria ranking index based on the particular measure of "closeness" to the "ideal" solution (Opricovic & Tzeng, 2004). The VIKOR method is developed on the basis of L_p-*metric*:

$$L_{p,i} = \left\{ \sum_{j=1}^{n} \left[w_j \frac{(d_{ij}^+ - d_{ij})}{(d_{ij}^+ - d_{ij}^-)} \right]^p \right\}^{1/p} \qquad 1 \le p \le \infty; i = 1,2,\cdots,m \tag{3.16}$$

Within the VIKOR method, $L_{1,i}$ as S_i and $L_{\infty,i}$ as R_i are used to formulate the ranking measure. The solution obtained by $\min S_i$ is with a maximum group utility ("majority" rule), and the solution obtained by $\min R_i$ is with a minimum individual regret of the "opponent". The compromise solution F^c is a feasible solution that is the "closest" to the ideal F^* and compromise means an agreement established by mutual concessions, as illustrated in Figure 3.1(b).

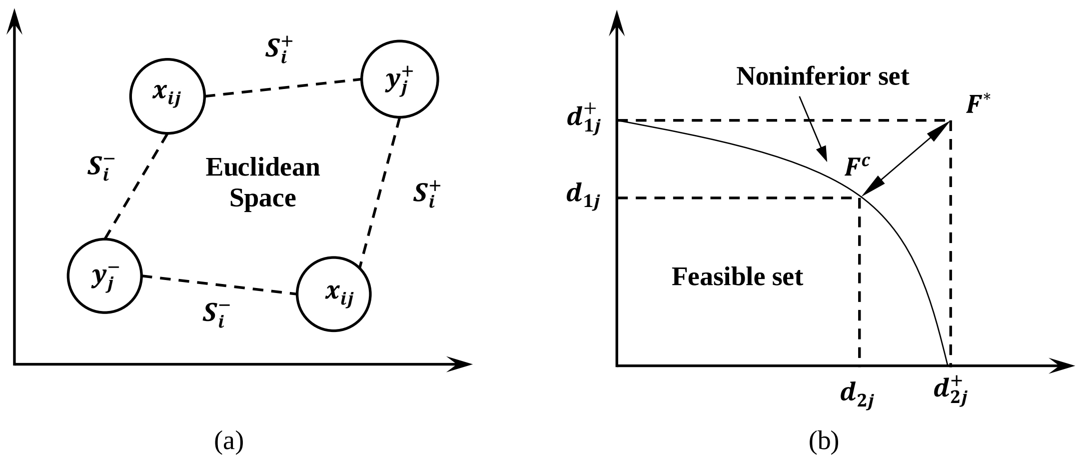


Figure 3.1 (a) Compromise to ideal and negative ideal solutions (TOPSIS), (b) Compromise to ideal solutions (VIKOR)

The compromise ranking algorithm VIKOR has the following steps:

Step 1: To Compute the values S_i and R_i.

$$S_i = \sum_{j=1}^{n} w_j \cdot f_{ij} \qquad i = 1,2,\cdots,m \tag{3.17}$$

$$f_{ij} = \frac{d_{ij}^+ - d_{ij}}{d_{ij}^+ - d_{ij}^-} \qquad i = 1,2,\cdots,m; j = 1,2,\cdots,n \tag{3.18}$$

where, d_{ij}^+ is the best value of d_{ij} and d_{ij}^- is the worst value of d_{ij} (for benefit criteria, higher value is the best and for the cost criteria, lower value is the best).

$$R_i = \max_j \left[w_j \cdot f_{ij}\right] \qquad i = 1,2,\cdots,m; j = 1,2,\cdots,n \tag{3.19}$$

Step 2: To Compute the value of Q_i.

$$Q_i = v\frac{S_i - S^-}{S^+ - S^-} + (1-v)\frac{R_i - R^-}{R^+ - R^-} \tag{3.20}$$

where, S^+ and S^- are the best and worst value of S_i and R^+ and R^- are the best and worst value of R_i. v is introduced as weight of the strategy of "the majority of criteria" (or "the maximum group utility"), here $v = 0.5$.

Step 3: To rank the alternatives, sorting by the values S_i, R_i and Q_i, in decreasing order. The results are three ranking lists.

The same problem of performance evaluation of the gear materials is solved following the expressions (3.17) to (3.20) and the results are tabulated in Table 3.6.

Table 3.6. Overall performance of the gear materials following VIKOR

(a_i)	e_1 (max) ($w_1 = 0.47$)		e_2 (max) ($w_2 = 0.33$)		e_3 (min) ($w_3 = 0.08$)		e_4 (min) ($w_4 = 0.12$)		S_i	R_i	Q_i	Rank
	d_{i1}	$w_i f_{i1}$	d_{i2}	$w_2 f_{i2}$	d_{i3}	$w_3 f_{i3}$	d_{i4}	$w_4 f_{i4}$				
a_1	515	0.147	149	0.325	7.84	0.079	1.30	0.000	0.551	0.325	0.403	3
a_2	560	0.000	156	0.308	7.85	0.080	2.00	0.031	0.420	0.308	0.243	2
a_3	515	0.147	147	0.330	7.80	0.075	2.85	0.069	0.620	0.330	0.479	4
a_4	448	0.366	150	0.323	7.80	0.075	3.50	0.098	0.861	0.366	0.780	7
a_5	431	0.421	285	0.000	7.30	0.021	3.30	0.089	0.531	0.421	0.577	6
a_6	416	0.470	167	0.282	7.10	0.000	4.00	0.120	0.872	0.470	1.000	8
a_7	464	0.313	167	0.282	7.10	0.000	4.00	0.120	0.716	0.313	0.535	5
a_8	559	0.003	192	0.222	7.10	0.000	4.00	0.120	0.346	0.222	0.000	1

3.2.5 ELECTRE (Elimination et Choice Translating Reality)

The ELECTRE method (Elimination et Choice Translating Reality) was originally introduced by Benayoun et al. (1966). ELECTRE uses the concept of an 'outranking relationship'. The outranking relationship of $a_1 \rightarrow a_2$ says that even though two alternatives a_1 and a_2 do not dominate each other mathematically, the decision maker accepts the risk of regarding a_1 as almost surely better than a_2. Through the successive assessments of the outranking re1ationships of the other alternatives, the dominated alternatives defined by the outranking relationship can be eliminated. A *'concordance matrix'* is built, comparing paired alternatives, where outranking exists if there is a strong supremacy in criteria, and there is another *'discordance matrix',* which opposes the former in the sense that it opposes the supremacy of one alternative over another. To justify a supremacy of one alternative over another, a threshold value is considered. The ELECTRE method takes the following steps:

Step 1: To calculate the weighted normalized decision matrix as in TOPSIS following the expression (3.10).

Step 2: To calculate the concordance matrix.

The relative value of the concordance set is measured by means of the concordance index. The concordance index between a_k and a_l is equal to the sum of the weights associated with those criteria where $x_{kj} > x_{lj}$.

$$p_{kl} = \sum_{j|x_{kj}>x_{lj}} w_j \qquad j = 1, \cdots n;\ k, l = 1, \cdots, m; k \neq l \tag{3.21}$$

Step 3: To calculate the discordance matrix:

The discordance index measures the worseness of a_k over a_l.

$$q_{kl} = \frac{\max_{j|x_{kj}<x_{lj}} |x_{kj} - x_{lj}|}{\max_j |x_{kj} - x_{lj}|} \qquad j = 1, \cdots n;\ k, l = 1, \cdots, m; k \neq l \tag{3.22}$$

A higher value of q_{kl} implies that, for the discordance criteria, a_k is less favourable than a_l.

Step 4: To determine the concordance dominance matrix.

This matrix can be calculated with the aid of a threshold value for the concordance index. a_k will only have a chance of dominating a_l, if its corresponding concordance index p_{kl} exceeds at least a certain threshold value $\overline{p}$, i.e.,

$$p_{kl}^d = \begin{cases} 1, & if\ p_{kl} \geq \overline{p} \\ 0, & if\ p_{kl} \leq \overline{p} \end{cases} \tag{3.23}$$

$$\overline{p} = \frac{\sum_{k=1}^{m} \sum_{l=1}^{m} p_{kl}}{m(m-1)} \tag{3.24}$$

Step 5: To determine the discordance dominance matrix.

This matrix is constructed in the same way as concordance dominance matrix on the basis of a threshold value $\overline{q}$ to the discordance indices. The elements of the discordance dominance matrix can be calculated as,

$$q_{kl}^{d} = \begin{cases} 0, & if\ q_{kl} \geq \overline{q} \\ 1, & if\ q_{kl} \leq \overline{q} \end{cases} \tag{3.25}$$

$$\overline{q} = \frac{\sum_{k=1}^{m} \sum_{l=1}^{m} q_{kl}}{m(m-1)} \tag{3.26}$$

Step 6: To determine the aggregate dominance matrix.

Aggregate dominance matrix is the intersection of the concordance dominance matrix (p_{kl}^{d}) and discordance dominance matrix (q_{kl}^{d}).

$$h_{kl} = p_{kl}^{d} \cdot q_{kl}^{d} \tag{3.27}$$

The aggregate dominance matrix gives the partial-preference ordering of the alternatives. If $h_{kl} = 1$, then a_k is preferred to a_l for both the concordance and discordance criteria, but a_k still has the chance of being dominated by the other alternatives.

A numerical example of same gear materials is performed and results are tabulated in Table 3.7 to Table 3.9 for clear understanding where the alternatives a_2, a_5, and a_8 show the most dominating nature. Simple ELECTRE does not encourage the ranking but ranking can be done by analysing the dominance diagram shown in Table 3.9.

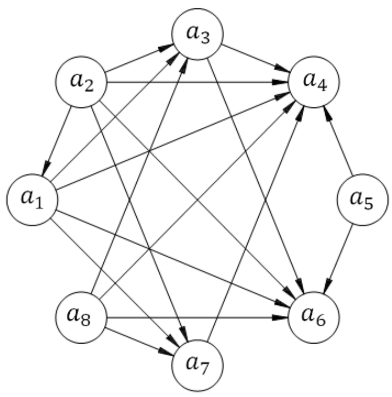
Table 3.7 Concordance and discordance matrix of the gear materials following ELECTRE

Concordance index, $p_{k,l}$									Discordance index, $q_{k,l}$								
$\boldsymbol{a_{k,l}}$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	$\boldsymbol{a_{k,l}}$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
a_1		0.20	0.92	0.59	0.59	0.59	0.59	0.12	a_1		1.000	0.050	0.034	1.000	0.314	0.314	0.771
a_2	0.80		0.92	0.92	0.59	0.59	0.59	0.59	a_2	0.600		0.067	0.027	1.000	0.146	0.219	0.885
a_3	0.08	0.08		0.67	0.59	0.59	0.59	0.12	a_3	1.000	1.000		0.091	1.000	0.394	0.765	1.000
a_4	0.41	0.08	0.33		0.47	0.59	0.12	0.12	a_4	1.000	1.000	1.000		1.000	1.000	1.000	1.000
a_5	0.41	0.41	0.41	0.53		0.92	0.45	0.45	a_5	0.322	0.518	0.315	0.069		0.013	0.145	0.717
a_6	0.41	0.41	0.41	0.41	0.08		0.53	0.20	a_6	1.000	1.000	1.000	1.000	1.000		1.000	1.000
a_7	0.41	0.41	0.41	0.88	0.55	0.47		0.20	a_7	1.000	1.000	1.000	0.545	1.000	0.000		1.000
a_8	0.88	0.41	0.88	0.88	0.55	0.80	0.80		a_8	1.000	1.000	0.517	0.162	1.000	0.000	0.000	

Table 3.8 Concordance and discordance dominance matrix of the gear materials

Concordance dominance, p_{kl}^{d} ($\overline{p} = 0.5$)									Discordance dominance, q_{kl}^{d} ($\overline{q} = 0.661$)								
$\boldsymbol{a_{k,l}}$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	$\boldsymbol{a_{k,l}}$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
a_1		0	1	1	1	1	1	0	a_1		0	1	1	0	1	1	0
a_2	1		1	1	1	1	1	1	a_2	1		1	1	0	1	1	0
a_3	0	0		1	1	1	1	0	a_3	0	0		1	0	1	0	0
a_4	0	0	0		0	1	0	0	a_4	0	0	0		0	0	0	0
a_5	0	0	0	1		1	0	0	a_5	1	1	1	1		1	1	0
a_6	0	0	0	0	0		1	0	a_6	0	0	0	0	0		0	0
a_7	0	0	0	1	1	0		0	a_7	0	0	0	1	0	1		0
a_8	1	0	1	1	1	1	1		a_8	0	0	1	1	0	1	1	

Table 3.9 Aggregate dominance matrix of the gear materials following ELECTRE

Aggregate dominance, $h_{k,l}$									Dominance diagram	Rank
$\boldsymbol{a_{k,l}}$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8		
a_1		0	1	1	0	1	1	0		4
a_2	1		1	1	0	1	1	0		1
a_3	0	0		1	0	1	0	0		5
a_4	0	0	0		0	0	0	0		8
a_5	0	0	0	1		1	0	0		3
a_6	0	0	0	0	0		0	0		7
a_7	0	0	0	1	0	0		0		6
a_8	0	0	1	1	0	1	1			2

3.2.6 Previous works on material selection

There is an ample of literature where varieties of MADM methods are discussed, modified, and applied in the field of material selection (Table 3.10). Shahinur et al. (2017) judiciously introduced the fuzzy logic to select the material for vehicle body and the aluminium alloys are chosen as the best materials. Cryogenic storage tank materials are analysed by WPM (Dehghan-Manshadi et al., 2007), TOPSIS (Jahan et al., 2012), MOORA (Karande & Chakraborty, 2012), Fuzzy logic (Khabbaz et al., 2009) and the preeminent material is considered as Austenitic steel (SS 301- FH). Flywheel materials are evaluated by, TOPSIS (Jee & Kang, 2000), VIKOR, ELECTRE (Chatterjee et al., 2009), TOPSIS (Jahan et al., 2012) and the analyses show the best material is Kevler 49-epoxy FRP. Gear materials are divisioned by Extended PROMETHEE II (Chatterjee & Chakraborty, 2012), TOPSIS (Milani et al., 2005), MULTIMOORA (Hafezalkotob et al., 2016) and raise the best material as Carburised steel. Metallic bipolar plate materials are interpreted by VIKOR (Rao, 2008), TOPSIS [Shanian & Savadogo, 2006) and the optimal choice shows the Austenitic stainless steel 316. Meanwhile, with time and the advancement of technology, new materials take place besides the traditional materials. For example, shape memory alloy (smart material) that can memorize the shape at various operating conditions and behave accordingly. Unlike traditional MADM approaches, Huang (2002) introduced a series of performance index charts under various operating conditions to select the preeminent material for actuators and suggested the NiTi popularly known as Nitinol as a good choice.

Table 3.10 A brief chart of MADM methods used in material selection

MADM methods	Engineering components from various engineering field													
	Cryogenic storage tank			Flywheel			Gear			Metallic bipolar plates		Femoral comp.	Impulse turbine blade	Thin film solar cell
TOPSIS	✓			✓			✓			✓			✓	✓
VIKOR					✓						✓	✓		
ELECTRE						✓								
PROMOTHEE								✓						
COPRAS-G		✓												
MOORA			✓						✓					
Reference papers	Jahan et al., 2012	Chatterjee et al., 2011	Karande et al.,2012	Jee & Kang, 2000	Chatterjee et al., 2009	Chatterjee et al., 2009	Milani et al.,2004	Chatterjee et al., 2012	Hafezalkotob et al., 2016	Shanian et al., 2006	Rao, 2008	Bahraminasab et al., 2011	Thakker et al., 2008	Gupta, 2011

From the above discussion and from Table 3.10, it can be concluded that the TOPSIS and VIKOR are widely used. Most of the cases, different types of normalization process are suggested and implemented (Fayazbakhsh et al., 2009), but the basic structure of overall performance analysis of the alternatives in all processes is same. Cables et al. (2016) proposed a method, RIM (reference ideal method) where the only 'ideal point' is considered as a reference point (in case of TOPSIS, there is 'ideal point' and 'negative ideal point') and the outcomes is compared with TOPSIS and VIKOR methods. Lourenzutti and Krohling (2014) introduced the Hellinger distance in TOPSIS (H-TOPSIS) whereas the original TOPSIS is based on Euclidean distance.

