THERMAL ANALYSIS OF ENGINE WALL USING VARIATIONAL ITERATIVE METHOD

THESIS SUBMITTED IN PARTIAL FULFILMENTS OF THE REQUIREMENT FOR THE
DEGREE OF MASTER OF ENGINEERING IN AUTOMOBILE ENGINEERING UNDER
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DECLARATION OF ORIGINALITY AND COMPLIANCE OF ACADEMIC ETHICS

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CERTIFICATE OF RECOMMENDATION.

This is to certify that the thesis entitled " **THERMAL ANALYSIS OF ENGINE WALL USING VARIATIONAL ITERATIVE METHOD** " is a bona-fide work carried out by ANURAG ANAND under our supervision and guidance in partial fulfilment of the requirements for awarding the degree of Master of Engineering in Automobile Engineering under Department of Mechanical Engineering, Jadavpur University during the academic session 2020-2022.

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CERTIFICATE OF APPROVAL

The foregoing thesis entitled "**THERMAL ANALYSIS OF ENGINE WALL USING VARIATIONAL ITERATIVE METHOD**" is hereby approved as a creditable study of an engineering subject carried out and presented in a satisfactory manner to warrant its acceptance as a prerequisite for the degree of "Master of Automobile Engineering**"** under Department of Mechanical Engineering, Jadavpur University, Kolkata 700032, for which it has been submitted. It is understood that by this approval the undersigned do not necessarily endorse or approve any statement made, opinion expressed or conclusion drawn there in but approve the thesis only for the purpose for which it is submitted.

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CONTENTS

1. NOMENCLATURE

$\dot{q}$	Heat flux (W/m^2)
$\dot{Q}$	Heat transfer rate (W)
$\dot{g}$	Heat generation rate
k	Thermal conductivity(W/m-K)
A	Cross sectional area (m^2)
t	Time (s)
h	Convective heat transfer coefficient (W/m^2K)
T_s	Surface temperature (K)
T_∞	Ambient Temperature (K)
T	Absolute Temperature (K)
Fo	Fourier Number $(\alpha t / L^2)$
θ^*	Dimensionless variable of Temperature
X	Dimensionless variable of length $(\frac{x}{L})$

GREEK SYMBOLS

ε	Emissivity
σ	Stefan Boltzmann constant (W/m^2K^4)
λ	Lagrange multiplier
α	Thermal diffusivity ($m^2.s^{-1}$)
μ	Dynamic viscosity

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3. ABSTRACT

This thesis presents the analysis of the temperature distribution and various parameters affecting the heat transfer in an internal combustion engine cylinder wall using a semi-analytical method known as the Variational iterative method. VIM is a well-known method for solving linear and non-linear ordinary and partial differential equations. The VIM method uses the Lagrange multiplier to obtain optimal values of the parameters in a correction functional. The method usually gives rapidly convergent successive approximations to the exact solution if any solution possible exists for that differential equation. The analysis of the engine wall has been done based on Dirichlet boundary conditions. The results and expressions of temperature distribution with time and distance obtained from the variational iterative method have been validated with another analytical method used to solve partial differential equations, i.e. the separation of variables method. The heat conduction equation used in this thesis work is one-dimensional without volumetric heat generation.

4. INTRODUCTION

4.1 IC ENGINE CYLINDER WALL HEAT TRANSFER

An internal combustion engine is a heat engine in which the combustion of fuel occurs with an oxidiser which is usually air in a combustion chamber.

The Internal combustion engine is the most reliable and efficient form of powerplant which is in heavy use in automobile industry and transportation sector. Major advantages of IC engine being:

- Low overall working temperature because the peak temperature is achieved for a short duration of time thus keeping a check on thermal stresses inside the engine.
- Easy availability of fuel.

