Abstract

Title: EUCLIDEAN OPERATOR RADIUS INEQUALITIES OF HILBERT SPACE OPERATORS AND THEIR APPLICATIONS

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The numerical radius $w(T)$ of a bounded linear operator T defined on a complex Hilbert space $\mathcal{H}$, is defined as the radius of the smallest circular disc with centre at the origin that contains the numerical range, i.e, $w(T) = \sup\{|\langle Tx, x\rangle| : x \in \mathcal{H}, \|x\| = 1\}$. Recall that the numerical range $W(T)$ of T is defined as the subset of complex plane $\mathbb{C}$ whose elements are the image of the unit circle of Hilbert space $\mathcal{H}$ under the continuous mapping $x \longmapsto \langle Tx, x\rangle$ from $\mathcal{H}$ to $\mathbb{C}$, i.e., $W(T) = \{\langle Tx, x\rangle : x \in \mathcal{H}, \|x\| = 1\}$. There are several generalizations of numerical radius, one of them is Euclidean operator radius. Euclidean operator radius of any d-tuple operator $\mathbf{T} = (T_1, T_2, \ldots, T_d) \in \mathbb{B}^d(\mathcal{H})$ is defined as $w_e(\mathbf{T}) = \sup\left\{\left(\sum_{k=1}^{d} |\langle T_k x, x\rangle|^2\right)^{\frac{1}{2}} : x \in \mathcal{H}, \|x\| = 1\right\}$, where $\mathbb{B}^d(\mathcal{H})$ is the collection of all d-tuple of bounded linear operators defined on $\mathcal{H}$. The main focus of this thesis is to develop stronger lower and upper bounds of the numerical radius and Euclidean operator radius using various technique. Applying Euclidean operator radius inequalities we obtain various numerical radius inequalities which are finer than existing numerical radius inequalities. Among many inequalities, we obtain improvements and generalizations of the inequalities $\frac{1}{4}\|T^*T + TT^*\| \leq w^2(T) \leq \frac{1}{2}\|T^*T + TT^*\|$. Then we obtain new bounds for the zeros of a complex monic polynomial $p(z)$ of higher degree by applying the bounds of numerical radius developed here. Next we study the Euclidean operator radius inequalities of a pair of bounded linear operators which improve existing ones. Then we present bounds for the numerical radius of bounded linear operators which generalize and improve on the well-known numerical radius bounds using Euclidean operator radius inequalities. We also study generalized Euclidean operator radius inequalities and their applications. Next we obtain power inequality for d-tuple operator and applying these power inequality we obtain the Euclidean operator radius bounds for product of two d-tuple operators. Finally we obtain bounds for Euclidean operator radius of d-tuple operator and of $n \times n$ operator matrix whose entries are d-tuple operators.

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