3.2.7 Observations

We have to evaluate the performance of the alternatives under uncertainty. Uncertainty enters into the problem formulation due to complexity, lack of understanding of causes and effects and lack of information. The mechanical properties of engineering materials exhibit variability, fracture and fatigue properties show greater variability. Uncertainty also exists in the model or process itself, or about future events that will influence the outcome of a decision. Therefore, we have to proceed in rational way that we can maximize the overall objectives. Rationality means to balance between the customers' requirements and design parameters. In order to achieve a rational choice, the following observations are considered:

- *Structuring the problem*: The first step in design is to establish the specification the problem. Readers want to know that under which specific conditions the alternatives are chosen. This information is absent in selecting the gear material using TOPSIS by Melani et al. [20] or Chatterjee et al. [21] using PROMETHEE-II. In this situation the selected material may or may not assure the optimal design. An isolated gear has no identity; it should be analysed when it meshes with the other. In order to design the gear for stress and, it is necessary to know the applied forces through gears and it is necessary to know gear specification in order to determine the forces that will be transmitted to the shaft. The primary design parameters to specify a gear are material, module and face width those are interrelated within the gearbox size constraints. Optimal design is a feasible solution that optimizes the overall objective function or maximizes the utility. All the methods are structured with strong mathematical background but it is important that how well-organized way we are implementing these methods. Therefore, problem should be analysed within a structured framework.

- *Knowledge*: In the above-mentioned framework, no doubt, MAUT is well suited to evaluate the alternatives under uncertainty. In the available traditional MADM approaches, we consider the known knowledge or observed utility of the alternatives. In the frame of uncertainty when our knowledge is incomplete, unobserved utility which is stochastic in nature should be considered partially to analyse the problem.
- *Belief*: Decision making depends on the decision maker's belief. Belief is the function of knowledge and confidence. "Bird flies," it is a belief. "Bird is flying," we can visualize i.e., a confidence in our knowledge and this confidence gives us a relax under uncertainty. For example, in TOPSIS method, the closeness between two points are shown by a ratio, it is a belief. There is a lack of confidence, we cannot visualize it. In general perception, the closeness in between two points is a distance parameter.

Chapter 4

Development of the notion

4.1 Introduction

Engineering design encompasses a wide range of activities whose goal is to determine all attributes of a product before it is manufactured. A strong capability to engineer industrial and consumer products is needed by any nation to stay competitive in an increasingly global economy. Good engineering design know-how results in lower time to market, better quality, lower cost, lower use of energy and natural resources, and minimization of adverse effects on the environment. Engineering decision making theory recognizes that the ranking produced by using a criterion has to be consistent with the engineer's objectives and preferences. The theory offers a rich collection of techniques and procedures to reveal preferences and in this research work, it has been tried to introduce them into the proposed models of decision-making in Materials selection by evaluating their performance. This Chapter provides a material selection framework and two new methods for performance evaluation of the alternatives. One of them is newly developed method and all the approaches with overcoming the previous are termed as:

1. Normative-prescriptive approach (NPA)
2. Discrete choice analysis (DCA)
3. Nearest neighbour search (NNS)

4.2 Normative-prescriptive approach (NPA)

Hazelrigg (1998), proposed a framework known as Decision-Based Design (DBD) to select an alternative among the set of alternatives by assigning utility function to each alternative (Mastron & Mistree, 1998). It is a normative approach based on Neumann-Morgenstern (1947) utility axioms to ensure a rational choice that are explicitly considered. von Neumann and Morgenstern (1947) axiomatized expected utility theory by showing that, if a set of apparently normatively appealing axioms hold, alternative actions can be ranked by their expected utilities. The expected utility of an alternative action is the weighted average of the utilities of the possible outcomes where the weights are the objective probabilities of each outcome.

Objective probability is decided through observations. it is too costly, time consuming, or technologically infeasible to make the observations, or in principle because that quantity in which we are interested, such as the probability of a rare event or condition occurring in the future, cannot be observed. Savage (1954) proposed Bayesian view of probability in which probability describes an individual's "degree of belief." This is also known as subjective probability. Savage's (1954) subjective expected utility model allows the derivation of a decision maker's own subjective probabilities for events, which are then used to compute the subjective expected utility of each alternative (Figure 4.1). Instead, there are many other ways of performing a normative or prescriptive "decision analysis." The Analytical Hierarchy process, for example, is designed to guide a decision, and therefore, it can be considered prescriptive even though it does not rely on the norms of the von Neumann axioms (Elisabeth, 2007).

Utility of $\boldsymbol{x_i}$

Subjective probability of $\boldsymbol{x_i}$
(Savage's model)

$$U(x_i) = \sum_i u(x_i) \cdot P(x_i)$$

Overall utility of $\boldsymbol{x_i}$

Objective probability of $\boldsymbol{x_i}$
(von Neumann-Morgenstern model)

Figure 4.1 Utility model

However, the sole attention is to satisfy the customer by maximizing the utility value. In this model we introduce Suh's Design Axioms (SDA), Multiple Attribute Utility Theory (MAUT), and Analytical Hierarchy Process (AHP) in DBD framework in order to maintain the normativity (Elisabeth, 2007). That is, a systematic quantitative approach is prescribed to raise a rational choice. The objectives of this model are:

- To implement SDA to generate the alternatives where customer requirements are mapped to functional requirements which are the function of design parameters.
- To employ the MAUT and AHP to select the best alternative among the alternatives where AHP assigns weightage to the attributes and MAUT assigns the utility value to the alternatives.
- To execute the SDA, MAUT and AHP in DBD framework (Figure 4.2) to select the best material and geometrical variables simultaneously to ensure a rational design where the design space is discrete.

The details of *What* and *How* model (Figure 4.2) is described to the next section.

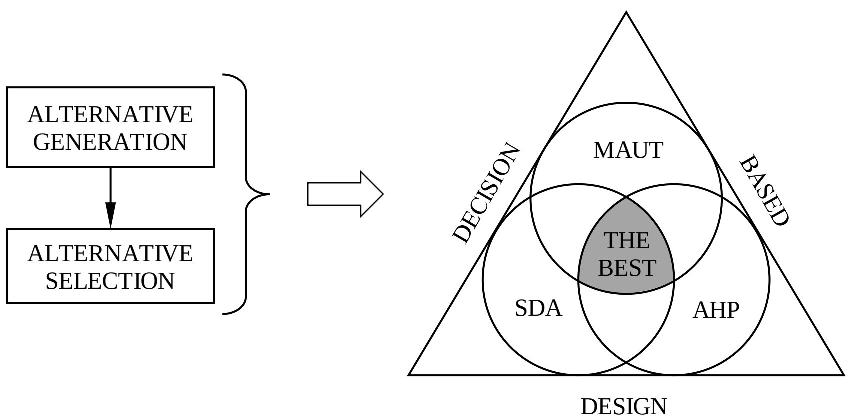


Figure 4.2 *What* and *how*

The performance of the material is changed when it takes a human created shape and shape is given on the basis of performance of materials. A systematic approach is necessary to integrate performance and shape which is shown in Figure 4.3. *System* *criteria* ($A(x)$) are the measure of effectiveness in the form of functional requirements; in turn it is in the form of engineering attributes that will generate a set of alternatives. *Engineering attributes* (e_j) are performance parameters or material properties such as, tensile strength, modulus of elasticity or poisson's ratio. *Functional requirements* (FR) are set according to the *customer requirements* (CR) which can be split up in demands and wishes. *Demands* must be met under all circumstances and are the function of design parameters (DP) or geometrical variables (x_i). A set of alternatives (m_i) are generated within the range x_l and x_u. Recalling the expression (2.1), we can write,

$$\text{FR} = f\{\text{material}(m_i),\text{geometry}(x_i)\} \qquad x_l \leq x_i \leq x_u \tag{4.1}$$

Wishes are taken into consideration whenever necessary and converted into engineering attributes of the selected alternatives. Engineering materials are multi-attributed and MAUT is employed to determine the utility value of the alternatives under uncertainty and risk that raises a rational choice. In material selection point of view the *rationality* is the balancing between customer's demands and wishes that maximizes the overall utility value. Two types of multi-attribute utility theory are used; additive and multiplicative utility theory (Keeney & Raiffa, 1993).

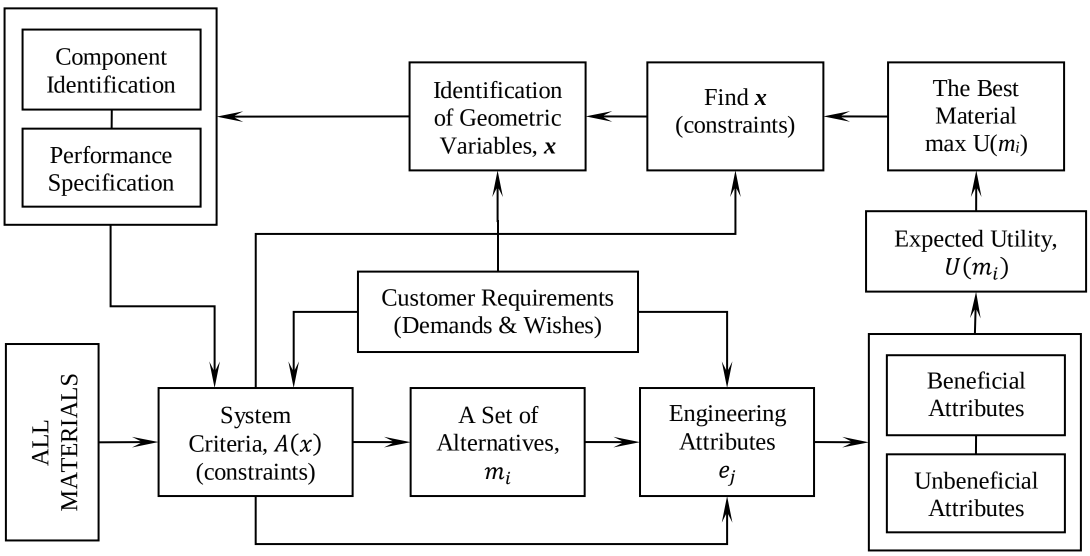


Figure 4.3 Decision-Based Design-Driven Material Selection Flow Diagram

Steps are followed in the evaluating stage as:

Step 1: A absolute set of alternatives (A) with performance ratings (D) is decided according to the design requirements. Performance is the measure of effectiveness in the form of attributes (E). Attributes are set according to the demands and desires (wishes) of the design requirements and can be split up in benefit and cost attributes.

$$A = \{a_i | i = 1,2,\cdots,m\} \tag{4.2}$$

$$E = \left\{ e_j \middle| \underbrace{j = 1,2,\cdots,n-r}_{\text{benefit attribute}} ; \underbrace{j = n-r+1,\cdots,n}_{\text{cost attribute}} \right\} \tag{4.3}$$

$$D = \left\{ d_{ij} \middle| i = 1,2,\cdots,m; j = 1,2,\cdots,n \right\} \tag{4.4}$$

Step 2: To find the utility value of j^{th} attribute for i^{th} alternative.

$$u_{ij} = \frac{d_{ij} - d_{ij}^{-}}{d_{ij}^{+} - d_{ij}^{-}} \tag{4.5}$$

where, d_{ij}^{+} is the best value of d_{ij} and d_{ij}^{-} is the worst value of d_{ij} (for benefit criteria, higher value is the best and for the cost criteria, lower value is the best).

Step 3: To find the overall additive utility value of the respective alternative.

$$U(a_i) = \sum_{j=1}^{n} w_j(e_j) \cdot u_{ij}(d_{ij}) \qquad i = 1,2,\cdots,m \tag{4.6}$$

where, w_j is the weightage to the attributes assigned by AHP such that, $\sum_{j=1}^{n} w_j = 1$ and maximum of $U(a_i)$ assures the best material.

This approach can be regarded as deterministic choice theory and choice is made taking the observed data with certainty, i.e. there is some lack of addressing the risk and uncertainty. A choice model should be structured to consider the lack of information about the alternatives by considering the unobserved utilities. This issue is discussed and overcome to the next section.

4.3 Discrete choice analysis (DCA)

The above-discussed method is a deterministic approach considering only the observed performance ratings or utilities of the alternatives. In the choice analysis, unobserved attributes, unobserved taste variation, and variability in observed utilities should be considered (Wassenaar and Chen 2003). Discrete Choice Analysis (DCA) platform accommodates these unobserved factors or random disturbance along with the deterministic part. In this section, the Conditional Logit (CLGT) under DCA platform is introduced that addresses the acceptance of an alternative in terms of choice probability under risk and uncertainty.

The discrete choice theory is generally fallen in rational choice theory and was popular in transportation and state to state migration problem (Davies, Greenwood, & Li, 2001). Wassenger et al. (2001) implemented Discrete Choice Analysis (DCA) in DBD framework to solve motor design problem and used economic benefit to the producer as a single criterion in alternative selection. Basic background of a discrete choice analysis is the choice from a set of mutually exclusive and collectively exhaustive alternatives based on utility maximization (Ben-Akiva & Lerman, 1985). In an operational model, the utility (U) is considered as the sum of observed component (u) which is the deterministic function of independent variables and unobserved or error component (ε) which is stochastic in nature. The utility of j alternative for an individual i is given by

$$U_{ij} = u_{ij} + \varepsilon_{ij} \tag{4.7}$$

Discrete choice can be categorized as Binary Choice where the numbers of alternatives are two and Multinomial Choice where the numbers of alternatives are more than two. Discrete choice analysis has many models. Very briefly the multinomial choice can be further categorized as Multinomial Logit (MNLGT) where the choice is based on characteristics of individuals across alternatives and Conditional Logit (CLGT) where the choice is based on characteristics of the attributes of alternatives across alternatives. The attention of this article is on conditional logit developed by McFadden (1974).

Conditional logit is a random utility model. If an individual i faces j choice, then the utility of j alternative following the expression (4.7) is given by,

$$U_{ij} = \beta \cdot x_{ij} + \varepsilon_{ij} \tag{4.8}$$

where, x_{ij} is a vector of attributes of alternatives and β is the coefficient of x_{ij} which is constant across alternatives. According to McFadden unobserved or error vector ε_{ij} are independent and identically distributed (IID) with the Gumbel distribution that produces a closed-form probabilistic choice model. The probability of individual i choosing alternative j among a set of alternatives J is given by,

$$P_{ij} = \frac{\exp\left(\beta \cdot x_{ij}\right)}{\sum_{k=1}^{J} \exp(\beta \cdot ij_{ik})} \tag{4.9}$$

Conditional Logit (CLGT) is popular in the field of demographic problems, i.e. transportation or state to state migration, but can easily be implemented in the field of material selection (Das et al., 2016). The main objective of this section is to implement the conditional logit in the multi-attributed rational decision-making framework of material selection to ensure a rational choice. Rationality has different meanings in different perspectives. To ensure rationality in choice is to analyse in a systematic way (Fransen & Bucciarelli, 2005). From the material selection point of view in engineering design, the rationality is the balancing between the customer's demands and desires that maximizes the overall utility in the form of probability.

The shape is given to a product according to the performance of the material. Performance is the measure of effectiveness in the form of design requirements; in turn, it is in the form of engineering attributes such as tensile strength, modulus of elasticity or Poisson's ratio. Design requirements are set according to the customer requirements which can be split up in demands and desires. Demands must be met under all circumstances. Desires are taken into consideration whenever necessary and converted into engineering attributes. Engineering attributes are further divided in benefit and cost attributes. The material selection flow diagram is same as shown in Figure 4.3.

In CLGT analysis, an individual (i) choose an alternative (j) with the highest utility (U_{ij}) which is the sum of deterministic component (u_{ij}) and stochastic component (ε_{ij}) as shown in expression (4.7). The deterministic component is the function of the observed variable (x_{ij}) and tastes variation coefficient β as shown in expression (4.8) and β is constant across alternatives. This assumption about β is advantageous to implement the CLGT in MADM framework. In the MADM framework, decision maker's taste is constant across alternatives and can be considered as equal to 1 in respect of each attribute. As the material selection is a MADM problem, therefore, for i alternative in respect of j attribute, the choice can be expressed as expression (4.9). In every decision-making process, decision maker's (customer or designer) taste parameters are considered those are varied attribute to attribute according to design requirements and socio-economic and demographic pattern of the customers. This is termed as weightage of the attributes. All considerations are described step-wise below.