An IC engine is composed of: -

- Piston
- Cylinder
- Inlet valve
- Exhaust valve
- Spark plug
- Crank shaft
- Cam shaft
- Crank

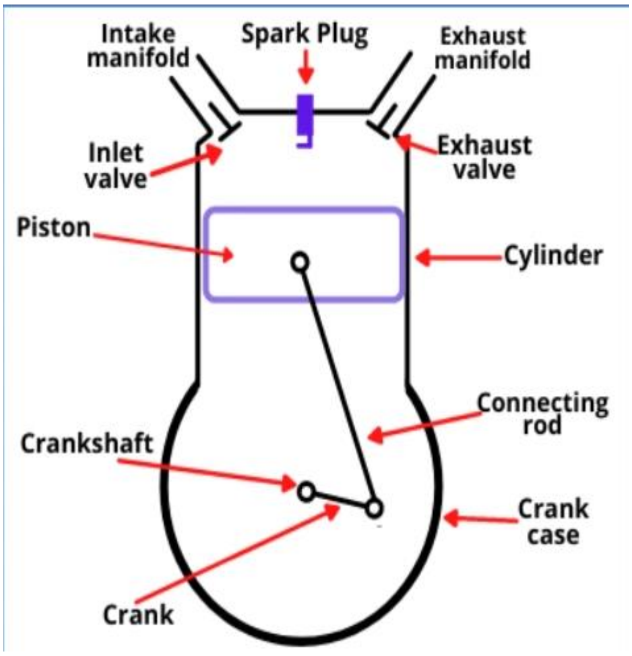

Figure 1 IC engine components

A cylinder is the part of IC engine in which there is movement of piston also known as stroke. It is where the combustion process takes place between the gasoline fuel and air mixture mixed in a proportionate amount depending upon the engine load. Cylinder of the IC engine is composed of a cylinder block which is considered to be the main body of engine. This is supposed to be the main structure responsible for supporting and holding the other components together. The cylinder block is also responsible for providing mounting points to the other engine components such as the valves, ports spark plug etc.

The manufacturing process used for cylinder block is casting. The material that is used for making cylinder blocks nowadays is either Cast iron or aluminium depending upon the working conditions like temperature and pressure. Even we see that for a multi cylinder engine also the whole cylinder block unit is casted as single unit only. The cylinder head is mounted tightly on top of cylinder block with the help of fastening devices like bolts and studs. The Cylinders are provided with proper cooling system like fins, water jacket etc. to take away any sort of excessive heat caused during the prolonged operation and to avoid any mechanical failure due to exposure to increased temperatures.

Apart from that, in order to seal the surfaces cylinder gaskets are used even between the cylinder head and the cylinder block. The bottom most part of cylinder is called crankcase.

In IC engine, there is basically two ways where heat is transferred,

- Piston heat transfer
- Cylinder wall heat transfer

The heat transfer is important because it is necessary to keep the overall temperatures of the working parts under material design limits such as the piston crown and exhaust valve.

There are three modes of heat transfer mechanisms in the cylinder wall,

- Conduction Heat transfer

- Convection heat transfer
- Radiation heat transfer

The Conduction heat transfer is type of transfer in which the energy transport is in the form of molecular motion and interaction between the molecules.

Rate of heat conduction through a medium in any given direction for the cartesian or polar axis is proportional to the temperature difference across the medium and the area normal to the direction of heat flow but is inversely proportional to the distance in that direction. This relationship is expressed in the differential form by Fourier's law of heat conduction for one-dimensional heat conduction as,

$$\dot{q} = \left(\frac{Q}{A}\right) = -K\,\frac{dT}{dx}$$

For a multidimensional system, Heat conduction is defined by governing differential equations in a more generalised form for rectangular, cylindrical and spherical coordinate systems given for constant thermal conductivity which are defined as,

- Rectangular coordinate system

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{g}}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

- Cylindrical coordinate system

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{g}}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

- Spherical coordinate system

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{\dot{g}}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

