ABSTRACT

This thesis deals with the ferroresonance which is a complex phenomenon. Ferroresonance may appear in a power system when transmission line capacitance is connected in series with the transformer at no load arising out of various line configurations that have suitable matching of the parameters of both. Although ferroresonance could be found due to abnormal situations like single pole switching in 3 phase, presence of PT with grading capacitance of C.B. but on the occurrence of that resulting in sustained ferroresonant over voltages and currents across the power system equipment; which may cause outages of distressed transformer along with line equipment- surge arrester, etc. So ferroresonance research activities are not possible in the commercial power system.

So over the years, there have been ferroresonance research activities like efforts to forecast or confirm the occurrence of ferroresonance in susceptible networks and mitigation measures. The studies pertain to the recreation of ferroresonance in a laboratory environment and the development of mathematically suitable transformer models for simulations and analysis using the computer. Nevertheless, the complexity of a non-linear mathematical equation representing the saturable core of a transformer making **ferroresonance a nonlinear dynamical system** still poses a challenge for researchers to understand the stability of the system under ferroresonance. So the study on ferroresonance and transformer experiencing it applying nonlinear dynamical mathematics to determine how the stability of the system changes, by a bifurcation, chaos, having variations of system(circuit) parameters is still under demand.

So the STUDIES ON NONLINEAR DYNAMICS OF FERRORESONANCE IN TRANSFORMERS have been performed: Experiment on Ferroresonance, designed and developed in a laboratory set up along with analysis on it using a nonlinear dynamical system technique and Finite Element Method.

The works are done in a nutshell:

i) **Laboratory Experimental Study and Verification of Ferroresonance Simulated with Rudenburg's Method.**

ii) **Investigation on the Occurrence of Ferroresonance with the Variation of Core Loss of a Transformer using Nonlinear Dynamic Model of the Transformer.**

iii) **Investigation on the Stability of an Electric Power Circuit under Ferroresonance Based on Nonlinear Dynamical Model of Transformer.**

iv) **Verification of Period-Doubling Behavior of Ferroresonance Circuit with the Jacobian Matrix and Eigenvalues.**

v) **Finite Element Method Applied under Ferroresonance.**

This thesis work is divided into ten (10) chapters. The chapter-wise organization of this thesis is as follows:

Chapter 1 gives an overall introduction to the thesis. It discusses the current scenario of the ferroresonance study and the necessity of the proposed work. This is explained in a nutshell with the basic works that have been performed under the current thesis.

Chapter 2 on the definition of ferroresonance. It discusses the effects of ferroresonance that have been listed in different publications. The study says that the circuit consists of saturable inductance, capacitance, and an alternating voltage source is prone to ferroresonance. How this combination can be achieved in the power system that also discussed in this chapter. In 1950, a German engineer R. Rudenberg presented a graphical analysis of ferroresonance which is the stepping stone of ferroresonance analysis. **This chapter provides an overview of Rudenberg's analysis.** Analysis shows, how the system jumps into ferroresonance from normal value while changing any of the circuit parameters. At the end of this chapter, a comprehensive study is made on the different types of works on ferroresonance that have been performed for decades. Four main categories have been discussed - examples & case studies, experimental investigations, damping, and analysis of ferroresonance.

Chapter 3 describes the details of **a laboratory experiment** that has been developed and performed to observe ferroresonance in a controlled environment. At first, the magnetic characteristic of the transformer core under the ferroresonance test is extracted from the open circuit test of the transformer. The series capacitance value is designed and calculated using R. Rudenberg's graphical method. To protect the digital oscilloscope a potential divider made of capacitor is used. To prevent the line equipment from extended exposure to overvoltage at ferroresonance, an auto cut-off switch is used which cuts off the circuit after a pre-defined time. The ferroresonance incident has been captured successfully. Time domain Waveform of capacitor voltage and transformer voltage at different source voltages are taken on the creation of ferroresonance at various line conditions.

Chapter 4 explains the process of **building up the simulation model of ferroresonance** incorporating experimental transformer magnetic characteristics. A wide experimental study on ferroresonance in a real setup is difficult and also hazardous for the equipment used as it involves power frequency and very high overvoltages. So for further study of ferroresonance, a suitable simulation model is developed. The B-H loop data of the transformer obtained from the experiment is used to find an approximated mathematical equation to represent the transformer core magnetism for the ferroresonance circuit model. Then that mathematical nonlinear differential equation for **ferroresonance is simulated in a MATLAB software** platform. The ferroresonance result obtained from the simulation model is compared with the experimental results where both the results match with a minor deviation.