Step 1: A set of alternatives (A) with performance ratings (D) in the form of attributes (E)

$$A = \{a_i | i = 1,2,\cdots,m\} \tag{4.10}$$

$$E = \left\{ e_j \middle| \underbrace{j = 1,2,\cdots,n-r}_{\text{benefit attribute}} ; \underbrace{j = n-r+1,\cdots,n}_{\text{cost attribute}} \right\} \tag{4.11}$$

$$D = \left\{ d_{ij} \middle| i = 1,2,\cdots,m; j = 1,2,\cdots,n \right\} \tag{4.12}$$

Step 2: Weightage to the attributes (E)

$$W = \left\{ w_j \middle| j = 1,2,\cdots,n \right\} \tag{4.13}$$

Step 3: Normalization of the performance ratings matrix (D)

$$r_{ij} = \frac{d_{ij}}{\sqrt{\sum_{i=1}^{m} {d_{ij}}^2}} \qquad j = 1,2,\cdots,n \tag{4.14}$$

Step 4: The choice probability of alternative *i* with respect to attribute *j* following (3) can be formulated by considering the product $\beta \cdot x_{ij}$ as a normalized value of the attribute as:

$$P_{ij} = \frac{\exp(\pm r_{ij})}{\sum_{i=1}^{m} \exp(\pm r_{ij})} \qquad j = 1,2,\cdots,n \tag{4.15}$$

A positive sign is for the benefit attributes such as modulus of elasticity or tensile strength and the negative sign is for the cost attributes such as cost or density.

Step 5: As the materials are multi-attributed, the weighted sum probability of the respective alternative is given by (Train, 2003),

$$P(a_i) = \sum_{j=1}^{n} w_j \cdot P_{ij} \qquad i = 1,2.\cdots,m \tag{4.16}$$

The highest value of $P(a_i)$ addresses the best choice among the alternatives. The relative weight w_j of j^{th} attribute in expression (7) can be assigned by using the analytical hierarchy process (AHP) or by digital logic (DL).

Decision making is the process to choose an appropriate alternative based on the belief of the decision maker. A designer point of view, a straightforward approach is visual or spatial relationship in the choice parameter that can give a justification in that belief which have been tried to the next section.

4.4 Nearest neighbor search (NNS)

Some of the methods (Cables et al., 2016; Lourenzutti & Krohling, 2014) can be regarded in some certain extent as a nearest neighbor search (NNS) approaches (Barrena et al., 2010). Nearest neighbor search is the finding of a point (a) in a given set which is nearer to a reference point or query point (q) in multi-dimensional Euclidean space. In the above mentioned TOPSIS-based methods, the closeness or nearness between two points is decided by a ratio lies in between 0 and 1 where the higher value dictates the close proximity. All the methods are structured with strong mathematical background, but if we consider the alternative as a point in the metric space, there is some lack of spatial relationship evidence from 'closeness' point of view. From mathematical point of view, the closeness between two points can be described as a distance parameter, $d(a,q) \to 0$ as $a \to q$ (Arkhangel'skii & Fedorchuk,1990).

At this point, very specifically, the aim of this model is to select the preeminent material in design based on NNS on the basis of $d(a,q) \to 0$ in the Cartesian plane under MADM framework through the eyes of TOPSIS. If the points (alternatives) are mapped in the 2-dimensional plane, then we can easily visualize and compare the points with the query point. The designers rather enjoy the spatial relationship which gives a confidence and courage under uncertainty.

The nearest neighbor search (NNS) is generally pronounced in the field of machine learning, pattern recognition, geometrical information systems (GIS) and many others (Papadopoulos & Manolopoulos, 2005; Moghtadaiee & Dempster, 2015). It deals with spatial data in an effective and efficient way. The data can be characterized as points or lines or higher entities oriented in 2-dimensional or multi-dimensional space. The scenario is same in the sense of material selection where the material is considered as a n-dimensional point in the above mentioned MADM-methods. There is a wide range of NNS-methods.

The proposed approach does not consider the traditional NNS-methods, but the basic definition of the NNS i.e. searching the nearness of a set of points ($a_{i=1,2,\cdots,m}$) to a query point (q) by means of a distance parameter in n-dimensional Euclidean space ($a_{i,}q \in \mathbb{R}^n$) shown in Figure 4.4. From the material selection point of view, the absolute nearness or similarity between a_i (alternatives) and q (reference alternative) depends upon the length of the position vectors ($|\overrightarrow{oa_i}|$, $|\overrightarrow{oq}|$) and the angle between the vectors (θ_i). Cosine similarity is a popular approach (Xia et al., 2015; Kou & Lin, 2014) to find the θ_i that investigates the similarity among alternatives and given by,

$$\cos\theta_i = \frac{\overrightarrow{oa_i} \cdot \overrightarrow{oq}}{|\overrightarrow{oa_i}| \cdot |\overrightarrow{oq}|} \qquad (i = 1,2,\cdots,m) \qquad (4.17)$$

Therefore, the comparison between a_i and q is the function of the length and orientation of the alternatives and should be mapped in Cartesian plane $((|\overrightarrow{oa_i}|, \theta_i, |\overrightarrow{oq}|): \mathbb{R}^n \rightarrow \mathbb{R}^2)$ in respect of the query vector to investigate the exact nearness between a_i and q by means of the distance parameter (Figure 4.4). Actually, these distance parameters are the dissimilarity functions of the alternatives and the less value of the dissimilarity function dictates the choice which is very nearer to the query point. Now the question is that which distance parameter should be considered, Euclidean distance (l_2 norm) or Manhattan distance (l_1 norm). In Cartesian plane, the Manhattan distance is the sum of the projected length of a Euclidean distance along the axes. The proximity of any two points depends upon the adjustment of the coordinates along the axes which preserve the absolute similarity information. Therefore, our decision favours the Manhattan distance. From Cartesian plane in Figure 4.4, the coordinates of a_i and q are $a_i = (|\overrightarrow{oa_i}| \cos(\theta_i + \varphi), |\overrightarrow{oa_i}| \sin(\theta_i + \varphi))$ and $q = (|\overrightarrow{oq}| \cos\varphi, |\overrightarrow{oq}| \sin\varphi)$. The Manhattan distance between a_i and q is given by,

$$d_M(a_i, q) = \overline{a_i b_i} + \overline{b_i q} \qquad i = 1,2, \cdots, m \tag{4.18}$$

$$\overline{a_i b_i} = \left| |\overrightarrow{oq}| \cos\varphi - |\overrightarrow{oa_i}| \cos(\theta_i + \varphi) \right| \tag{4.19}$$

$$\overline{b_i q} = \left| |\overrightarrow{oq}| \sin\varphi - |\overrightarrow{oa_i}| \sin(\theta_i + \varphi) \right| \tag{4.20}$$

The most favourable orientation of $|\overrightarrow{oq}|$ in Cartesian plane is the equi-inclination with the axes, i.e. $\varphi = 45°$ that gives the optimal position of q and the maximum equivalent Manhattan distance of the length $|\overrightarrow{oq}|$ $(d_M(o, q) = \overline{oh} + \overline{hq})$. In Cartesian plane of Figure 4.4, the line $\overline{oa_i}$ can be either any side of $\overline{oq}$ and θ_i may or may not be greater or less than $45°$, but the ultimate Manhattan distance $(d_M(a_i, q))$ will be the same in all cases and the generalise Manhattan distance between a_i and q following the expressions (4.18) to (4.20), is given by,

$$d_M(a_i, q) = \frac{1}{\sqrt{2}} \left\{ \left| |\overrightarrow{oq}| - |\overrightarrow{oa_i}| (\cos\theta_i - \sin\theta_i) \right| + \left| |\overrightarrow{oq}| - |\overrightarrow{oa_i}| (\cos\theta_i + \sin\theta_i) \right| \right\} \tag{4.21}$$

In Cartesian plane of Figure 4.4, the line $\overline{oa_i}$ can be either any side of $\overline{oq}$ and θ_i can be greater than 45°, but the ultimate Manhattan distance $(d_M(a_i, q))$ will be the same. The minimum value of the expression (4.21) addresses the preeminent alternative.

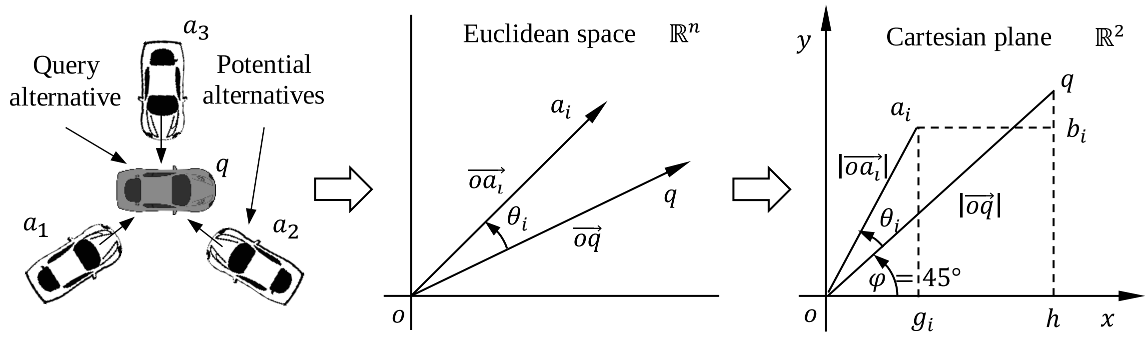


Figure 4.4 Spatial representation of alternatives and choice parameter

A set of alternatives (A) with performance ratings (D) is decided according to the design requirements. Performance is the measure of effectiveness in the form of attributes (E). Attributes are set according to the demands and desires of the design requirements and can be split up in benefit and cost attributes. The steps are followed to capture the preeminent alternative:

Step 1: A set of alternatives with performance ratings in the form of attributes.

$$A = \{a_i | i = 1,2,\cdots,m\} \tag{4.22}$$

$$E = \left\{ e_j \middle| \underbrace{j = 1,2,\cdots,n-r}_{\text{benefit attribute}}; \underbrace{j = n-r+1,\cdots,n}_{\text{cost attribute}} \right\} \tag{4.23}$$

$$D = \left\{ d_{ij} \middle| i = 1,2,\cdots,m; j = 1,2,\cdots,n \right\} \tag{4.24}$$

Step 2: Weightage to the attributes

$$W = \left\{ w_j \middle| j = 1,2,\cdots,n \right\} \tag{4.25}$$

Step 3: Normalization of the performance ratings matrix

$$r_{ij} = \frac{d_{ij}}{\sqrt{\sum_{i=1}^{m} {d_{ij}}^2}} \qquad j = 1,2,\cdots,n \tag{4.26}$$

Step 4: Weighted normalization of the performance ratings matrix

$$x_{ij} = r_{ij} \cdot w_j = \frac{d_{ij}}{\sqrt{\sum_{i=1}^{m} {d_{ij}}^2}} \cdot w_j \qquad j = 1,2,\cdots,n \tag{4.27}$$

Step 5: Query alternative with performance ratings

$$q = \{y_j(\max x_{ij}) | j = 1,2,\cdots,n-r;\ y_j(\min x_{ij}) | j = n-r+1,\cdots,n\} \tag{4.28}$$

Step 6: Mapping of the alternatives and query alternative in Euclidean space

$$\overrightarrow{oa_i} = \sum_{j=1}^{n} x_{ij} \cdot \widehat{e_j} \qquad i = 1,2,\cdots,n \tag{4.29}$$

$$|\overrightarrow{oa_i}| = \sqrt{\sum_{j=1}^{n} {x_{ij}}^2} \qquad i = 1,2,\cdots,n \tag{4.30}$$

$$\overrightarrow{oq} = \sum_{j=1}^{n} y_j \cdot \widehat{e_j} \tag{4.31}$$

$$|\overrightarrow{oq}| = \sqrt{\sum_{j=1}^{n} {y_j}^2} \tag{4.32}$$

Step 7: Cosine similarity following the expressions (4.17) and (4.29) to (4.32)

$$\cos\theta_i = \frac{\sum_{j=1}^{n} x_{ij} \cdot y_j}{\sqrt{\sum_{j=1}^{n} {x_{ij}}^2} \cdot \sqrt{\sum_{j=1}^{n} {y_j}^2}} \qquad i = 1,2.\cdots,m \tag{4.33}$$

$$\sin\theta_i = \sqrt{1-\cos^2\theta_i} \qquad\qquad i = 1,2.\cdots,m \tag{4.34}$$

Step 8: Calculation of Manhattan distance following the expressions (4.18) to (4.20) and (4.29) to (4.34) or directly from the expressions (4.21) and (4.29) to (4.34)

$$d_M(a_i,q) = \frac{1}{\sqrt{2}}\left\{\left||\overrightarrow{oq}| - |\overrightarrow{oa_i}|(\cos\theta_i - \sin\theta_i)\right| + \left||\overrightarrow{oq}| - |\overrightarrow{oa_i}|(\cos\theta_i + \sin\theta_i)\right|\right\} \tag{4.35}$$

The alternatives are ranked according to the minimum value of the Manhattan distance $(d_M\left(a_{i=1,2,\cdots m}, q\right)$. The steps followed and respective input/output of the proposed method are shown in Figure 4.5.

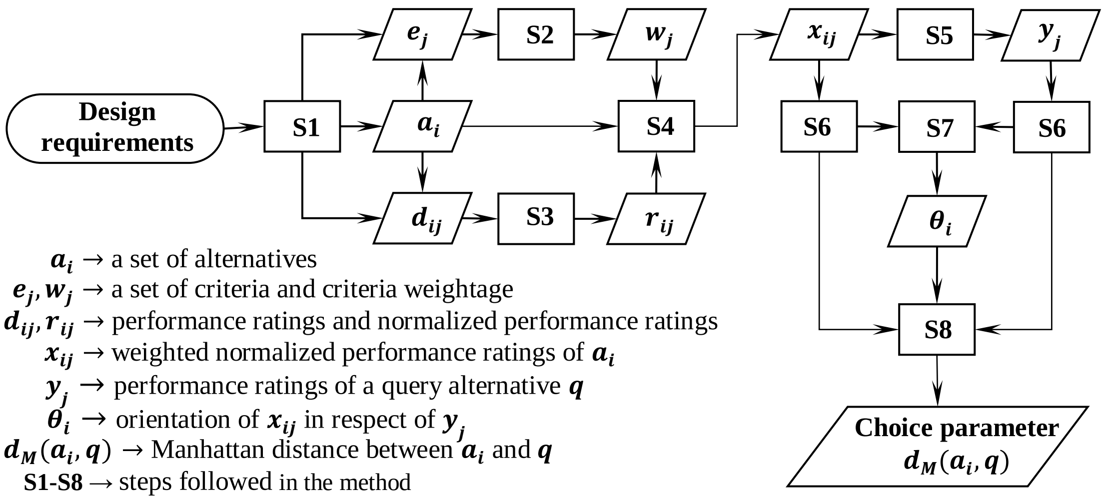


Figure 4.5 Nearest neighbor search-based material selection flow diagram

This is a new developed method in cognitive way where we can visualize the difference among the alternatives. It is easily understood, take minimum time having computational ability. In the next chapter some case studies are conducted to check the suitability of the above-mentioned methods.

Chapter 5

Implementation of the notion

5.1 Introduction

In Chapter 4 we have proposed three models to select a preeminent material based on normative-prescriptive approach, discrete choice analysis, and nearest neighbour search. In this chapter, we will implement these proposed models judiciously. Any model itself does not guarantee the right choice. It is important how precisely the problem is structured from the setup of objectives of the design from the beginning and the selection of appropriate criteria from the objectives.

The sole attention of a designer is to develop a product by choosing a preeminent material that must fulfill the functional requirements (FRs) as well as the customer requirements (CRs). The CRs are the variables that characterize the design in the customer domain in terms of demands and desires. The FRs characterize the design in the functional space that describes the intended behaviour of the device. The first work of a designer is to identify the CRs and to convert them into the feasible FRs. A product is a physical embodiment (Shape and material) of a feasible design that will fulfill the FRs. In typical mechanical design, the functional requirement can be represented as,

$$FR = f\{\text{material}(\text{attribute}), \text{shape}(\text{geometry})\}$$

where the material is characterized by mechanical properties (attribute). Appropriate combinations of these properties will dictate the suitability of a material for a specific application. Therefore, a systematic in-depth analysis should be carried out in material selection. Taking the past works, an empirical inquiry is conducted to investigate the reliability and validity of the proposed method in real-world practice.

Four case studies are presented in this chapter:

1. Two stage spur gear reduction unit (alternative generation and evaluation).
2. Cryogenic storage tank (alternative evaluation).
3. Flywheel (alternative evaluation).
4. Human powered aircraft (alternative evaluation).

In the first case study, we will start from the alternative generation followed by alternative evaluation where we will apply our proposed models. The next consecutives case studies are taken from the past works and the proposed models are applied. Each of the case studies starts by an introduction that provides background information about the product under consideration, followed by an analysis of its functional requirements and design.