The second mode of heat transfer is Convective heat transfer, under which the energy is transported by means of mass fluid motion of the surrounding medium. The convection heat transfer highly depends on the fluid properties like dynamic viscosity (μ), thermal conductivity(k), density and specific heat (Cp) as well as the fluid velocity. It also depends on the geometry and the roughness of the solid surface, in addition to the type of fluid flow (such as being streamlined or turbulent). Thus, the convection heat transfer relations are found rather complex because of the dependence of convection on so many variables. However, Newton Law of cooling define that the heat flux i.e. the heat transfer per unit area is proportional to the temperature difference between the object and the surrounding medium.

$$\dot{q} = \frac{Q}{A} = h*(T_s - T_\infty)$$

where the convection heat transfer coefficient (h) is determined by utilizing a dimensionless number known as Nusselt number, which itself has different values with different geometry and flow conditions.

The other mode of heat transfer is the radiation heat transfer which is a means of energy transportation that involves carrying energy by the electromagnetic waves coming out of the surface or a volume at a temperature "T". The radiation mode of heat transfer does not require any medium and can happen in vacuum also. Radiation heat transfer is governed by Stefan Boltzmann's law which states that radiation energy emitted by black body per unit time and per unit area is proportional to the fourth power of the absolute temperature. The energy emitted was known as blackbody emissive power and is denoted by E_b.

Inside the IC engine, the heat transfer in the cylinder follows a certain order,

- Firstly, the Forced convection heat transfer of the hot combustion gases through the boundary layer inside cylinder.

- After that the conduction heat transfer occurring across the cylinder wall and after that forced convection outside the cylinder wall.

4.2 BOUNDARY CONDITIONS:

Boundary conditions are form of constraints in the form of mathematical equations which exert a certain condition on a given problem in the form of partial differential equations. There are several types of boundary conditions such as Neumann, Dirichlet, Cauchy, Mixed and Robin of which Dirichlet and Neumann are mostly used while solving partial differential equation.

- **Dirichlet**: This boundary condition gives the value of function for the boundary or extremes. For example, in order to specify Dirichlet boundary conditions for any domain [m, n], gives the unknown values at the endpoints of the domain i.e. m and n.

- **Neumann**: This boundary condition is similar to the Dirichlet, except for the fact that the boundary condition here defines the derivative of the function that is to be known. For example, we could define $f'(m) = \lambda$ which defines a Neumann boundary condition at the right extreme of the interval domain [m, n].

- **Robin**: This boundary condition is a combination of the value of a function and its derivative which is given weightage.

- **Mixed**: This boundary condition is similar to the Robin boundary condition, except that the parts of the boundary conditions are specified differently. For example, on the interval [m, n], the unknown $f'(x)$ at $x = m$ could be governed by a Neumann condition and on the other hand unknown f(x) at x = n will be governed by a Dirichlet condition.

- **Cauchy**: This boundary condition is similar to the Robin boundary condition, except that while the Robin boundary condition involves only one constraint while on the other hand the Cauchy boundary condition implies two constraints.

5 VARIATIONAL ITERATIVE METHOD

The variational iteration method (VIM) was introduced firstly by Ji-Huan He for giving solution to diverse range of problems whose mathematical models tend to give form of a differential equation or system of differential Equations. The idea behind development of VIM is construction of an iteration method based on a correction functional in which a generalized Lagrange multiplier is included. The value of the lagrange multiplier is chosen depending upon the variational theory. This is done so that each iteration improves the solution's accuracy. The initial approximation or condition (trial function) usually includes unknown coefficients which can be determined to justify any boundary and initial conditions. The method usually gives rapidly convergent successive approximations to the exact solution if any solution possible exists for that differential equation. VIM has been applied successfully to many different problems.

In this section we are going to get an idea about the Variational iterative method.

Consider, a partial differential equation,

$$L_t\, u(x,t) + L_x\, u(x,t) + Nu(x,t) = g(x,t) \qquad \textbf{(1)}$$

Where,

u is a function of two independent variables x and t

L_t is the linear operator which involves partial derivatives w.r.t 't' only

L_x is the linear operator which involves partial derivatives w.r.t 'x' only

N is non linear operator and g(x,t) is a continuous function.

