Study of certain types of summability methods and Approximation theory

Abstract

This thesis is a detailed work on generalization of Korovkin approximation process and related summability theory and some new types of convergence. As we know after the exploration of Korovkin approximation theorem numerous paths have opened for researcher to investigate the Korovkin set. Also summability theory is a very interesting part of mathematics as it helps to approximate the limit of divergent sequences. We have combined these two branches and derived some abundant theorems which have significant depth and utilities.

This Thesis contains four main chapters. In Chapter 2, we have established Korovkin type approximation theorems for positive linear operators on $UC_*[0,\infty)$, the Banach space of all real valued uniform continuous functions on $[0,\infty)$ with the property that $\lim_{x\to\infty} f(x)$ exists finitely for any $f \in UC_*[0,\infty)$ using the notion of $A^{\mathcal{I}}$-statistical convergence and $A^{\mathcal{I}}$-summability method for real sequences and examined the Korovkin set on $UC_*([0,\infty)\times[0,\infty))$. In Chapter 3, we have approximated a sequence of positive convolution operators on $C[a,b]$, the Banach space of all real valued continuous functions on $[a,b]$ endowed with the supremum norm $||f|| = \sup_{x\in[a,b]} |f(x)|$ for $f \in C[a,b]$, based on the notion of $A^{\mathcal{I}}$-summability and $A^{\mathcal{I}}$-statistical convergence and calculated corresponding rates of convergence. In Chapter 4, we have visited on topological spaces and introduced statistical $A_{\mathcal{T}}$-strong convergence and $A^{\mathcal{I}}_{\mathcal{T}}$-strong convergence, both of which are generalizations of $A_{\mathcal{T}}$-strong convergence in Hausdorff topological spaces via a certain class of special functions. Similar to the classic scenario, we find some correlations between $A^{\mathcal{I}}$-statistical convergence and $A^{\mathcal{I}}_{\mathcal{T}}$-strong convergence. Additionally, we obtain a characterization of $A^{\mathcal{I}}$-statistical convergence. In Chapter 5, we have introduced new forms of convergence, namely, $\mathcal{I}^*_\alpha$-ue, $\mathcal{I}^*_\alpha$-ud, $\mathcal{I}^*_\alpha$-sue and $\mathcal{I}\alpha$-equal convergence and follow up some associated findings adding to the lattice features of the classes made up of all those real valued functions defined on a metric space (X,d).

P. Das 25.08.23
(Signature of the Supervisor)

Sudipta Dutta 25.08.2023
(Signature of the Co-Supervisor)

Professor
DEPARTMENT OF MATHEMATICS
Jadavpur University
Kolkata – 700 032, West Bengal

Rima Ghosh 25.08.2023
Signature of the Research Scholar (Index No:171/18/Maths/26)

Asst. Professor (W.B.E.S.)
Dept. Of Mathematics
Govt. Gen. Degree College
Manbazar - II Purulia
Govt. of W.B