5.2 Case Study 1: Two stage spur gear reduction unit

Gear reducers are used in all industries, they reduce speed and increase torque. We will find them between the prime mover (i.e.: electric motor, gas, diesel or steam engine, etc.) and the driven equipment: conveyors, mills, paper machines, elevators, screws, agitators, etc.). An industrial gearbox is defined as a machine for the majority of drives requiring a reliable life and factor of safety, and with the pitch line velocity of the gears limited to below 25 m/s, as opposed to mass produced gearboxes designed for a specific duty and stressed to the limit, or used for very high speeds etc., e.g. automobile, aerospace, marine gearboxes. The two types of tooth that can be used for both parallel and angled drives are straight or helical (spiral). Spur gears are easier to manufacture and inspect than helical gears, and they can be rectified more easily at the assembly stage if required. It is a general practice to design a gear under specific condition with preassigned material (Budynas, 2006; Bhandari, 2010). In this case study, a finite set of materials have been generated and these are strictly ferrous materials followed by proposed material selection methods

(NPA, DCA, and NNS) described in Chapter 4 to select the preeminent alternative. Let us consider a two-stage spur gear reduction unit problem and can be solved in a systematic way as,

Customer requirements

Engineering design is the process to formulize the customer requirements which in turn can be split up into demands and wishes. A typical customer requirement for 2-stage spur gear reduction unit can be stated as:

Demands	Wishes
Power to be transmitted 10 kw	To minimize the weight
Input speed 1440 r.p.m.; Output speed 120±2 r.p.m.	To minimize the cost
Distance between two shafts about 230 mm	

Functional requirements

From the above-mentioned customer requirements, demands are translated into functional requirements. An isolated gear has no identity; it should be analysed when it meshes with the other. In gear design, two types of failure are generally considered. First the bending failure due to the tangential load through the pitch point that causes bending stress (Lewis equation), and second the surface fatigue failure known as pitting due to the load along the line of action through the pitch point that causes contact stress (Hertz theory) (Budynas, 2006).

Gear tooth under bending:

Ultimately the bending stress σ^b in N/mm^2 considering various factors is given by,

$$\sigma^b = (fs)\left(\frac{C^s}{C^v}\right)\left(\frac{F^t}{mbY}\right) \qquad 8m \le b \le 12m \tag{5.1}$$

where, fs is the factor of safety, C^s is the service factor, C^v is the velocity factor, m is the module in mm, b is the face width in mm, Y is the Lewis form factor and F^t is the tangential force at the pitch point in N, given by,

$$F^t_{pw} = \frac{P \times 10^3}{V_{pw}} \tag{5.2}$$

where, P is the power to be transmitted in kw and V_{pw} is the linear velocity at the pitch point in m/s, given by,

$$V_{pw} = \frac{2\pi r_k N_k}{60 \times 10^3} \qquad \{k = p \text{ or } w\} \tag{5.3}$$

where, r is the pitch circle radius of the gear in mm and N is the speed of the shaft in r.p.m. on which the gear is mounted. According to Earle Buckingham, endurance limit stress (in case of gear tooth it is bending stress) is approximately one third of the ultimate tensile strength σ^{ut}. Therefore, the ultimate tensile strength can be rewritten with combining the expressions (5.1) to (5.3) as,

$$\sigma_k^{ut} = 3(fs)\left(\frac{C^s}{C_{pw}^v}\right)\left(\frac{P \times 10^3}{mbY_k}\right)\left(\frac{60 \times 10^3}{2\pi r_k N_k}\right) \qquad \{k = p, w\} \tag{5.4}$$

It is the minimum ultimate tensile strength required to select the gear material to resist the failure under bending which is associated with the geometric configuration (function of width *b*) of the gear and power to be transmitted. Suffix *p*, *w*, and *pw* indicate the pinion, wheel, and common for pinion and wheel both at the pitch point respectively.

Gear tooth under pitting:

The contact stress σ^c in N/mm^2, between to teeth are in mesh is given by,

$$\sigma^c = \left\{\left(\frac{\frac{F^t}{\cos\varphi}}{\pi b}\right)\frac{\frac{1}{r_p \sin\varphi} + \frac{1}{r_w \sin\varphi}}{\frac{1-\nu_p{}^2}{E_p} + \frac{1-\nu_w{}^2}{E_w}}\right\}^{1/2} \tag{5.5}$$

where, E_p, E_w are the modulus of elasticity in N/mm^2 and ν_p, ν_w are the poisson's ratio of pinion and wheel respectively, F^t is the tangential force exerted by the tooth at the pitch point, r_p and r_w are the pitch circle radius of pinion and wheel respectively, and φ is the pressure angle. According to G Niemann, for ferrous materials, contact stress in N/mm^2 related with Brinell Hardness Number is given by,

$$\sigma^c = 0.27(9.81)(\text{BHN}) \tag{5.6}$$

If pinion and wheel are made up of same material i.e., $E_p = E_w$ and $\nu_p = \nu_w$ and combining the expressions (5.5) and (5.6), then

$$BHN_{pw} = 0.15\left\{\left(\frac{F^t}{b\cos\varphi\sin\varphi}\right)\left(\frac{1}{r_p}+\frac{1}{r_w}\right)\left(\frac{E_k}{1-\nu_k{}^2}\right)\right\}^{1/2} \tag{5.7}$$

It is the minimum hardness required for the material against pitting. It is also associated with the geometric configuration of the gears and material properties. From the above discussion it can be concluded that the system demands required ultimate tensile strength and surface hardness in terms of BHN as well as both should be maximized.

Identification of Design Parameters

A simplified line diagram of two stages speed reduction unit having spur gear with involute gear teeth shown in Fig. 3 where,

r_1, r_2, r_3, r_4 are the pitch circle radius in mm of gear 1, 2, 3, and 4 respectively with $r_1 + r_2 = r_3 + r_4$,

N_1, N_2, N_3, N_4 are the speeds in r.p.m. of gear 1, gear 2, gear 3, and gear 4 respectively,

T_1, T_2, T_3, T_4 are the number of teeth of gear 1, gear 2, gear 3, and gear 4 respectively,

V_{12}, V_{34} are the pitch line velocities in m/s at the pitch point P_{12} and P_{34} respectively,

F^t_{12}, F^t_{34} are the tangential forces in Newton at P_{12} and P_{34} respectively.

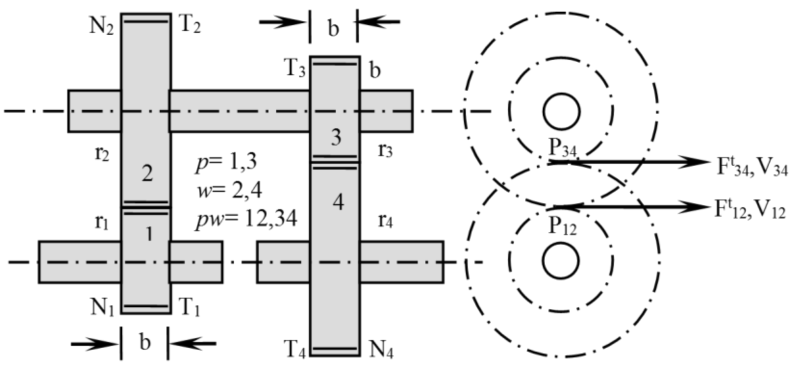


Figure 5.1 Two stage spur gear reduction unit

Number of teeth, speed of the gears, module, pitch circle radius, Lewis form factor, and velocity factor has been tabulated (Table 5.1) following the general toothed gear formulae (Budynas, 2006; Bhandari, 2010) for the given specification. It is necessary to establish the primary specification before select the material and the centre distance between the shafts ($r_1 + r_2 = r_3 + r_4$) takes the new value of 240 mm. An isolated gear has no identity; it should be analyzed when it meshes with the other.

Table 5.1 Design Parameters of Case study 1

Gear-1	Gear-2	Gear-3	Gear-4
Pressure angle, ϕ=20° full-depth			
$T_1 = 18$	$T_2 = 62$	$T_3 = 18$	$T_4 = 62$
$N_1 = 1440$	$N_2 = 418$	$N_3 = 418$	$N_4 = 121$
Module, $m = 6$			
$r_1 = 54$	$r_2 = 186$	$r_3 = 54$	$r_4 = 186$
$V_{12} = 8.13$		$V_{34} = 2.36$	
$F_{12}^t = 1229.55$		$F_{34}^t = 4237.29$	
$C_{12}^v = 0.27$		$C_{34}^v = 0.56$	
$Y_1 = 0.309$	$Y_2 = 0.432$	$Y_3 = 0.309$	$Y_4 = 0.432$
Factor of safety, $fs = 1.5$ Service factor, $C^s = 1.5$			

Selection of Materials

An isolated gear has no identity; it should be analysed when it meshes with the other. Materials can be selected separately for pinion and wheel. As the gearbox acts as a single unit and the nature of the all elements inside the gearbox are same therefore materials are selected in the same platform for the sake of economic production.

Pre-selection of materials

At initial stage the potential materials are selected based on max-min required ultimate tensile strength because Brinell hardness number is related with the material properties and at this point materials are not known to us.

$$\sigma_{\max}^{ut} = \sigma_k^{ut}|_{b=8m} \qquad \{k = p, w\} \tag{5.8}$$

$$\sigma_{\min}^{ut} = \sigma_k^{ut}|_{b=12m} \qquad \{k = p, w\} \tag{5.9}$$

Attributes are chosen according to the nature of the functional requirements following the expressions (5.4) and (5.7), i.e. Ultimate tensile strength, Modulus of elasticity, Brinell hardness number, Poisson's ratio, and to the customer wishes, i.e. density, cost. The results of the expressions (5.8) and (5.9) are tabulated in Table 5.2 and accordingly a set of potential materials in Table 5.3 and it is typically restricted to ferrous materials (Walsh, 2000).

Table 5.2 Induced stresses in the gears, $k = 1,2,3,4$ of Case study 1

For $fs = 1.5$ and $C^s = 1.5$	Ultimate tensile strength in N/mm^2			
	Pinion, k=1	Wheel, k=2	Pinion, k=3	Wheel, k=4
$\sigma_{\min}^{ut}$	231	165	383	274
$\sigma_{\max}^{ut}$	346	248	574	411

Table 5.3 Potential materials based on $165 \leq \sigma^{ut} \leq 574$ of Case study 1

Potential materials	**HT**	$\boldsymbol{\sigma^{ut}}$	***E***	**BHN**	ν	ρ	***c***
Plain carbon and Low alloy steel							
AISI 1020	A	395	200	111	0.30	7.87	1.15
AISI 1040	A	515	200	149	0.30	7.84	1.30
AISI 4130	A	560	200	156	0.30	7.85	2.00
Stainless steel							
AISI 304	A	515	193	147	0.30	7.80	2.85
AISI 330	A	550	197	136	0.30	8.00	3.70
AISI 405	A	448	200	150	0.30	7.80	3.50
Grey cast iron							
ASTM class 30	AC	205	104	180	0.26	7.10	3.30
ASTM class 40	AC	275	125	220	0.26	7.20	3.30
ASTM class 60	AC	431	152	285	0.26	7.30	3.30
Ductile cast iron							
Grade 60-40-18	A	416	169	167	0.29	7.10	4.00
Grade 65-45-12	AC	464	168	167	0.31	7.10	4.00
Grade 80-55-06	N	559	168	192	0.31	7.10	4.00

σ^{ut}=Ultimate tensile strength in MPa, E=Modulus of elasticity in GPa, BHN=Brinell hardness number, ν=Poisson's ratio, ρ=Density in g/cc, c =Cost in US $/kg. HT: Heat Treatment, A: Annealed, QT: Quench & Tempered, N: Normalized, AC: As Cast.

Screening

At this stage the alternatives for pinion and wheel are selected on the basis of minimum ultimate tensile strength and minimum common Brinell hardness number at the pitch point against a desired value of face width.

$$\mathrm{BHN_{min}} = BHN_{ipw}\big|_{b=12m} \qquad \{i = 1,2,\cdots\} \tag{5.10}$$

A set of candidate materials,

$$A_i = \{a_i(\sigma^{ut},\mathrm{BHN})|i = 1,2,\cdots\} \quad \text{if } \sigma^{ut} \geq \sigma^{ut}_{\min} \text{ and BHN} \geq \mathrm{BHN_{min}} \tag{5.11}$$

The results of expressions (5.10) and (5.11) are tabulated in Table 5.4. Before starting ranking process further short listing is necessary from economical production point of view. In this paper it is assumed that all gears are made up of same material. Therefore, the materials those are common for all gears in Table 5.4 that will be shortlisted and Table 5.4 shows the shortlisted candidate materials as: AISI 1040 (a_1), AISI 4130 (a_2), AISI 304 (a_3), AISI 405 (a_4), ASTM class 60 (a_5), Grade 60-40-18 (a_6), Grade 65-45-12 (a_7), and Grade 80-55-06 (a_8). These shortlisted candidate materials will be evaluated by the proposed methods (NPA, DCA, and NNS) in the consecutive sections.

Table 5.4 Screened candidate materials of Case study 1

Potential Materials (σ^{ut},BHN)	Screening Criteria		Candidate Materials			
	$\mathrm{BHN}_{12}\|_{b=12m}$ Lower Limit	$\mathrm{BHN}_{34}\|_{b=12m}$ Lower Limit	Pinion (k=1) $\sigma_1^{ut} \geq 231$	Wheel (k=2) $\sigma_2^{ut} \geq 165$	Pinion (k=3) $\sigma_3^{ut} \geq 383$	Wheel (k=4) $\sigma_4^{ut} \geq 274$
AISI 1020 (395,111)	80	148	✓	✓	×	×
AISI 1040 (515,149)	80	148	✓	✓	✓	✓
AISI 4130 (560,156)	80	148	✓	✓	✓	✓
AISI 304 (515,147)	78	146	✓	✓	✓	✓
AISI 330 (550,136)	80	147	✓	✓	×	×
AISI 405 (448,150)	80	149	✓	✓	✓	✓
ASTM class 30 (205,180)	57	106	×	✓	×	×
ASTM class 40 (275,220)	62	116	✓	✓	×	×
ASTM class 60 (431,285)	69	129	✓	✓	✓	✓
Grade 60-40-18 (416,167)	73	137	✓	✓	✓	✓
Grade 65-45-12 (464,167)	73	137	✓	✓	✓	✓
Grade 80-55-06 (559,192)	73	137	✓	✓	✓	✓

5.2.1 Normative-prescriptive approach (NPA)

In this proposed approach, it is required to assign the utility value to the respective attribute of the shortlisted alternatives which can be categorized as beneficial (higher values are desirable) and non-beneficial (lower values are desirable) and to find out the overall utility value (Expected Utility) of the respective alternatives following the sections 3.2.1 and 3.2.2 where these shortlisted alternatives are already evaluated. All attributes are not essential for evaluation stage, knowing more does not guarantee the good decision. To make a decision we need to know the problem, the need and purpose of the decision, the criteria of the decision, their sub-criteria, stakeholders and affected groups (Satty, 2000). The set of attributes should be kept minimal for simplicity. The attributes are chosen as:

- *Benefit criteria*: Ultimate tensile strength, σ^{ut} (e_1) and Brinell hardness number, BHN (e_2).
- *Cost criteria*: Density, ρ (e_3) and cost, c (e_4) to balance between customers' requirements and design requirements.

In section 3.2.1, weightage to the criteria is assigned by the AHP process (Table 3.3) and in section 3.2.2, the overall utility value of each alternative is assigned by the MAUT method. However, we would like to present again that furnished result here in Table 5.5.

Table 5.5 Overall performance of the Case study 1 following the NPA

Alternative (a_i)		e_1 (max) $(w_1 = 0.47)$		e_2 (max) $(w_2 = 0.33)$		e_3 (min) $(w_3 = 0.08)$		e_4 (min) $(w_4 = 0.12)$		$U(a_i)$	Rank
		d_{i1}	u_{i1}	d_{i2}	u_{i2}	d_{i3}	u_{i3}	d_{i4}	u_{i4}		
a_1	AISI 1040	515	0.687	149	0.014	7.84	0.013	1.30	1.000	0.449	4
a_2	AISI 4130	560	1	156	0.065	7.85	0.000	2.00	0.741	0.580	2
a_3	AISI 304	515	0.687	147	0	7.80	0.067	2.85	0.426	0.379	5
a_4	AISI 405	448	0.222	150	0.021	7.80	0.067	3.50	0.185	0.139	7
a_5	ASTM class 60	431	0.104	285	1	7.30	0.733	3.30	0.259	0.469	3
a_6	Grade 60-40-18	416	0	167	0.145	7.10	1.000	4.00	0.000	0.128	8
a_7	Grade 65-45-12	464	0.333	167	0.145	7.10	1.000	4.00	0.000	0.284	6
a_8	Grade 80-55-06	559	0.993	192	0.326	7.10	1.000	4.00	0.000	0.654	1

5.2.2 Discrete choice analysis (DCA)

Here we will directly proceed from the ranking or evaluation process following the expressions (4.10) to (4.16) and results are tabulated in Table 5.6.