According to Variational Iterative Method, the Correctional function in terms of x and t for the equation (1) is defined as,

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda(x,\xi).\{L_t u(x,\xi) + L_x u(x,\xi) + Nu(x,\xi) - g(x,\xi)\}d\xi$$

for n =1,2,3………

where, $u_n(x,t)$ is the correctional functional but ù(x,t) is considered as restricted variation, i.e. $\delta u_n = 0$ and λ is lagrange multiplier which can be obtained and identified optimally using the variational theory. After the determination of the lagrange multiplier λ, we now obtain the successive approximation $u_{n+1}(x,t)$ of u(x,t)
after applying initial condition $u_0(x,t)$.
After the successive approximation we now find the solution of equation (1) by,

$$u(x,t) = \lim_{n\to\infty} u_n(x,t)$$

For a variational variation method, it consists of various steps:

1. Determine how many iterations.
2. Creating correction functional using lagrange multiplier.
3. Setting up initial conditions.
4. Verifying the boundary value conditions.
5. Transforming the ordinary differential equation using VIM
6. Solve the integral form
7. Checking the solution.
8. Repeat from step 3 to step 7 for each iteration.
9. Continue until a desired iteration is achieved.

5.1 LAGRANGE MULTIPLIER

Lagrange multiplier is a term introduced in the field of algebra and calculus. It is widely known in case of optimization and variational calculus. Inokuti et al. suggested the method of General Lagrange multiplier [5]. To understand general Lagrange multiplier concept for partial differential equations, let us take an algebraic equation,

$$f(x) = 0 \text{ for all } x \epsilon R \qquad (1)$$

If we consider "x_n" as an approximate root of the equation, then

$$f(x_n) \neq 0 \qquad (2)$$

In order to improve the accuracy, we rewrite the above equation known as correction equation,

$$x_{n+1} = x_n + \lambda . f\left(x_n\right) \qquad (3)$$

where, λ is Lagrange multiplier

To identify the Lagrange multiplier λ, we can write eqn. (3) as,

$$\frac{dx_{n+1}}{dx_n} = 0$$

After differentiating we get,

$$1 + \lambda . f'(x_n) = 0$$

Which implies,

$$\lambda = \frac{(-1)}{f'(x_n)}$$

Similarly, for different kinds of equation we get different types of lagrange multiplier. In Integral calculus the lagrange multiplier also depends upon the order of the given partial differential equation.

However, for n^{th} order of time derivative in differential equation,

$$\lambda = \left(\frac{(-1)^n}{(n-1)!}\right).(t-x)^{n-1}$$

Let's discuss some of the following examples to understand this method in a better way.

5.1.1 WAVE EQUATION

Let us consider a first order one dimensional wave equation.

$$\frac{\partial u}{\partial t} + c\left(\frac{\partial^2 u}{\partial x^2}\right) = 0,$$

For c > 0

Boundary conditions are:

$u(0,t) = \sin(\frac{-c\pi t}{l})$

$u_x(0,t) = \frac{\pi}{l}\cos(\frac{-c\pi t}{l})$

$u_t(x,0) = (\frac{-c\pi}{l})\cos(\frac{\pi x}{l})$

and initial conditions we consider as,

$u(x,0) = \sin(\frac{\pi x}{l})$

Now we write the correction functional for the above wave equation of the form,

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda[(\frac{\partial u_n(x,t)}{d\tau}) + c(\frac{\partial 2u_n(x,t)}{dx^2})]d\tau$$

The stationary conditions for the above equation are,

$\lambda'(\tau) = 0$ which gives, $1 + \lambda(\tau) = 0$ which gives the solution, $\lambda = -1$

Now substituting the value of λ in the above correction functional we have

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t [(\frac{\partial u_n(x,t)}{d\tau}) + c(\frac{\partial 2 u_n(x,t)}{dx^2})] d\tau$$