Chapter 5 continues the study with the MATLAB model for ferroresonance developed in the previous chapter. Here some circuit parameters like **supply voltage, supply frequency, series capacitance, core loss resistance of transformer, degree of transformer core saturation, initial flux linkage** in the core of the transformer are varied and the voltage across series capacitor and transformer for ferroresonance are observed. For each of the cases it has been found at a certain point when the circuit parameter and other conditions match, voltages jump to a high value at ferroresonance. Using Rudenberg's graphical analysis it has been shown how the stable operating point of the system switches from the 1^{st} quadrant to the 3^{rd} quadrant of V-I characteristic and builds up ferroresonance oscillation.

Chapter 6 briefs the nonlinear dynamical method and its application for ferroresonance in the thesis work. It describes the use of a phase plane diagram in the analysis of a nonlinear system. For a periodic function for nonlinear ferroresonance, the **'Phase-Plane'** analysis produces **'Limit Cycles'**. The method to determine the '**Stability**' of a nonlinear ferroresonance system is to provide a small '**Perturbation**' at a steady state and if the system returns to its previous 'Phase Plane' path then the system will be called **'Stable'** otherwise **'Unstable'**. So a **'Stability domain of nonlinear ferroresonance'** has been studied. Mathematically, the linearization method that is used to determine the **'Stability of a Fixed Point'** cannot be applied to the **'Stability of Limit Cycles'**. The **'Stability of Limit Cycles'** problem has been converted into a 'Stability of a Fixed Point' problem by using the concept of **'The Poincare method and Floquet theory for the**

stability of periodic orbit. The study of the Poincare plot also reveals how a system '**Bifurcate from one Stable state to another Stable state'**.

Chapter 7 shows the simulation study of the model ferroresonance system used in Chapter 5. In this case, the system is under ferroresonance and the supply voltages are increased in steps. The Poincare plots of different ferroresonance system voltages are also observed. At first system generates a single point on the Poincare plane. As the supply voltage is increased, it generates two points, then four, eight and so on leading to **the Period-Doubling Behavior of Ferroresonance**. At a very high supply voltage, the Poincare plot shows a **'Strange Attractor'** which changes with the change of initial condition. The behavior of the system at this stage is moving to '**Chaos**'. The bifurcation plot, which holds all the Poincare plots for all supply voltages in a single diagram, shows a period-doubling behavior of the system. The calculation of **'The Feigenbaum Constant'** also establishes the fact that the system will ultimately land in chaos after **successive period doubling**. In the end, the bifurcation study with series capacitance as the **'Bifurcation'** parameter is also made.

Chapter 8 deals with the stability of the system at different **periodic oscillation**s as obtained in the **bifurcation diagram** in the previous chapter. For determining the stability at the period-1 region of the bifurcation diagram, a small perturbation is injected in the flux linkage while the system is in a steady state. Poincare plots and phase plane diagram shows that the system iterates back to its previous operating condition. To verify this condition mathematically, **the Jacobian matrix was calculated with the Poincare values near perturbation**. The magnitude of the **Eigenvalues of the Jacobian matrix** falls within the unity value. A similar investigation was carried out in the period-2 region and mathematical calculation shows the **system lost its period-1 stability**. **For period-2 oscillation**, the relations $x_{(i+2)} = x_i$ and $y_{(i+2)} = y_i$ hold, and the **Eigenvalues of the Jacobian matrix fall within the value 1**. Analysis with **period-4 oscillation** is also performed and verified.

Chapter 9 gives the details of **finite element analysis done on the ferroresonance** circuit. To know the **magnetic field distribution in the core of the transformer** during ferroresonance conditions, the finite element method is used. The **transformer model is built up in the Comsol Multiphysics** simulator. The core characteristic equation that was used in the previous chapters is injected into the core properties. Then to build up the ferroresonance circuit, **the magnetic field model of the transformer is coupled with the electric circuit** model consisting of an alternating voltage source, capacitor in series, and resistance in parallel which will act as core loss resistance. The current developed in the circuit is used as the excitation parameter of the primary of the transformer keeping the secondary of the transformer open. The simulation was done with different supply voltages. At a certain supply voltage ferroresonance is observed. **The results match with the experimental outputs** given in Chapter 3. **The magnetic flux density distribution during ferroresonance is also observed.**

Chapter 10, the last chapter, contains the conclusion and prospects of the present work. The study of ferroresonance in real systems is not possible because it leads to the destruction of the equipment. So a laboratory study of ferroresonance is performed in this research work on a miniature scale. To study the system behavior under ferroresonance, simulation, and numerical analysis has been done with the background of nonlinear mathematics. **Numerical simulations and analysis showed how the ferroresonance circuit lost its stability moving from a stable period-one response to a period-two response and towards chaos through a period double bifurcation**.

The prospects of the present work are: to implement a parallel connected capacitor in the ferroresonance circuit by both computer simulations and experimental testing, to study in detail the chaotic behavior of the system, to carry out finite element analysis with a 3-phase transformer which is used in power system.

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