Table 5.6 Overall performance of Case study 1 following DCA

Alternative (a_i)		e_1 (max) ($w_1 = 0.47$)			e_2 (max) ($w_2 = 0.33$)			e_3 (min) ($w_3 = 0.08$)			e_4 (min) ($w_4 = 0.12$)			$P(a_i)$	Rank
		d_{i1}	r_{i1}	P_{i1}	d_{i2}	r_{i2}	P_{i2}	d_{i3}	r_{i3}	P_{i3}	d_{i4}	r_{i4}	P_{i4}		
a_1	AISI 1040	515	0.371	0.127	149	0.290	0.118	7.84	0.370	0.123	1.30	0.141	0.151	0.1268	4
a_2	AISI 4130	560	0.403	0.132	156	0.303	0.120	7.85	0.370	0.123	2.00	0.217	0.140	0.1280	2
a_3	AISI 304	515	0.371	0.127	147	0.286	0.118	7.80	0.368	0.123	2.85	0.309	0.128	0.1238	5
a_4	AISI 405	448	0.322	0.121	150	0.292	0.118	7.80	0.368	0.123	3.50	0.380	0.119	0.1202	7
a_5	ASTM class 60	431	0.310	0.120	285	0.554	0.154	7.30	0.344	0.126	3.30	0.358	0.122	0.1318	1
a_6	Grade 60-40-18	416	0.299	0.119	167	0.325	0.122	7.10	0.335	0.127	4.00	0.434	0.113	0.1198	8
a_7	Grade 65-45-12	464	0.334	0.123	167	0.325	0.122	7.10	0.335	0.127	4.00	0.434	0.113	0.1218	6
a_8	Grade 80-55-06	559	0.402	0.131	192	0.373	0.128	7.10	0.335	0.127	4.00	0.434	0.113	0.1278	3

5.2.3 Nearest neighbor search (NNS)

The performance ratings, normalized performance ratings weighted normalized performance ratings are tabulated in Table 5.7 following the expressions (4.22) to (4.27). From the Table 5.7 and following the expression (4.28), the query alternative with performance ratings is given by,

$$q = [0.189, 0.183, 0.027, 0.017]$$

The overall performances of the alternatives are evaluated through the expressions (4.29) to (4.35) and the results are tabulated in Table 5.8.

5.2.4 Results and Discussion

Some approaches show the Carburized Steel as a gear material [Milani et al., 2005; Chatterjee & Chakraborty, 2012). Carburized Steel is well-known gear material but it is not new to choose it as a gear material. Ultimate selection of gear material depends on functional requirements i.e. forces acting on it as well as the required factor of safety and the analysis directs us towards the grey cast iron or ductile cast iron. Ductile cast iron as a gear material is becoming very popular in automotive industries specially austempered ductile iron (ADI) due to its high strength to weight ratio, good damping quality and recyclability (Guesser et al., 2012; Davis, 2005). In Table 5.9, the proposed methods (DCA and NNS) give consistent results with TOPSIS (Jahan et al., 2012). ELECTRE raises the most dominating alternatives ASTM class 60 (a_5), Grade 80-55-06 (a_8), and AISI 4130 (a_2) from the dominance diagram (Table 3.7). It is the limitation of the ELECTRE method that it does not encourage the ranking. It is the decision-making paradox that different methods give different rankings due to their own philosophy and mathematical foundation. We need to examine some another case studies for better understanding in the next consecutive sections.

Table 5.7 Performance ratings, normalized, and weighted normalized performance ratings of Case study 1 following the NNS

Alternative (a_i)		e_1 (max) ($w_1 = 0.47$)			e_2 (max) ($w_2 = 0.33$)			e_3 (min) ($w_3 = 0.08$)			e_4 (min) ($w_4 = 0.12$)		
		d_{i1}	r_{i1}	x_{i1}	d_{i2}	r_{i2}	x_{i2}	d_{i3}	r_{i3}	x_{i3}	d_{i4}	r_{i4}	x_{i4}
a_1	AISI 1040	515	0.371	0.174	149	0.290	0.096	7.84	0.370	0.030	1.30	0.141	0.017
a_2	AISI 4130	560	0.403	0.189	156	0.303	0.100	7.85	0.370	0.030	2.00	0.217	0.026
a_3	AISI 304	515	0.371	0.174	147	0.286	0.094	7.80	0.368	0.029	2.85	0.309	0.037
a_4	AISI 405	448	0.322	0.152	150	0.292	0.096	7.80	0.368	0.029	3.50	0.380	0.046
a_5	ASTM class 60	431	0.310	0.146	285	0.554	0.183	7.30	0.344	0.028	3.30	0.358	0.043
a_6	Grade 60-40-18	416	0.299	0.141	167	0.325	0.107	7.10	0.335	0.027	4.00	0.434	0.052
a_7	Grade 65-45-12	464	0.334	0.157	167	0.325	0.107	7.10	0.335	0.027	4.00	0.434	0.052
a_8	Grade 80-55-06	559	0.402	0.189	192	0.373	0.123	7.10	0.335	0.027	4.00	0.434	0.052

Table 5.8 Overall performance evaluation of Case study 1 following the NNS

Candidate Material		$\lvert\overrightarrow{oa_i}\rvert$	$\lvert\overrightarrow{oq}\rvert$	$\overrightarrow{oa_i}\cdot\overrightarrow{oq}$	$\cos\theta_i$	$\sin\theta_i$	$\overline{a_i b_i}$	$\overline{b_i q}$	$d_M(a_i, q)$	Rank
a_1	AISI 1040	0.2016	0.2650	0.0515	0.9640	0.2658	0.0879	0.0121	0.0999	5
a_2	AISI 4130	0.2178	0.2650	0.0554	0.9590	0.2835	0.0833	0.0040	0.0873	3
a_3	AISI 304	0.2037	0.2650	0.0516	0.9562	0.2928	0.0919	0.0075	0.0994	4
a_4	AISI 405	0.1875	0.2650	0.0478	0.9623	0.2721	0.0959	0.0237	0.1196	7
a_5	ASTM class 60	0.2394	0.2650	0.0625	0.9851	0.1718	0.0497	0.0084	0.0582	1
a_6	Grade 60-40-18	0.1863	0.2650	0.0478	0.9684	0.2495	0.0927	0.0270	0.1196	8
a_7	Grade 65-45-12	0.1988	0.2650	0.0509	0.9655	0.2603	0.0882	0.0150	0.1033	6
a_8	Grade 80-55-06	0.2331	0.2650	0.0599	0.9693	0.2458	0.0681	0.0129	0.0810	2

Table 5.9 Rank comparison of Case study 1

Candidate Material		NPA	DCA	NNS	Pahl & Beitz (Table 2.11)	TOPSIS (Table 3.5)	VIKOR (Table 3.6)	ELECTRE (Table 3.7)
a_1	AISI 1040	4	4	5	4	4	3	4
a_2	AISI 4130	2	2	3	2	3	2	1 or 2 or 3
a_3	AISI 304	5	5	4	5	5	4	5
a_4	AISI 405	7	7	7	7	8	7	8
a_5	ASTM class 60	3	1	1	3	1	6	1 or 2 or 3
a_6	Grade 60-40-18	8	8	8	8	7	8	7
a_7	Grade 65-45-12	6	6	6	6	6	5	6
a_8	Grade 80-55-06	1	3	2	1	2	1	1 or 2 or 3

5.3 Case study 2: Cryogenic storage tank

Cryogenic storage tank also referred as cryogenic liquid container and cryogenic storage dewar is basically a double walled vacuum vessel shown in Figure 5.2. It carries the liquid nitrogen, oxygen, hydrogen, argon, and helium gas with less than 110 K/–163 °C temperature. Therefore, the cryogenic tank must be designed that can withstand against brittle fracture at low temperature. In this regard, the customer requirements can be stated as:

Demand: Cryogenic tank to store the liquid nitrogen gas; and

Desire: Minimum weight for transporting.

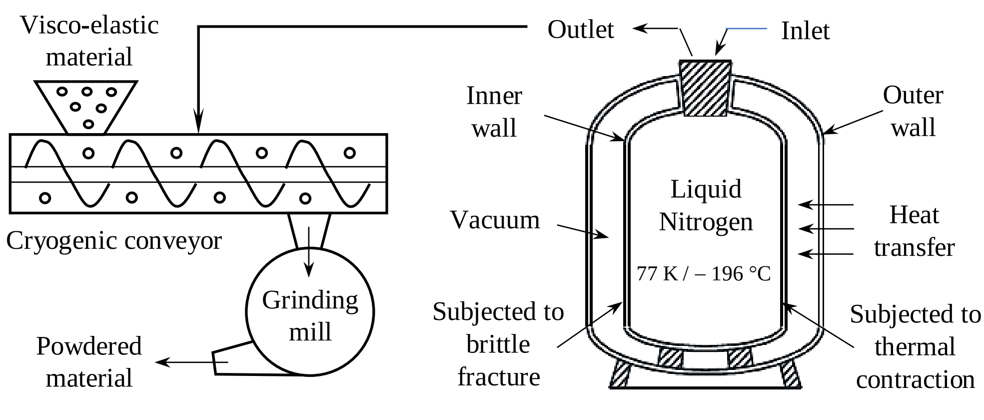


Figure 5.2 Cryogenic storage tank and an industrial application

To meet the customer requirements, the design requirements of the cryogenic tank can be stated as (Farag, 2014; Flynn, 2005):

- *Fracture toughness*: The boiling temperature of liquid nitrogen gas is about −196 °C. At this temperature, the materials lose their ductile behaviour and become brittle in nature. Therefore, materials should be sufficiently tough to resist the brittle fracture. The metals having face-centered cube (fcc) lattice are suitable because of their relatively insensitiveness at low temperature.
- *Heat transfer*: The flow of heat through the wall of cryogenic tank is generally conduction type. Materials having low thermal conductivity are preferable.
- *Thermal stress*: Due to low temperature, the inner wall is subjected to contraction that causes thermal stresses. Therefore, Materials having low coefficient of thermal expansion are suitable.
- *Thermal diffusivity*: perfect thermal insulation is not possible practically. Materials should be chosen that they can release the heat as quickly as possible. Diffusivity measures the rate of transfer of heat, which is inversely proportional to the specific heat of the materials.
- *Transportation*: Materials having lower specific gravity are suitable for transportation.

From the above-mentioned discussion and multi-criteria decision-making point of view, all criteria or attributes can be categorized as:

- *Benefit attribute*: Toughness index (e_1); Yield stress (e_2); Young's modulus (e_3);
- *Cost attribute*: Specific gravity (e_4); Coefficient of thermal expansion (e_5); Thermal conductivity (e_6); Specific heat (e_7).

Some of the candidate materials under uncertainty having face-centered cube (fcc) lattice are given in Table 5.10 with their performance ratings. In previous literature, MOORA (Karande et al., 2012), WPM (Manshadi et al., 2007), TOPSIS (Jahan et al., 2012), and Fuzzy Logic (Khabbaz et al., 2009) have been used to select the preeminent material. In this thesis, the proposed methods (DCA and NNS) will be applied to the consecutive sections and to compare the results. The relative weights to the criteria are assigned following the previous works (Khabbaz et al., 2009; Jahan et al., 2012) and given by,

$$w_{j=1,\cdots,7} = [0.28, 0.14, 0.05, 0.24, 0.19, 0.05, 0.05]$$

Table 5.10 Performance ratings (d_{ij}) of Case study 2

Sl. No.	Candidate Material	Toughness index	Yield stress (MPa)	Young's modulus (GPa)	Specific gravity	Coeff. of thermal exp. (10^{-6}/°C)	Thermal conductivity (cal/cm²/cm/°C/s)	Specific heat (cal/g/°C)
1.	Al 2014 – T6	75.50	420.00	74.20	2.80	21.40	0.37	0.16
2.	Al 5052 – O	95.00	91.00	70.00	2.68	22.10	0.33	0.16
3.	SS 301 – FH	770.00	1365.00	189.00	7.90	16.90	0.04	0.08
4.	SS 310 – 3/4H	187.00	1120.00	210.00	7.90	14.40	0.03	0.08
5.	Ti–6Al – 4V	179.00	875.00	112.00	4.43	9.40	0.02	0.09
6.	Inconel 718	239.00	1190.00	217.00	8.51	11.50	0.31	0.07
7.	70Cu – 30Zn	273.00	200.00	112.00	8.53	19.90	0.29	0.06

5.3.1 Discrete choice analysis (DCA)

Normalized performance ratings are tabulated in Table 5.11 following the expression (4.14). The final outcome, i.e. the choice probabilities and weighted sum probabilities of the alternatives are tabulated in Table 5.12 following the expressions (4.15) and (4.16).

Table 5.11 Normalized performance ratings (r_{ij}) of Case study 2 following DCA

Candidate Material (a_i)		e_1	e_2	e_3	e_4	e_5	e_6	e_7
a_1	Al 2014 – T6	0.0841	0.1787	0.1841	0.1604	0.4720	0.5651	0.5636
a_2	Al 5052 – O	0.1058	0.0387	0.1737	0.1535	0.4874	0.5040	0.5636
a_3	SS 301 – FH	0.8575	0.5808	0.4690	0.4526	0.3727	0.0611	0.2818
a_4	SS 310 – 3/4H	0.2083	0.4765	0.5211	0.4526	0.3176	0.0458	0.2818
a_5	Ti–6Al – 4V	0.1993	0.3723	0.2779	0.2538	0.2073	0.0244	0.3170
a_6	Inconel 718	0.2662	0.5063	0.5385	0.4876	0.2536	0.4734	0.2466
a_7	70Cu – 30Zn	0.3040	0.0851	0.2779	0.4887	0.4389	0.4429	0.2113

Table 5.12 Choice probability (P_{ij}) and weighted sum probability ($P(a_i)$) of Case study 2 following DCA

Candidate Material (a_i)		Choice probability, P_{ij}							$P(a_i)$	Rank
		e_1 (+ve)	e_2 (+ve)	e_3 (+ve)	e_4 (−ve)	e_5 (−ve)	e_6 (−ve)	e_7 (−ve)		
		$w_1 = 0.28$	$w_2 = 0.14$	$w_3 = 0.05$	$w_4 = 0.24$	$w_5 = 0.19$	$w_6 = 0.05$	$w_7 = 0.05$		
a_1	Al 2014 – T6	0.1125	0.1216	0.1199	0.1709	0.1276	0.1070	0.1146	0.1309	5
a_2	Al 5052 – O	0.1150	0.1057	0.1186	0.1721	0.1257	0.1138	0.1146	0.1295	7
a_3	SS 301 – FH	0.2438	0.1818	0.1594	0.1276	0.1409	0.1772	0.1519	0.1755	1
a_4	SS 310 – 3/4H	0.1274	0.1638	0.1679	0.1276	0.1489	0.1799	0.1519	0.1425	4
a_5	Ti–6Al – 4V	0.1262	0.1476	0.1317	0.1556	0.1663	0.1838	0.1466	0.1481	2
a_6	Inconel 718	0.1350	0.1688	0.1709	0.1232	0.1587	0.1173	0.1574	0.1434	3
a_7	70Cu – 30Zn	0.1402	0.1107	0.1317	0.1231	0.1319	0.1210	0.1630	0.1301	6

5.3.2 Nearest neighbour search (NNS)

The weighted normalized performance ratings are tabulated in Table 5.13 following the expressions (4.22) to (4.27). From the Table 5.13 and following the expression (4.28), the query alternative with performance ratings is given by,

$$q = [0.2401, 0.0813, 0.0269, 0.0369, 0.0394, 0.0012, 0.0106]$$

The overall performances of the alternatives are evaluated through the expressions (4.29) to (4.35) and tabulated in Table 5.14.