From the above boundary conditions we can understand that the solution

contains $\sin(\frac{\pi x}{l})$,

So, we can choose the initial conditions as,

$u_0(x,t) = u(x,t) = \sin(\frac{\pi x}{l})$

Now considering the iterations from n = 0 and above, we get

$u_0(x,t) = \sin(\frac{\pi x}{l})$

$u_1(x,t) = \sin(\frac{\pi x}{l}) - \cos(\frac{\pi x}{l}) (\frac{c\pi t}{l})$

$u_2(x,t) = \sin(\frac{\pi x}{l})[1 - \frac{1}{2!} (\frac{c\pi t}{l})^2] - \cos(\frac{\pi x}{l}) (\frac{c\pi t}{l})$

$u_3(x,t) = \sin(\frac{\pi x}{l})[1 - \frac{1}{2!} (\frac{c\pi t}{l})^2] - \cos(\frac{\pi x}{l}) [(\frac{c\pi t}{l}) - \frac{1}{3!} (\frac{c\pi t}{l})^3]$

Then the n^{th} approximation is,

$u_n(x,t) = \sin(\frac{\pi x}{l})[1 - \frac{1}{2!} (\frac{c\pi t}{l})^2 + \frac{1}{4!} (\frac{c\pi t}{l})^4 - \frac{1}{6!} (\frac{c\pi t}{l})^6 + \ldots\ldots] - \cos(\frac{\pi x}{l}) [(\frac{c\pi t}{l}) - \frac{1}{3!} (\frac{c\pi t}{l})^3 + \frac{1}{5!}$

$(\frac{c\pi t}{l})^5 - \frac{1}{7!} (\frac{c\pi t}{l})^7 + \ldots..]$

Then the solution is given as,

$u(x,t) = \sin(\frac{\pi x}{l}) \cos(\frac{c\pi t}{l}) - \cos(\frac{\pi x}{l}) \sin(\frac{c\pi t}{l})$

$\quad = \sin[\pi(\frac{x-ct}{l})]$

5.2 APPLICATIONS OF VIM

This method is vastly applicable on higher as well as lower order of linear and nonlinear partial and ordinary differential equations. It has many applications like:

Boundary Value Problems of various-orders

- Boussinesq Equations
- Thomas-Fermi Model
- Unsteady Flow of Gas through Porous Medium
- Boundary Layer Flows
- Blasius Problem
- Goursat Problems and Laplace Problems Heat and Wave Like Models
- Burger Equations, Couple Burger equations and Parabolic Equations
- Helmholtz Equations, Fisher's Equations
- Schrödinger Equations
- Sine-Gordon Equations, Telegraph Equations, Flierl Petviashivili Equations
- Lane-Emden Equations
- Emden-Fowler Equations

6 LITERATURE REVIEW

To solve problems related to various phenomenon related to physics and engineering we need some mathematical tools in the form of operators and equations to observe and study the physical interpretation. Generally, these physical problems yield partial differential equations with given boundary and initial conditions. To solve these partial differential equations various numerical methods have been developed over the years. In order to study the physical interpretation, we need the solutions of these partial differential problems. However, it is not possible to get the exact solution of some pde problems. But using these numerical methods we can find the approximate solutions. Some of the numerical methods developed earlier to solve these partial differential problems were adomian decomposition method, homotopy perturbation method etc. from these methods approximate solutions could be obtained but the solutions could not converge to exact solutions in some cases.

