

# ON CERTAIN VERSIONS OF CONTINUITY AND RELATED PROPERTIES ASSOCIATED WITH CAUCHY AND QUASI-CAUCHY SEQUENCES

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## Abstract

The main objective of this thesis is to investigate various aspects of both metric space and uniform space concerning uniform continuity by using the notions of Cauchy regularity and ward continuity. In the initial three chapters, our main emphasis lies on exploring Cauchy regularity. There are two well-known notions related to uniform continuity, UC space and a more general concept of straight space. A space  $X$  is called UC if every real-valued continuous function on  $X$  is uniformly continuous and a space  $X$  is called straight if for any real-valued continuous function  $f$  on  $X$  and a closed cover  $X = C_1 \cup C_2$  of  $X$ , the restrictions of  $f$  on both  $C_1, C_2$  are uniformly continuous implies  $f$  is uniformly continuous. Hence straight space is a kind of generalization of UC space and one can see an additive type property of uniform continuity in a straight space. In Chapter 4, we define two types of straight spaces using Cauchy regularity, namely pre-straight and pre(\*)-straight and we show that class of all straight spaces is basically the intersection of class of all pre-straight and class of all pre(\*)-straight spaces. The concept of straightness is closely linked with certain versions of connectedness. To investigate this direction in our context in Chapter 3, we introduce a new type of connectedness, namely Cauchy connected space and present some relations of Cauchy connected space with various types of straight spaces. In Chapter 5, we analyze another perspective of Cauchy regularity specifically its preserving properties. We find several conditions under which a precompactness- and Cauchy connectedness- preserving function is Cauchy regular and discuss the role of a pre-straight space as a generalization of complete space.

In Chapter 6, initially, we work on several types of completeness in uniform space, which strictly lies in between compactness and completeness. We find some results on metrizability in line with the well-known results given by Čech, which state that a metrizable space  $X$  is completely metrizable iff  $X = \bigcap_{n=1}^{\infty} G_n$ , where each  $G_n$  is an open subspace of  $\beta X$ . At the end of this chapter we look into the notion of BqC sequence in metric space and connect it with quasi-Cauchy sequence. Finally, in the last chapter, we discuss about some variants of Cauchy regular functions.

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