Table 5.13 Weighted normalized performance ratings (x_{ij}) of Case study 2 following NNS

Candidate Material		e_1 (max)	e_2 (max)	e_3 (max)	e_4 (min)	e_5 (min)	e_6 (min)	e_7 (min)
a_1	Al 2014 – T6	0.0235	0.0250	0.0092	0.0385	0.0897	0.0283	0.0282
a_2	Al 5052 – O	0.0296	0.0054	0.0087	0.0369	0.0926	0.0252	0.0282
a_3	SS 301 – FH	0.2401	0.0813	0.0234	0.1086	0.0708	0.0031	0.0141
a_4	SS 310 – 3/4H	0.0583	0.0667	0.0261	0.1086	0.0603	0.0023	0.0141
a_5	Ti–6Al – 4V	0.0558	0.0521	0.0139	0.0609	0.0394	0.0012	0.0159
a_6	Inconel 718	0.0745	0.0709	0.0269	0.1170	0.0482	0.0237	0.0123
a_7	70Cu – 30Zn	0.0851	0.0119	0.0139	0.1173	0.0834	0.0221	0.0106

Table 5.14 Overall performance evaluation of Case study 2 following NNS

Candidate Material		$\lvert\overrightarrow{oa_i}\rvert$	$\overrightarrow{oa_i}\cdot\overrightarrow{oq}$	$\lvert\overrightarrow{oq}\rvert$	$\cos\theta_i$	$\sin\theta_i$	$\overline{a_ib_i}$	$\overline{b_iq}$	$d_M(a_i,q)$	Rank
a_1	Al 2014 – T6	0.1113	0.0132	0.2608	0.4547	0.8906	0.2187	0.0785	0.2972	6
a_2	Al 5052 – O	0.1111	0.0131	0.2608	0.4521	0.8920	0.2190	0.0788	0.2978	7
a_3	SS 301 – FH	0.2861	0.0718	0.2608	0.9623	0.2720	0.0448	0.0653	0.1101	1
a_4	SS 310 – 3/4H	0.1555	0.0267	0.2608	0.6584	0.7527	0.1948	0.0293	0.2241	4
a_5	Ti–6Al – 4V	0.1074	0.0220	0.2608	0.7854	0.6190	0.1718	0.0778	0.2496	5
a_6	Inconel 718	0.1674	0.0308	0.2608	0.7055	0.7087	0.1848	0.0170	0.2018	2
a_7	70Cu – 30Zn	0.1700	0.0295	0.2608	0.6654	0.7465	0.1942	0.0147	0.2089	3

5.3.3 Results and discussion

For low temperature, less than –163 °C, the face-centered cube (fcc) materials are widely used. The austenitic steel SS 301 – FH is ranked 1 material in Table 5.12 and Table 5.14 which is a consistent result with the previous works and real-world practice. Austenitic steel still now is very popular and widely used liquid nitrogen or hydrogen storage tank (Godula-Jopek et al., 2012). Now it is the time to check the consistency of the ranking of the materials in Table 5.12 and Table 5.14 with previous works as shown in Table 5.15. Sometimes second option is very important in the ranking. In (Dehghan-Manshadi et al., 2007; Jahan et al., 2012), the titanium alloy (Ti-6Al-4V) got 2nd rank. Ti-6Al-4V is excellent in the aerospace industry, but for low temperature embrittlement cases, titanium alloys are very weak (Flynn, 2005) whereas Inconel is a good alternative than Titanium alloy as a cryogenic storage tank material.

Table 5.15 Rank comparison of Case study 2

Candidate Material		**DCA**	**NNS**	MOORA (Karande et al., 2012)	WPM (Manshadi et al., 2007)	TOPSIS (Jahan et al., 2012)	Fuzzy Logic (Khabbaz et al., 2009)
a_1	Al 2014 – T6	5	6	7	5	4	6
a_2	Al 5052 – O	7	7	6	7	5	7
a_3	SS 301 – FH	1	1	1	1	1	1
a_4	SS 310 – 3/4H	4	4	4	4	6	4
a_5	Ti–6Al – 4V	2	5	5	2	2	2
a_6	Inconel 718	3	2	2	3	3	3
a_7	70Cu – 30Zn	6	3	3	6	7	5

5.4 Case study 3: Flywheel

The flywheel is an energy storage device used to maintain the minimum fluctuation of speed of a machine. Energy density, i.e. ratio of the energy (e) and mass (m) is an important parameter used to qualify the energy storage device. The energy density is proportional to the ratio between maximum stress (σ_u) the material can withstand and its density (ρ) (Genta, 1985; Ashby, 1999) given by,

$$\frac{e}{m} = K \cdot \frac{\sigma_u}{\rho} \tag{5.12}$$

where, K is known as 'shape factor' depends on the geometrical configuration of the flywheel and the choice of flywheel material depends on the specific strength (σ_u/ρ). Composite materials are widely used due to its excellent specific strength. As the flywheel is subjected to the cyclic loading, the fatigue strength (σ_{limit}) is considered instead of ultimate strength (σ_u) (Jee & Kang, 2000; Chatterjee et al., 2009).

Another important phenomenon is about the burst containment which is associated with the rotor. All materials have a fatigue life when they are subjected to cyclic loading or may have inconsistencies in the structure and the fracture may take place even below the strength limit. In some literatures (Jee & Kang, 2000), this fracture phenomenon is addressed by the ratio (K_{1c}/ρ) of fracture toughness (K_{1c}) and density (ρ), but there is no clear explanation of choosing density in fracture toughness. According to Griffith crack theory, the critical strain energy release rate (G_{1c}) that propagates the crack is given by (Broek, 1984),

$$\left.\begin{aligned} G_{1c} &= \frac{\pi {\sigma_c}^2 a}{E} \\ \text{with } K_{1c} &= \sigma_c \cdot \sqrt{\pi a} \end{aligned}\right\} \xrightarrow{yields} G_{1c} = \frac{{K_{1c}}^2}{E} \tag{5.13}$$

where, a is the half crack length ($2a$), σ_c is the critical stress at which the fracture takes place and K_{1c} is known as critical stress intensity factor or popularly fracture toughness. We would like to consider the K_{1c} as a material selection attribute instead of K_{1c}/ρ. If the fracture happens, the wheel breaks up into the number of fragments and the kinetic energy of the wheel is transferred to surrounding through these fragments that cause the damage to the system. In case of monolithic metal, the number of fragments is three or four, whereas for composite materials, there are many tiny fragments. On that ground, the composite materials are the better choice. One of the popular materials for the flywheel is Kevlar but it is costly. However, following the previous literature (Jee & Kang, 2000), the specific strength ($\sigma_{\text{limit}}/\rho$), fracture toughness to density ratio (K_{1c}), fragmentability, and cost are considered as material attributes for the flywheel. All attributes can be summarized as:

- *Benefit attribute*: Specific strength (e_1); Fracture toughness (e_2); Fragmentability (e_3);
- *Cost attribute*: Cost (e_4).

Some of the potential materials under uncertainty with their performance ratings are tabulated in Table 5.16 where basically Al-alloys, Ti-alloys, and Composite materials have been considered. In previous literature, TOPSIS (Jee et al.,2000), ELECTRE II (Chatterjee et al.,2009), and VIKOR (Chatterjee et al.,2009) have been used to select the best material. Some material attributes are rather ordinal like fragmentability, creep resistance or corrosion resistance and can be converted into cardinal by following the 5-points *Likert Scale* (Likert, 1932) as: Very weak – 1; Weak – 2; Average – 3; Good – 4; Excellent – 5. The priority to the attributes are assigned following the previous works (Jee & Kang, 2000; Chatterjee et al., 2009) and given by,

$$w_{j=1,2,\cdots,n} = [0.4, 0.3, 0.1, 0.2]$$

5.4.1 Discrete choice analysis (DCA)

Normalized performance ratings are tabulated in Table 5.17 following the expression (4.14). The final outcome, i.e. the choice probabilities and weighted sum probabilities of the alternatives are tabulated in Table 5.17 following the expressions (4.15) and (4.16).

Table 5.16 Performance ratings matrix (d_{ij}) of case study 3

Candidate Material ($a_{i=1,2,\cdots,7}$)		Specific Strength, σ_{limit}/ρ (MJ/kg)	Fracture toughness, K_{1c} (MPa m$^{1/2}$)	Fragmentability	Cost (US $/ton)
a_1	300M	100.00	68.90	Poor (2)	4200
a_2	2024 – T3	49.65	38.00	Poor (2)	2100
a_3	7050 – T73651	78.00	35.40	Poor (2)	2100
a_4	Ti – 6Al – 4V	108.88	123.00	Poor (2)	10500
a_5	E glass – epoxy FRP	70.00	20.00	Excellent (5)	2735
a_6	S glass – epoxy FRP	165.00	50.00	Excellent (5)	4095
a_7	Carbon – epoxy FRP	440.25	35.00	Fairly good (4)	35470
a_8	Kevlar 29 – epoxy FRP	242.86	40.00	Fairly good (4)	11000
a_9	Kevlar 49 – epoxy FRP	616.44	50.00	Fairly good (4)	25000
a_{10}	Boron – epoxy FRP	500.00	46.00	Good (3)	315000

Table 5.17 Overall performance of the Case study 3 following the DCA

Alternative (a_i)		e_1 (+ve) ($w_1 = 0.40$)		e_2 (+ve) ($w_2 = 0.30$)		e_3 (+ve) ($w_3 = 0.10$)		e_4 ($-$ve) ($w_4 = 0.20$)		$P(a_i)$	Rank
		r_{i1}	P_{i1}	r_{i2}	P_{i2}	r_{i3}	P_{i3}	r_{i4}	P_{i4}		
a_1	300M	0.1028	0.0850	0.3797	0.1093	0.1800	0.0884	0.0130	0.1088	0.09737	7
a_2	2024 – T3	0.0510	0.0807	0.2093	0.0921	0.1800	0.0884	0.0065	0.1095	0.09067	10
a_3	7050 – T73651	0.0803	0.0831	0.1950	0.0908	0.1800	0.0884	0.0065	0.1095	0.09123	9
a_4	Ti – 6Al – 4V	0.1120	0.0858	0.6780	0.1472	0.1800	0.0884	0.0330	0.1066	0.10865	2
a_5	E glass – epoxy FRP	0.0720	0.0824	0.1103	0.0835	0.4510	0.1159	0.0085	0.1093	0.09146	8
a_6	S glass – epoxy FRP	0.1698	0.0909	0.2757	0.0985	0.4510	0.1159	0.0130	0.1088	0.09924	4
a_7	Carbon – epoxy FRP	0.4528	0.1206	0.1930	0.0906	0.3610	0.1059	0.1115	0.0986	0.10576	3
a_8	Kevlar 29 – epoxy FRP	0.2498	0.0985	0.2203	0.0932	0.3610	0.1059	0.0345	0.1065	0.09922	5
a_9	Kevlar 49 – epoxy FRP	0.6340	0.1446	0.2757	0.0985	0.3610	0.1059	0.0785	0.1019	0.11835	1
a_{10}	Boron – epoxy FRP	0.5143	0.1283	0.2537	0.0963	0.2710	0.0968	0.9895	0.0410	0.09808	6

5.4.2 Nearest neighbour search (NNS)

Following the same steps like case study1, the weighted normalized performance ratings are tabulated in Table 5.18 and the overall performances of the alternatives are systematized in Table 5.19 according to the steps 6 to 7 formalized in section 4.4. From Table 5.18 the query alternative with performance ratings is given by,

$$q = [0.2536, 0.2034, 0.0451, 0.0013]$$

Table 5.18 Weighted normalized performance ratings (x_{ij}) of Case study 3 following NNS

Candidate Material (a_i)		e_1 (max) ($w_1 = 0.40$)		e_2 (max) ($w_2 = 0.30$)		e_3 (max) ($w_3 = 0.10$)		e_4 (min) ($w_4 = 0.20$)	
		r_{i1}	x_{i1}	r_{i2}	x_{i2}	r_{i3}	x_{i3}	r_{i4}	x_{i4}
a_1	300M	0.1028	0.0411	0.3797	0.1139	0.1800	0.0180	0.0130	0.0026
a_2	2024 – T3	0.0510	0.0204	0.2093	0.0628	0.1800	0.0180	0.0065	0.0013
a_3	7050 – T73651	0.0803	0.0321	0.1950	0.0585	0.1800	0.0180	0.0065	0.0013
a_4	Ti – 6Al – 4V	0.1120	0.0448	0.6780	0.2034	0.1800	0.0180	0.0330	0.0066
a_5	E glass – epoxy FRP	0.0720	0.0288	0.1103	0.0331	0.4510	0.0451	0.0085	0.0017
a_6	S glass – epoxy FRP	0.1698	0.0679	0.2757	0.0827	0.4510	0.0451	0.0130	0.0026
a_7	Carbon – epoxy FRP	0.4528	0.1811	0.1930	0.0579	0.3610	0.0361	0.1115	0.0223
a_8	Kevlar 29 – epoxy FRP	0.2498	0.0999	0.2203	0.0661	0.3610	0.0361	0.0345	0.0069
a_9	Kevlar 49 – epoxy FRP	0.6340	0.2536	0.2757	0.0827	0.3610	0.0361	0.0785	0.0157
a_{10}	Boron – epoxy FRP	0.5143	0.2057	0.2537	0.0761	0.2710	0.0271	0.9895	0.1979

Table 5.19 Overall performance evaluation of Case study 3 following NNS

Candidate Material		$\lvert\overrightarrow{oa_i}\rvert$	$\overrightarrow{oa_i}\cdot\overrightarrow{oq}$	$\lvert\overrightarrow{oq}\rvert$	$\cos\theta_i$	$\sin\theta_i$	$\overline{a_ib_i}$	$\overline{b_iq}$	$d_M(a_i, q)$	Rank
a_1	300M	0.1224	0.0344	0.3282	0.8563	0.5165	0.2027	0.1132	0.3159	7
a_2	2024 – T3	0.0685	0.0188	0.3282	0.8362	0.5484	0.2181	0.1650	0.3831	9
a_3	7050 – T73651	0.0691	0.0209	0.3282	0.9216	0.3881	0.2060	0.1681	0.3741	8
a_4	Ti – 6Al – 4V	0.2092	0.0536	0.3282	0.7807	0.6249	0.2090	0.0241	0.2331	3
a_5	E glass – epoxy FRP	0.0629	0.0161	0.3282	0.7799	0.6259	0.2252	0.1695	0.3947	10
a_6	S glass – epoxy FRP	0.1161	0.0361	0.3282	0.9474	0.3200	0.1805	0.1280	0.3085	6
a_7	Carbon – epoxy FRP	0.1948	0.0594	0.3282	0.9291	0.3698	0.1550	0.0531	0.2081	2
a_8	Kevlar 29 – epoxy FRP	0.1253	0.0404	0.3282	0.9824	0.1868	0.1616	0.1285	0.2901	4
a_9	Kevlar 49 – epoxy FRP	0.2696	0.0828	0.3282	0.9358	0.3525	0.1209	0.0135	0.1344	1
a_{10}	Boron – epoxy FRP	0.2967	0.0691	0.3282	0.7096	0.7046	0.2310	0.0646	0.2956	5

5.4.3 Results and discussion

The result in Table 5.19 gives some interesting points and a clear delineation of past and present. As the technology is changing rapidly, new challenging alternatives are taking place in the market, despite being costly a designer or decision maker bends towards the Composite materials due to its excellent specific strength. The ranking in Table 5.19 is compared with the previous works as shown Table 5.20. There is no confusion about ranked 1 material, Kevlar 49 due to its excellent specific strength and consistent with the real-world practice (Genta, 1985). Carbon-epoxy FRP also holds the 2nd position in all methods. Composite materials reinforced with glass fibre, aramid or carbon are similar in specific strength point of view and the choice among the differs by their cost mainly.

Table 5.20 Rank comparison of Case study 3

Candidate Material		**DCA**	**NNS**	TOPSIS (Jee et al.,2000)	ELECTRE II (Chatterjee et al.,2009)	VIKOR (Chatterjee et al.,2009)
a_1	300M	7	7	5	10	9
a_2	2024 – T3	10	9	9	9	10
a_3	7050 – T73651	9	8	7	8	8
a_4	Ti – 6Al – 4V	2	3	6	6	6
a_5	E glass – epoxy FRP	8	10	8	7	7
a_6	S glass – epoxy FRP	4	6	3	3	5
a_7	Carbon – epoxy FRP	3	2	4	2	2
a_8	Kevlar 29 – epoxy FRP	5	4	2	4	4
a_9	Kevlar 49 – epoxy FRP	1	1	1	1	1
a_{10}	Boron – epoxy FRP	6	5	10	5	3

5.5 Case study 4: Spar of human powered aircraft

Most successful human powered aircrafts generally weigh between 30 and 45 kilograms (Figure 5.3). As the aircraft totally powered by human muscle effort, it must be carefully designed to give it a minimum weight. The wings make up a major part of the weight of any aircraft, and human-powered aircraft is no exception. The wings are designed to use a little material as possible so the wings are made just stiff enough to support the fuselage. Therefore, in this regard the customer requirements can be stated as:

- *Demand*: Minimum mass (m); and
- *Desire*: Minimum cost.