In 1999 He[1] [2] proposed the variational iterative method(VIM). This method is currently used to solve various linear and nonlinear physical problems to get the analytical and the approximate solutions which eventually converge to get the exact solution. Further this method was used to solve various linear, non-linear, homogenous and non-homogenous differential equations. The idea behind developing variational iterative method is to construct an iteration method which is based on a correction functional that has a generalized lagrange multiplier based on the nature of the physical problem. The value of lagrange multiplier depends upon the order of the partial differential equations and is chosen using variational theory. This is done so that the accuracy of the solution improves upon each successive iteration. The algorithm also involves use of an initial approximation (trial function) which includes unknown coefficients which can be determined to satisfy the boundary conditions. The VIM method can give rapidly convergent approximations of the exact solution if there is any solution possible, or atleast can give some approximations which can be further used for numerical purposes. This method can be used for bounded and unbounded domains as well. Wazwaz [3] used the VIM for solving the linear and non-linear integro-differential equations and the Volterra integral to explain the use of variational iterative method for solving homogenous and non-homogenous partial differential equations. He J.H.[4-6] developed VIM to solve linear, non-linear and boundary value problems. The result obtained was that the successive approximations of the partial differential equation with boundary and initial conditions obtained by this method are converging to exact solution. H. Ahmad [7-9] used variational iterative method for obtaining the solutions of the fifth order differential equations, wave like vibration equations and telegraph equations. Muhammad Munib Khan[10] applied VIM to obtain the solution for differential equation for studying equation of motion of the mathematical pendulum and duffing harmonic oscillator. AL-Fayadh [11]. E.Rama, K.Somaiah and K.Sambaiah [12] have used this method for obtaining solution of various types of problems like differential equations of first and second order, isoperimetric problem and Volterra integral equation of second kind. Various algorithms were developed by researchers using the Variational iterative method. A computational method was developed by Behzad Kafash et al [13] using VIM for determining the solution for the optimal control problems. Hijaz Ahmad and Tufail A Khan[14] used VIM for developing an algorithm for finding solution for the differential equations defined for equations of motion for simple and damped mass-spring systems. Using VIM J.H. He and H.Latifizadeh [15] developed a general numerical algorithm for solving non-linear differential equations. Wazwaz [16] solved linear and non-linear Schrodinger using variational iterative method. Applying VIM method Dehghan and Shakeri [17] for solving the Cauchy reaction-diffusion problem.

Ravi Kanth and Arana [18] used VIM to solve treating nonlinear singular boundary value problems. Rezazadeh et al. [19] applied VIM to parametric oscillation of an eletrostatically actuated microbeam. Yang and Chen [20] used VIM to obtain Choice of an optimal initial solution for a wave equation. Torvattanabun and Koonprasert [21] applied VIM to convergence and solve a First-Order Linear System of PDEs with Constant Coefficients. Torvattanabun and Koonprasert [22] used VIM to solve Eighth-Order Boundary Value Problems. Multistage variational iteration method (MVIM) was first introduced by Batiha et al.[23] on a class of nonlinear system of ordinary differential equations. This MVIM offers accurate solutions over a longer time frame (more stable) compared to the VIM. Some progressive work has also been done in the field of VIM to improve the accuracy and stability of the method. In 2007, Multistage Variational iterative method(MVIM) was first introduced by Batiha et al. [24] on a class of non linear system of ordinary differential equations. MVIM method has been developed to deliver accurate solutions with stability over a longer time frame.

6.1 OBJECTIVES OF PRESENT WORK

From the literature review we came to know that how variational iterative method provides a stable and accurate solution to solve linear and non-linear partial differential equations. Also, VIM takes less time to for giving convergent solutions over long time frame or successive approximations.

The thesis work aims to provide solution with different conditions by doing calculations with a newer approach for solving heat equations using the VIM modification. The heat equation considered here is a one-dimensional heat conduction equation with Dirichlet boundary conditions with different conditions.

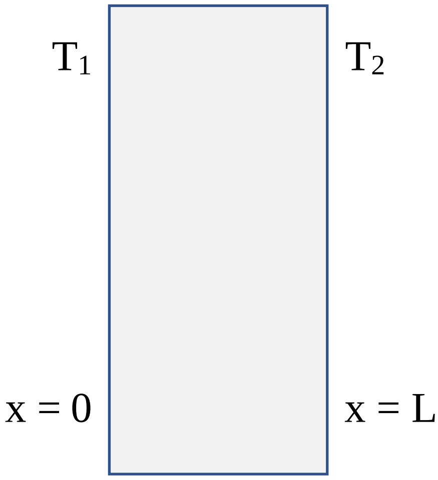

Figure 2 Cylinder wall of IC engine having thickness L

In the present work we consider an Internal combustion engine cylinder wall maintained at a temperature with different conditions such as-

- Temperature at ends is maintained at zero i.e $T_1 = 0$ and $T_2 = 0$.
- Temperature at ends is maintained at some constant value i.e T_1 and T_2 with $T_1 > T_2$.

