Title of the thesis: Ricci solitons and CPE conjecture on some differentiable manifolds.
Index No.: 59/18/Maths/25

Abstract

The aim of this doctoral thesis is to study Ricci solitons and CPE conjecture within the framework of various differentiable manifolds. The thesis consists of five chapters. An introduction of the Differential Geometry, Ricci soliton and critical point equation (shortly CPE) conjecture are presented in **first chapter**.

In the **second chapter**, we consider 3-dimensional trans-Sasakian manifold of type (α, β) to admit a Ricci soliton and characterize the covariant derivative of potential vector field along the Reeb vector field as well as the nature of the soliton. Later, we initiate the study of $* -\eta$ Ricci soliton and gradient almost $* -\eta$ Ricci soliton within the framework of Kenmotsu manifold and obtain some characteristics of the manifold and the potential vector field. Finally, we deliberate $* -\eta$ Ricci soliton admitting $(\kappa, \mu)'$-almost Kenmotsu manifold and proved that the manifold is Ricci flat and is locally isometric to $\mathbb{H}^{n+1}(-4) \times \mathbb{R}^n$.

In the **third chapter**, we establish some results regarding conformal η-Ricci soliton and conformal Ricci soliton on $(LCS)_n$ manifold satisfying some curvature conditions such as ξ-conharmonically semi-symmetric, ξ-concircularly semi-symmetric and ξ-quasi-conformally semi-symmetric and obtain some results regarding the nature of the manifold as well as the nature of the structural vector field ξ. Later, we initiate the study of conformal η-Ricci soliton and conformal almost η-Ricci soliton within the framework of para-Sasakian manifold and we are able to find some attributes of the manifold, the scalar curvature of the manifold and the potential vector field of the soliton. Further, we evolve the characterizations of the Kenmotsu manifold and the nature of the potential vector field when the manifold satisfies $*$-conformal η-Ricci soliton and gradient almost $*$-conformal η-Ricci soliton. Eventually, we have contrived $*$-conformal η-Ricci soliton admitting $(\kappa, \mu)'$-almost Kenmotsu manifold and proved that the manifold is Ricci flat and is locally isometric to $\mathbb{H}^{n+1}(-4) \times \mathbb{R}^n$.

In **fourth chapter**, we demonstrate the nature of the soliton if Kenmotsu manifold admits conformal almost Ricci soliton. We also able to observe some properties of the scalar curvature of the manifold and the soliton function, and potential vector field of the soliton. Then we prove that if an η-Einstein para-Kenmotsu manifold admits conformal Ricci soliton and $*$-conformal Ricci soliton, then it is Einstein. Further, we acquire that 3-dimensional paracosymplectic manifold is Ricci flat if the manifold satisfies conformal Ricci soliton where the soliton vector field is conformal. Next, we evolve the nature of scalar curvature when the 3-dimensional trans-Sasakian manifold of type (α, β), provided $\alpha \neq 0$ satisfies $*$-conformal Ricci soliton.

In the **fifth chapter**, we study the critical point equation (shortly CPE) conjecture and $*$-critical point equation (shortly $*$-CPE) conjecture within the framework of various contact metric manifolds. First, it is proved that if a compact Sasakian manifold admits CPE, then either the manifold is Einstein or the potential function is harmonic in an open subset. Later, it is shown that if the manifold satisfies $*$-CPE then the manifold is η-Einstein. Later, we establish that Kenmotsu manifold satisfying the CPE either becomes an Einstein manifold or the derivative of potential function along characteristic vector field satisfy a certain relation on the distribution of η. Next, we study CPE on $(\kappa, \mu)'$-almost Kenmotsu manifold and obtained that the manifold is Einstein. In case of 3-dimensional trans-Sasakian manifold, we get that either the manifold becomes α-Sasakian or it becomes Einstein.

Arindam Bhattacharyya
10.06.2022

Sumanjit Sarkar
10-06-2022.

Professor
DEPARTMENT OF MATHEMATICS
Jadavpur University
Kolkata – 700 032, West Bengal