The aerodynamic forces associated with powered aircraft are lift and drag. The main constraint on the aerodynamics of the HPA is the slow operating speed (about 6m/sec) that is caused by the lack of power to generate thrust; the average fit human makes approximately 0.2kW of power. Lift force is defined by the lift equation is given by

$$L = 0.5\rho V^2 SC_L \tag{5.16}$$

The weight force of the aircraft is equal and opposite to the lift force generated by the wing area (S). The density (ρ) of the air is fixed as for slow speed (V) and low altitudes air is assumed to be incompressible. So, there are only two parameters, the coefficient of lift (C_L) and the wing surface area (S) that can be varied to generate the required amount of lift. Surface area is the simplest parameter to vary because it can be done in the design stages and easily implemented into the finished product. For light, stiff beam shown in Figure 5.3 (as the spar is equivalent to a beam), when height is specified and width is free as in case of 'Wing Spars', then the functional requirements can be represented by (Ashby, 1999),

$$m = \sqrt{\frac{12S}{Cl}} \cdot l^3 \cdot \frac{1}{\left[\frac{E}{\rho}\right]} \tag{5.15}$$

where S is the stiffness, l and a is the length and depth of the beam respectively, E is the Young's modulus, ρ is the density and C is the constant depends on the end condition of the beam. It is very clear from the expression (5.15), for light stiff beam (as the spar is equivalent to a beam) the main criterion is specific stiffness (SS), E/ρ in MJ/kg [1]. Obviously, the related strength criteria are specific tensile strength (STS), σ_t/ρ in kJ/kg and specific compressive strength (SCS) σ_c/ρ, in kJ/kg. Besides above-mentioned benefit criteria, cost (COST) in \$/kg criterion is taken as a cost criterion. All criteria or attributes can be summarized as:

- *Benefit attribute*: Specific stiffness (e_1); Specific tensile strength (e_2); Specific compressive strength (e_3);
- *Cost attribute*: Cost (e_4).

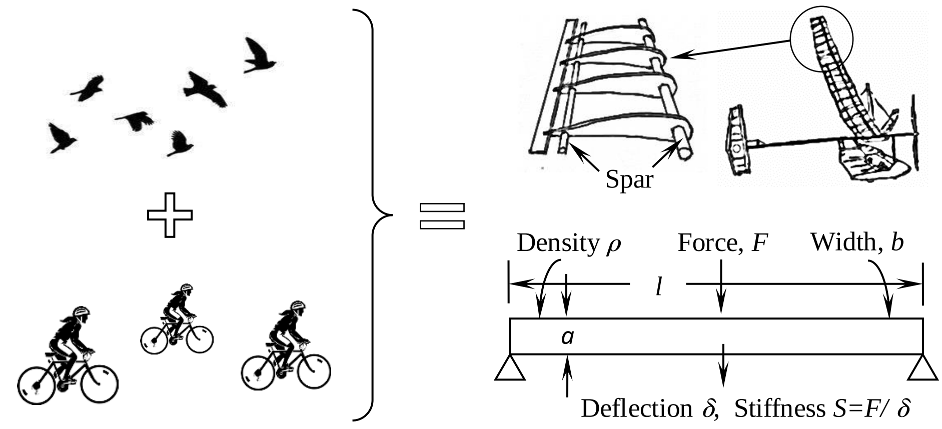


Figure 5.3 Human powered aircraft

Some of the potential materials under uncertainty with their performance ratings are tabulated in Table 5.21 (Manshadi, Mahmudi, & Abedian, 2007;) where basically Al-alloys, Ti-alloys, and Composite materials have been considered. In MADM approaches, the normalizing method also takes a vital role. In the previous literature, new normalizing methods have been introduced to examine the rankings e.g., WPM with Z-transformation normalization (Fayazbakhsh et al., 2009) and WPM with non-linear normalization (Manshadi et al., 2007). In these literatures, priority is given to the creep resistance attribute. For low speed HPA, the creep resistance has no influence. Therefore, in this case study, the creep resistance is omitted and the relative weights to the attributes have been assigned newly according to the functional requirements. Relative weights of the attributes are tabulated in Table 5.22 following the Section 3.2.1 where demand is much preferred than wish. It is necessary to check the consistency ratio (CR) of matrix A and it is found 0.004. If $CR \leq 0.10$, the degree of consistency is satisfactory (Munier, 2011).

5.5.1 Discrete choice analysis (DCA)

Normalized performance ratings are tabulated in Table 5.23 following expression (4.14). The final outcome, i.e. the choice probabilities and overall weighted sum probabilities of the alternatives are tabulated in Table 5.23 following expressions (4.15) and (4.16).

Table 5.21 Performance ratings matrix (d_{ij}) of Case study 4

Sl. No.	Candidate Material	SS (MJ/kg)	STS (kJ/kg)	SCS (kJ/kg)	COST ($/kg)
1.	Al 7075-T6	27.00	223.46	223.40	3.5
2.	Al 2024-T4	27.88	163.46	163.46	3.5
3.	Ti–6Al–4V	25.45	229.10	229.10	21.0
4.	Ti–2Fe–3Al	26.60	287.77	287.77	22.0
5.	E-glass73%-Epoxy	25.76	756.68	188.94	2.6
6.	E-glass56%-Epoxy	21.72	521.82	147.21	2.5
7.	E-glass65%-Polyester	10.88	188.88	50.00	2.5
8.	S-glass70%-Epoxy cont. fibers	29.52	995.26	260.66	9.0
9.	S-glass70%-Epoxy fabric	10.42	322.27	85.30	8.0
10.	Carbon 63%-Epoxy	98.57	1071.43	559.00	45.0
11.	Aramid 62%-Epoxy	59.92	950.00	217.40	20.0
12.	Balsa	31.18	129.54	79.54	6.0

Table 5.22 Relative weights of the attributes by AHP of Case study 4

Material attribute	e_1	e_2	e_3	e_4	$\bar{w}_i$	$\boldsymbol{w_i}$	v_i
Specific stiffness (e_1)	1	2	2	3	1.86	**0.42**	1.70
Specific tensile strength (e_2)	1/2	1	1	2	1	**0.23**	0.91
Specific compressive strength (e_3)	1/2	1	1	2	1	**0.23**	0.91
Cost (e_4)	1/3	1/2	1/2	1	0.53	**0.12**	0.49

Table 5.23 Overall performance of the Case study 4 following the DCA

Alternative (a_i)		e_1 (+ve) ($w_1 = 0.42$)		e_2 (+ve) ($w_2 = 0.23$)		e_3 (+ve) ($w_3 = 0.23$)		e_4 ($-$ve) ($w_4 = 0.12$)		$P(a_i)$	Rank
		r_{i1}	P_{i1}	r_{i2}	P_{i2}	r_{i3}	P_{i3}	r_{i4}	P_{i4}		
a_1	Al 7075-T6	0.1944	0.0787	0.1117	0.0719	0.2681	0.0838	0.0585	0.0943	0.0802	5
a_2	Al 2024-T4	0.2007	0.0792	0.0817	0.0698	0.1962	0.0781	0.0585	0.0943	0.0786	8
a_3	Ti–6Al–4V	0.1832	0.0778	0.1146	0.0721	0.2749	0.0844	0.3507	0.0704	0.0775	9
a_4	Ti–2Fe–3Al	0.1915	0.0784	0.1439	0.0743	0.3453	0.0906	0.3674	0.0693	0.0792	7
a_5	E-glass73%-Epoxy	0.1855	0.0780	0.3783	0.0939	0.2267	0.0805	0.0434	0.0958	0.0844	4
a_6	E-glass56%-Epoxy	0.1564	0.0757	0.2609	0.0835	0.1767	0.0765	0.0418	0.0959	0.0801	6
a_7	E-glass65%-Polyester	0.0783	0.0700	0.0944	0.0707	0.0600	0.0681	0.0418	0.0959	0.0728	12
a_8	S-glass70%-Epoxy	0.2125	0.0801	0.4976	0.1058	0.3128	0.0877	0.1503	0.0860	0.0885	2
a_9	S-glass70%-Epoxy	0.0750	0.0698	0.1611	0.0756	0.1024	0.0710	0.1336	0.0875	0.0735	11
a_{10}	Carbon 63%-Epoxy	0.7097	0.1317	0.5357	0.1099	0.6708	0.1254	0.7515	0.0472	0.1151	1
a_{11}	Aramid 62%-Epoxy	0.4314	0.0997	0.4750	0.1034	0.2609	0.0833	0.3340	0.0716	0.0856	3
a_{12}	Balsa	0.2245	0.0811	0.0648	0.0686	0.0954	0.0706	0.1002	0.0905	0.0769	10

5.5.2 Nearest neighbour search (NNS)

Following the same steps like case study1, the normalized and weighted normalized performance ratings are tabulated in Table 5.24 and the overall performances of the alternatives are systematized in Table 5.25 according to the steps 6 to 7 formalized in section 4.4. From Table 5.24, the query alternative with performance ratings is given by,

$$q = [0.2981, 0.1232, 0.1543, 0.0050]$$

Table 5.24 Normalized (r_{ij}) and weighted normalized performance ratings (x_{ij}) of Case study 4 following NNS

Candidate Material (a_i)		e_1 (max) ($w_1 = 0.40$)		e_2 (max) ($w_2 = 0.30$)		e_3 (max) ($w_3 = 0.10$)		e_4 (min) ($w_4 = 0.20$)	
		r_{i1}	x_{i1}	r_{i2}	x_{i2}	r_{i3}	x_{i3}	r_{i4}	x_{i4}
a_1	Al 7075-T6	0.1944	0.0816	0.1117	0.0257	0.2681	0.0617	0.0585	0.0070
a_2	Al 2024-T4	0.2007	0.0843	0.0817	0.0188	0.1962	0.0451	0.0585	0.0070
a_3	Ti–6Al–4V	0.1832	0.0770	0.1146	0.0263	0.2749	0.0632	0.3507	0.0421
a_4	Ti–2Fe–3Al	0.1915	0.0804	0.1439	0.0331	0.3453	0.0794	0.3674	0.0441
a_5	E-glass73%-Epoxy	0.1855	0.0779	0.3783	0.0870	0.2267	0.0521	0.0434	0.0052
a_6	E-glass56%-Epoxy	0.1564	0.0657	0.2609	0.0600	0.1767	0.0406	0.0418	0.0050
a_7	E-glass65%-Polyester	0.0783	0.0329	0.0944	0.0217	0.0600	0.0138	0.0418	0.0050
a_8	S-glass70%-Epoxy	0.2125	0.0893	0.4976	0.1145	0.3128	0.0719	0.1503	0.0180
a_9	S-glass70%-Epoxy	0.0750	0.0315	0.1611	0.0371	0.1024	0.0235	0.1336	0.0160
a_{10}	Carbon 63%-Epoxy	0.7097	0.2981	0.5357	0.1232	0.6708	0.1543	0.7515	0.0902
a_{11}	Aramid 62%-Epoxy	0.4314	0.1812	0.4750	0.1093	0.2609	0.0600	0.3340	0.0401
a_{12}	Balsa	0.2245	0.0943	0.0648	0.0149	0.0954	0.0220	0.1002	0.0120

5.5.3 Results and discussion

The result in Table 5.25 gives some interesting points and a clear delineation of past and present. First generation HPA was built of Balsa wood, even still it is used. As the technology is changing rapidly, new challenging alternatives are taking place in the market, despite being costly a designer or decision maker bends towards the Composite materials due to its excellent specific strength. The rankings of the proposed models are compared with the previous works as shown in Table 5.26. There is no confusion about ranked 1 material, Carbon 63%-Epoxy. The fit candidate always survives whatever the process. It is important that the consecutive rankings should be consistent with the real-world practice. There is an ambiguity about the ranking 2 by Manshadi et al. (2007). Generally, for low speed and minimum weight consideration, the Ti-alloys are not considered while selecting the materials for HPA.

Table 5.25 Overall performance evaluation of Case study 4 following the NNS

Candidate Material		$\lvert\overrightarrow{oa_i}\rvert$	$\lvert\overrightarrow{oq}\rvert$	$\overrightarrow{oa_i}\cdot\overrightarrow{oq}$	$\cos\theta_i$	$\sin\theta_i$	$\overline{a_ib_i}$	$\overline{b_iq}$	$d_M(a_i,q)$	Rank
a_1	Al 7075-T6	0.1057	0.3576	0.0371	0.9801	0.1985	0.1944	0.1647	0.3592	6
a_2	Al 2024-T4	0.0977	0.3576	0.0344	0.9859	0.1675	0.1963	0.1732	0.3695	8
a_3	Ti–6Al–4V	0.1113	0.3576	0.0362	0.9085	0.4180	0.2143	0.1485	0.3627	7
a_4	Ti–2Fe–3Al	0.1258	0.3576	0.0405	0.9012	0.4334	0.2113	0.1342	0.3454	5
a_5	E-glass73%-Epoxy	0.1280	0.3576	0.0420	0.9178	0.3970	0.2057	0.1338	0.3396	4
a_6	E-glass56%-Epoxy	0.0979	0.3576	0.0333	0.9499	0.3125	0.2087	0.1654	0.3742	10
a_7	E-glass65%-Polyester	0.0421	0.3576	0.0146	0.9730	0.2306	0.2308	0.2171	0.4478	12
a_8	S-glass70%-Epoxy	0.1630	0.3576	0.0519	0.8904	0.4551	0.2027	0.0978	0.3005	3
a_9	S-glass70%-Epoxy	0.0564	0.3576	0.0177	0.8767	0.4811	0.2371	0.1987	0.4358	11
a_{10}	Carbon 63%-Epoxy	0.3687	0.3576	0.1283	0.9730	0.2310	0.0594	0.0611	0.1205	1
a_{11}	Aramid 62%-Epoxy	0.2236	0.3576	0.0769	0.9624	0.2717	0.1437	0.0578	0.2015	2
a_{12}	Balsa	0.0987	0.3576	0.0334	0.9462	0.3236	0.2094	0.1642	0.3737	9

Table 5.26 Rank comparison of Case study 4

Candidate Material		**DCA**	**NNS**	WPM with Z-transformation normalization (Fayazbakhsh et al., 2009)	WPM with non-linear normalization (Manshadi et al., 2007)
a_1	Al 7075-T6	5	6	7	6
a_2	Al 2024-T4	8	8	8	8
a_3	Ti–6Al–4V	9	7	6	3
a_4	Ti–2Fe–3Al	7	5	3	2
a_5	E-glass73%-Epoxy	4	4	5	5
a_6	E-glass56%-Epoxy	6	10	9	9
a_7	E-glass65%-Polyester	12	12	12	12
a_8	S-glass70%-Epoxy cont. fibers	2	3	2	4
a_9	S-glass70%-Epoxy fabric	11	11	11	10
a_{10}	Carbon 63%-Epoxy	1	1	1	1
a_{11}	Aramid 62%-Epoxy	3	2	4	7
a_{12}	Balsa	10	9	10	11

5.6 Sensitivity analysis

The goal of sensitivity analysis is to investigate the impacts of the uncertainties to the model. In MADM problems, the attributes and weightage to the attributes are rather conflicting. All input data are considered as deterministic, but in reality, it is stochastic in nature (Wolters & Mareschal, 1995). Therefore, after solving the decision-making problem, a sensitivity analysis should be done to address the uncertainty in decision-making (Zavadskas, Turskis, Dėjus, & Viteikienė, 2007). A sensitivity analysis is an approach to examine how the dependent variables varies with the change in the independent variables. Memariani et al. (2009) proposed a method of sensitivity analysis of MADM problems, especially in SAW (simple additive weighting) method where the overall performance of an alternative varies with the change in attributes weightage. If the k attribute changes from w_k to $w_k{}'$, then the redistribution of the weightage to the other attributes is given by,

$$w_j{}' = \frac{1 - w_k{}'}{1 - w_k} \cdot w_j \qquad j = 1,2,\cdots,n; j \neq k \tag{5.17}$$

where $w_k{}'$ can be considered as stepwise increased or decreased of w_k by $c\%$ and given by,

$$w_k{}' = \left(1 \pm \frac{c}{100}\right) \cdot w_k \qquad k = 1,2,\cdots,n \tag{5.18}$$

Variability in data also takes place in normalized performance ratings. There is wide range of normalizing methods are adopted by the researchers in MADM approaches. Most effective and popular normalizing methods are vector normalizing and linear normalizing and can be expressed as:

$$\text{Vector normalization: } r_{ij} = \frac{d_{ij}}{\sqrt{\sum_{i=1}^{m} \left(d_{ij}\right)^2}} \qquad i = 1,2,\cdots,m; j = 1,2,\cdots,n \tag{5.19}$$

$$\text{Linear normalization: } r_{ij} = \frac{d_{ij}}{\sum_{i=1}^{m} d_{ij}} \qquad i = 1,2,\cdots,m; j = 1,2,\cdots,n \tag{5.20}$$

For example, in case of Case study 1 (Two stage spur gear reduction unit), if the weightage of the ultimate tensile strength (w_1) is decreased by 5%, we can get a set of weightages by following the expressions (5.17) and (5.18) as shown in Table 5.27. The normalized performance ratings are also tabulated in Table 5.28 following the expressions (5.19) and (5.20).