For these two cases we will be finding the exact solution for the temperature distribution by solving the partial differential equation for one dimensional heat conduction using successive approximations in variational iterative method starting with an initial guess or condition. The initial value will be governed by the situation/condition. After finding the exact value for temperature distribution we will be plotting the graph and showing the variation of temperature with varying value for thermal diffusivity. Also, after finding the exact solution for the temperature distribution from the heat conduction equation using VIM method, we will compare the results obtained with the expression for temperature distribution from separation of variables method to validate the results for these two cases as mentioned above. Apart from the validation and finding the solution proper emphasis has been given to analysis part also where the Temperature distribution has been plotted with respect to time and distance of the section from one end. Also, graphical analysis has been done to show Variation of heat transfer along the wall w.r.t time and distance(x).

6.2 ASSUMPTIONS

- Thermo-physical properties of the material and air is kept constant.
- Heat generation in the cylinder wall is neglected.
- The heat transfer analysis is considered one-dimensional. Hence temperature gradient along y and z directional is are neglected.
- Radiation and convection heat transfer is neglected.
- The material is homogenous and isotropic.
- Temperature at extreme ends do not change with time in case with T_1 and T_2 temperature at ends.
- Cross sectional area is constant.

6.3 MATHEMATICAL FORMULATION AND CALCULATIONS

Consider the one-dimensional heat conduction equation across the cylinder wall in dimensional form,

$$\frac{\partial T(x,t)}{\partial t} - \alpha\left(\frac{\partial^2 T(x,t)}{\partial x^2}\right) = 0 \quad \text{for } 0 \leq x \leq L,\ t > 0$$

Now we try to write this one-dimensional heat conduction equation in dimensionless form, so we can define dimensionless terms such as, for example,

$$\theta^* = \frac{T}{T_o},\ X = \frac{x}{L},\ Fo = \frac{t}{T}$$

Where θ^*,Fo , X are dimensionless parameters

Now, substituting the dimensionless parameters in the one-dimensional heat conduction equation we get,

$$\left(\frac{\partial^2 \theta^*(X,Fo)}{\partial X^2}\right) - \frac{\partial \theta^*(X,Fo)}{\partial Fo} = 0 \quad \text{for } 0 \leq X \leq 1,\ Fo > 0$$

Where, Fo is Fourier number and equal to ($\alpha t/L^2$).

The above equation is in the non-dimensionalized form of one-dimensional heat conduction equation.

The given Dirichlet boundary conditions here for the first condition is;

- **Temperature at ends is maintained at zero (ends are insulated) i.e.**

  $T_1 = T(0,t) = 0$

  ⇨ $\theta_1^* = \theta^*(0,Fo) = 0$

  For, $T_2 = T(1,t) = 0$

  ⇨ $\theta_2^* = \theta^*(1,Fo) = 0$

We first take initial conditions or initial approximation as,

$T(x,0) = T_o \sin(m\pi x/L)$, where m > 0

Writing in non-dimensional form we get,

$$\theta^*(X,0) = \frac{T(x,0)}{T_o} = \sin(m\pi X)$$

The Variational iterative method can now be written as for non-dimensional form as,

$$\theta^*_{n+1}(X,Fo) = \theta^*_n(X,Fo) + \int_0^{Fo} \lambda \left(\frac{(\partial \theta^*_n(X,Fo))}{\partial \xi} \right) - \left(\frac{(\partial^2 \theta^*_n(X,Fo))}{\partial X^2} \right) d\xi$$

