ABSTRACT

Title of the thesis: Study of certain selection principles in bornological spaces and its implications

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The thesis is concerned with the study of open covers and related selection principles in metric spaces using the idea of strong uniform convergence on bornology. This line of study was initiated by Caserta, Di Maio and Kočinac. In this thesis we present a comprehensive investigations of this study. We establish implications among various selection principles involving bornological covers resulting in Scheepers' like diagrams. Some interactive results between cardinalities of bornological bases and certain selection principles involving bornological covers are presented. For a bornology $\mathfrak{B}$ we introduce the notions of $\mathfrak{B}^s$-Hurewicz and $\mathfrak{B}^s$-Gerlits-Nagy properties and present several results. Of these two mainly the $\mathfrak{B}^s$-Hurewicz property has been characterized game theoretically, by an S_{fin}-type selection principle and also by using a Ramseyan partition relation. The $\mathfrak{B}^s$-Gerlits-Nagy property is also characterized using a Ramseyan partition relation. We also show their connection with the cardinalities of bornological bases.

The study of function spaces endowed with certain topologies from the perspective of selection principle have always been of much interest because certain classical properties of function spaces can be characterized in terms of selection principles involving certain classes of covers. Here we consider the function space $C(X,\mathbb{R})$ endowed with the topology $\tau^s_{\mathfrak{B}}$ of strong uniform convergence on $\mathfrak{B}$. Several properties like T-tightness, countable T-tightness, countable (strong) fan tightness for finite sets, Id-fan tightness, Fréchet-Urysohn for finite sets, the Reznichenko property etc. are characterized in terms of selection principles involving bornological covers. We also present a detailed investigation of sequences of dense and sequentially dense subsets of $(C(X,\mathbb{R}),\tau^s_{\mathfrak{B}})$.

Further more we also present game theoretic characterizations of certain selection principles and on $(C(X,\mathbb{R}),\tau^s_{\mathfrak{B}})$ several interactions between topological games related to discretely selective property, the Gruenhage game and certain games in X are presented.

A chapter is then devoted to a more general line of investigation where the study of bornological covers and related selection principles is done using the the idea of statistical convergence in place of usual convergence in metric spaces.

The thesis provides an elaborate study of certain selective properties, two players games and related topological properties in the bornological universe and which hopefully will be useful to the interested researchers to get a full view of this particular line of study.

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