Table 5.27 Set of weightages of the attributes of Case study 1 for sensitivity analysis

Set of weightages	Weightage decreased by c%	Ultimate tensile strength (e_1)	Brinell hardness number (e_2)	Density (e_3)	Cost (e_4)
		$w_1{}'$	$w_2{}'$	$w_3{}'$	$w_4{}'$
s_1	0%	0.470	0.330	0.080	0.120
s_2	5%	0.447	0.345	0.084	0.125
s_3	5%	0.424	0.359	0.087	0.130
s_4	5%	0.403	0.372	0.090	0.135
s_5	5%	0.383	0.384	0.093	0.140
s_6	5%	0.364	0.396	0.096	0.144
s_7	5%	0.345	0.408	0.099	0.148
s_8	5%	0.328	0.418	0.101	0.152
s_9	5%	0.312	0.428	0.104	0.156
s_{10}	5%	0.296	0.438	0.106	0.159

Table 5.28 Vector and linear normalization of Case study 1 for sensitivity analysis

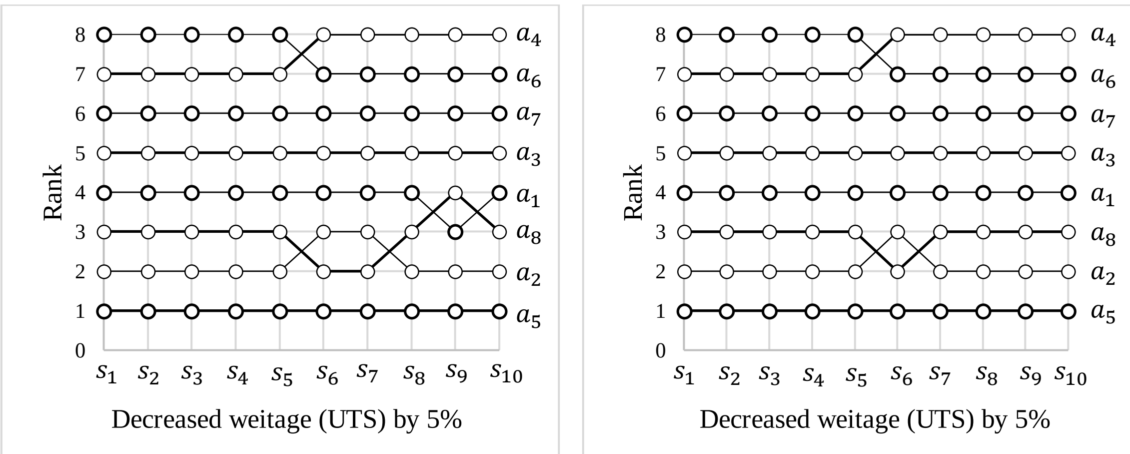
Alternatives ($a_{i=1,2,\cdots,8}$)		Vector normalization (r_{ij})				Linear normalization (r_{ij})			
		e_1	e_2	e_3	e_4	e_1	e_2	e_3	e_4
a_1	AISI 1040	0.371	0.290	0.370	0.141	0.132	0.105	0.131	0.052
a_2	AISI 4130	0.403	0.303	0.370	0.217	0.143	0.110	0.131	0.080
a_3	AISI 304	0.371	0.286	0.368	0.309	0.132	0.104	0.130	0.114
a_4	AISI 405	0.322	0.292	0.368	0.380	0.115	0.106	0.130	0.140
a_5	ASTM class 60	0.310	0.554	0.344	0.358	0.110	0.202	0.122	0.132
a_6	Grade 60-40-18	0.299	0.325	0.335	0.434	0.106	0.118	0.119	0.160
a_7	Grade 65-45-12	0.334	0.325	0.335	0.434	0.119	0.118	0.119	0.160
a_8	Grade 80-55-06	0.402	0.373	0.335	0.434	0.143	0.136	0.119	0.160

5.6.1 Sensitivity analysis of DCA

To test the stability of the ranking to changes in the weightage of the ultimate tensile strength of the gear materials, a set of 10 scenarios is defined. Each scenario is characterised by an incremental decrease (%) in the weightage to the ultimate tensile strength criterion. The 8 alternatives of Case study 1 are evaluated with respect to the 6 criteria, for each of the 10 scenarios. The evaluations are processed as usual same way before and the rankings for each scenario are presented in Table 5.29 and the graphical presentation of table 5.29 is shown in Figure 5.4 for better understanding of the impact of the different normalizing process as well as changes in the weightages. However, in Figure 5.4, the ranking patterns in both normalizing process with all scenarios are almost same and ranked 1 material (ASTM Grade 60) maintains the stability throughout the scenarios.

Table 5.29 Ranking variation of Case study 1 using DCA with the change in weightages

a_i	Ranking based on vector normalization										Ranking based on linear normalization									
	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
a_1	4	4	4	4	4	4	4	4	3	4	4	4	4	4	4	4	4	4	4	4
a_2	2	2	2	2	2	3	3	2	2	2	2	2	2	2	2	3	2	2	2	2
a_3	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
a_4	7	7	7	7	7	8	8	8	8	8	7	7	7	7	7	8	8	8	8	8
a_5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
a_6	8	8	8	8	8	7	7	7	7	7	8	8	8	8	8	7	7	7	7	7
a_7	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
a_8	3	3	3	3	3	2	2	3	4	3	3	3	3	3	3	2	3	3	3	3



(a) (b)

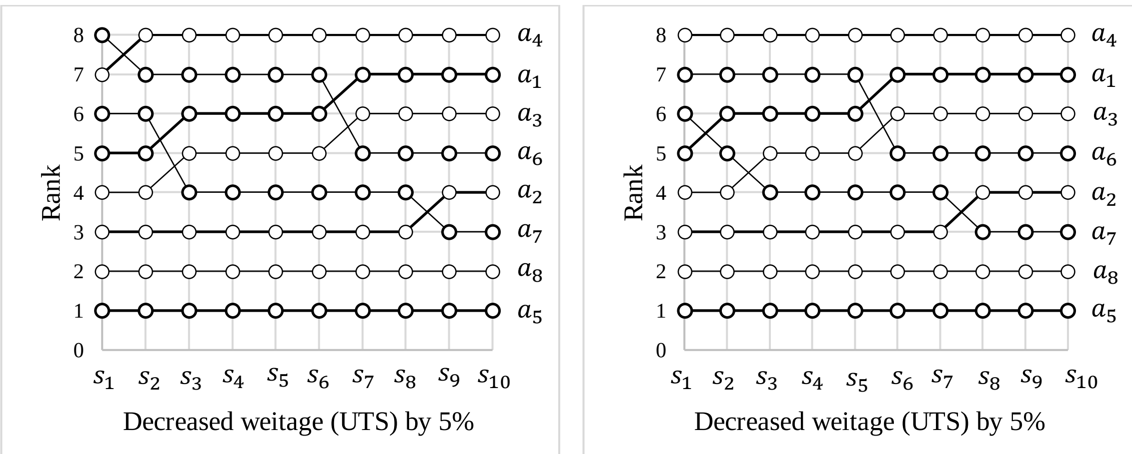
Figure 5.4 Sensitivity analysis of Case study 1 using DCA (a) based on vector normalization (b) based on linear normalization

5.6.2 Sensitivity analysis of NNS

The evaluations are processed as usual same way following NNS and the rankings for each scenario are tabulated in Table 5.30. The graphical presentation of Table 5.30 is shown in Figure 5.5 for better understanding of the impact of the different normalizing process as well as changes in the weightages. Better understanding of sensitivity analysis can be realized when the alternatives are in very tight competition. In the present analysis, a_7 (Grade 65-45-12) alternative finishes to rank 3 from rank 6 with change in scenarios. Comparing the Figure 5.4 and Figure 5.5 of sensitivity analysis for DCA and NNS, the NNS shows more sensitive than DCA as well as maintain the stability in ranking.

Table 5.30 Ranking variation of Case study 1 using NNS with the change in weightages

a_i	Ranking based on vector normalization										Ranking based on linear normalization									
	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
a_1	5	5	6	6	6	6	7	7	7	7	5	6	6	6	6	7	7	7	7	7
a_2	3	3	3	3	3	3	3	3	4	4	3	3	3	3	3	3	3	4	4	4
a_3	4	4	5	5	5	5	6	6	6	6	4	4	5	5	5	6	6	6	6	6
a_4	7	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
a_5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
a_6	8	7	7	7	7	7	5	5	5	5	7	7	7	7	7	5	5	5	5	5
a_7	6	6	4	4	4	4	4	4	3	3	6	5	4	4	4	4	4	3	3	3
a_8	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2



(a) (b)

Figure 5.5 Sensitivity analysis of Case study 1 using NNS (a) based on vector normalization (b) based on linear normalization

5.7 Observations

The design is a sequential decision-making process. Decision-making is the process by which the decision makers skim off an alternative and belief that will meet the destined objective function. It is the versatility of the MADM approaches blended with mathematics and cognitions that different decision-making methods yield different rankings considering the same input data. Whatever the methods, if the methods have definite logic, the fittest materials will always survive, but there should have some consistency among the rankings inside a particular method. The methodology gives an entire picture but we have to take the right decision from this picture. The following observations have been noted in the case studies.

- Throughout the case studies, the material properties are considered as discrete and average, but these are rather variable in nature. The true value cannot be known precisely (aleatory uncertainty). As the material selection is initiated at the conceptual stage of the design process, there is a further scope to analyse the suitability of the chosen material at the embodiment stage.
- In the above case studies, the proposed conditional logit (CLGT) method under DCA gives more consistent results when compared with the other MADM approaches. MADM is based on the observed attribute with certainty, but CLGT is based on both the observed and unobserved attributes that give us confidence under risk and uncertainty.
- Material selection takes place at the earliest stage of the design process when there is a lack of information and variability in the available information in the decision space. Under these circumstances, the proposed NNS gives a confidence in visualizing (geometrical) point of view and courage in ranking consistency point of view as compared with other rankings.

This thesis basically proposes two methods, Conditional Logit (CLGT) under Discrete Choice Analysis (DCA) and Nearest Neighbor Search (NNS) in the Decision-Based Design (DBD) framework. CLGT was developed to solve the socio demographic problems. One of the limitations of the proposed approach CLGT is associated with the IID assumption, i.e., the unobserved factors are not correlated over alternatives and also have the same variance for all alternatives. In the case of material selection problems, for example, in case study 4 (Human Powered Aircraft), the materials can be categorized as alloys and composites. In this situation, the unobserved factors may be correlated in a particular domain (say, alloys or composites). For the sake of simple calculation, the IID assumption is considered across candidate materials. Secondly, it is said, the material and geometric variables should be considered simultaneously for optimal design. Like most of the cases, in this thesis, the material is considered as a pre-assigned design parameter.

Chapter 6

Conclusions

6.1 Introduction

Engineering design is the judicious trade-off among shape, materials, and manufacturing that requires a wide range of decisions. Decision-making in engineering design allocates all the resources optimally while fulfilling the design objectives within economic constraints, quality constraints, safety constraints, environmental constraints etcetera under uncertainty. The intention in this thesis is to develop a strategy for supporting a designer to choose a preeminent material in the context of product design.

Several comprehensive and flexible procedures for performance evaluation of the engineering materials in engineering design have been defined and analysed in this thesis and many practical applications have been described. Based on the observations and critical review, the achievements and contributions reported in this thesis are presented in Section 6.2 followed by opportunities for future work in Section 6.3.

6.2 Contributions of the thesis

The main contributions of this thesis are given below to link and integrate the research findings in order to satisfy the aims and objectives as outlined in the Chapter 1.

The first research objective was to investigate the current knowledge regarding material selection processes those are applied in engineering design and to identify the major shortcomings. In Chapter 2, the design philosophy and the role of alternative generation followed by alternative evaluation in design are discussed. Typically, in the domain of alternative evaluation in engineering design, Pugh method and Pahl and Beitz method are well discussed. These methods are easy to understand and have computational ability but, there are some lack of precise ranking among the alternatives, a lack of joy of mathematical interpretation that guided us towards the MADM methods.

In Chapter 3, some of the popular MADM (multiple attribute decision-making) methods are discussed specifically, AHP, MAUT (SAW), TOPSIS, VIKOR, and ELECTRE. MADM methods are already well popular in the field of material selection. There is an ample of literature where the researchers around the world are continuously modifying the models to capture the right choice. Decision takes place in every stages of the design process. In general, MADM processes while selecting materials, decision maker has no control on decision process and absolutely depend upon the outcome of the analytical model except assigning the relative priorities to the attributes i.e., a *faith* takes place in implementation of decision making and there is a lack of *rationality*. To ensure a rational choice in order to allocate the all resources optimally, the problem should be precisely structured from the beginning i.e. mapping the customer requirements to functional requirements.

In Chapter 4, the performance evaluation strategies of the engineering materials are modelled to raise a preeminent material for a design rationally where stepwise strategies are prescribed as

- A structured rational decision-making framework based on DBD framework (decision-based design) is suggested to ensure a ration choice where the alternatives are evaluated by the traditional MAUT method under uncertainty. The entire approach is termed as Normative-prescriptive approach (NPA). In this approach, the alternatives are evaluated by their observed utility but, it has also unobserved utility that should be considered.
- At every stage of the design, a designer is supposed to make a rational decision. The rationality in the design is mapping the customer requirements to the useful

design requirements precisely from the available information, but rather this information is incomplete. Therefore, the decision problem is inductive in nature that requires an inductive logic. To consider the above mentioned unobserved utility, this thesis proposes the Conditional Logit from the domain of discrete choice analysis (DCA) in rational decision-making framework that assigns the probability to the alternatives. Probabilistic choice addresses the risk and uncertainty associated with unobserved taste variation and unobserved attribute. The advantage of using conditional logit is its closed-form formulation, i.e. no computer simulation is required.

- Decision making is the process to choose an appropriate alternative based on the belief of the decision maker. Belief is the function of knowledge and confidence. The above mentioned proposed approaches are modelled with structured knowledge but, the knowledge is always covered with aleatory and epistemic uncertainty. Therefore, a decision model should be structured in cognitive way to get the confidence under these uncertainties. A new method is developed in this thesis based on nearest neighbour search (NNS) which is a spatial approach in the Cartesian plane. If the points (alternatives) with multidimension (attributes) are mapped in the 2-dimensional plane and the nearness is compared with a reference or query point, then we can easily visualize the comparison of these points with the query point. In this proposed method, a spatial relationship is introduced to make the belief as a justified belief. A designer rather enjoys the spatial relationship which gives a confidence and courage to select a preeminent alternative under uncertainty.

In Chapter 5, various case studies are conducted to check the consistency in the raking by comparing with the other methods. In this research work, DCA is first applied in MADM framework to select the preeminent material in design. Discrete choice analysis (DCA) gives the good result but, nearest neighbour search (NNS) gives the confidence and courage under uncertainty in the geometrical point of view.

6.3 Future work

The performance of engineering materials strategy at the initial stage of the design process is proposed in this thesis to explore how to formulate a tool to support a designer's decision about material selection in the context of product design. However, there are limitations to the breadth and extent of this work, and these limitations offer opportunities for future work. In this section the simplifying assumptions made in this work are identified and the resulting opportunities for future work are outlined as:

- Pairwise comparisons of the alternatives are required to be undertaken, which can be quite time consuming if it is deemed necessary that all comparison to be performed, and especially if there are a considerable number of alternatives.
- The proposed CLGT approach assumes the IID assumption, i.e., the unobserved factors are not correlated over alternatives and also have the same variance for all alternatives. For example, in the case studies, the materials can be categorized as alloys and composites. In this situation, the unobserved factors may be correlated in a particular domain (say, alloys or composites). There is a further scope to think about this issue.
- In this thesis, like most of the cases, the material is considered as a pre-assigned design parameter. There is an opportunity to think about an issue of simultaneously selection of material and geometric variables for optimal design.
- A software-based decision support system (DSS) could help a decision maker to implement this approach easily and expeditiously. Hence, a computer-based DSS should be developed to integrate the classification procedures discussed in the thesis and assist in practical applications.

The research in the development of a material selection method was the prime objective and the model has been successfully applied in ranking engineering materials to provide the best solution for a product design. The research has opened up opportunities for further research in many other areas including supply chain management, vendor selection, business and marketing, human resources, and water resources management.

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