Starting with the initial iteration/approximation i.e n = 0,

$$\theta^*_1(X,F_o) = \theta^*_o(X,Fo) + \int_0^{F_o} \lambda \left(\frac{(\partial \theta^*_o(X,Fo))}{\partial \xi} \right) - \left(\frac{(\partial^2 \theta^*_o(X,Fo))}{\partial X^2} \right) d\xi$$

Based on variational theory we get the value for lagrange multiplier,

$\lambda = -1$

This gives the equation as,

$$\theta^*_1(X,Fo) = \theta^*_o(X,Fo) - \int_0^{Fo} \left(\frac{(\partial \theta^*_o(X,Fo)}{\partial \xi} \right) - \left(\frac{(\partial^2 \theta^*_o(X,Fo)}{\partial X^2} \right) d\xi \qquad \textbf{(1)}$$

We have assumed initial condition as $\theta^*(X,0) = \sin(m\pi X)$

$$\frac{\partial \theta^*_o(X,Fo)}{\partial \xi} = 0$$

$$\frac{\partial^2 \theta^*_o(X,Fo)}{\partial X^2} = -(m\pi)^2 * sin(m\pi X)$$

Putting the values in eqn.(1) we get,

$\theta^*_1(X,Fo) = \sin(m\pi X)(1 - (m\pi)^2.Fo)$

Now 2nd iteration i.e. for n=1 we get the value for $\theta^*(X,Fo)$ as,

$\theta^*_2(X,Fo) = \sin(m\pi X)(1 - (m\pi)^2 Fo + \frac{1}{2}*((m\pi)^2 Fo)^2)$

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Similarly, after successive ‘n’ iterations, we get

$\theta^*(X,Fo) = \sin(m\pi X)(1 - (m\pi)^2 Fo + \frac{1}{2}*((m\pi)^2 Fo)^2 - \frac{1}{6}*((m\pi)^2 Fo)^3 + \ldots..)$ ….(2)

Eqn.(2) shows a Taylor's expansion for e^{-x}

We get the equation (2) when we estimate the value of $\theta^*(X,Fo)$ for 'n' iterations,

So according to VIM algorithm,

$$\theta^*(X,Fo) = \lim_{n\to\infty} \theta_n^*(X,Fo)$$

The value of limit converges $\theta_n^*(X,Fo)$ to,

$$\theta^*(X,Fo) = sin(m\pi X) * e^{-(m\pi)^2.Fo}$$

- **Temperature at ends are maintained at constant values T_1 and T_2 ($T_1 > T_2$)**

Here we define dimensionless term $\theta^* = \frac{(T - T_2)}{(T_1 - T_2)}$

$T(0,t) = T_1$

⇨ $\theta_1^* = \theta^*(0,Fo) = 1$

For, $T(1,t) = T_2$

⇨ $\theta_2^* = \theta^*(1,Fo) = 0$

We first take initial conditions or initial approximation as,

Writing in non-dimensional form we get,

$\theta^*(X,0) = \theta_1^* - (\theta_1^* - \theta_2^*) X + \sin(m\pi X)$

Which equals to,

$\theta^*(X,0) = \theta_o^*(X,F_o) = 1 - X + \sin(m\pi X)$

The Variational iterative method can now be written as for non-dimensional form as,

$$\theta^*_{n+1}(X,Fo) = \theta^*_n(X,Fo) + \int_0^{Fo} \lambda \left(\frac{(\partial \theta^*_n(X,Fo)}{\partial \xi} \right) - \left(\frac{(\partial^2 \theta^*_n(X,Fo)}{\partial X^2} \right) d\xi$$

Starting with the initial iteration/approximation i.e n = 0,

$$\theta^*_1(X,F_o) = \theta^*_o(X,Fo) + \int_0^{F_o} \lambda \left(\frac{(\partial \theta^*_o(X,Fo)}{\partial \xi} \right) - \left(\frac{(\partial^2 \theta^*_o(X,Fo)}{\partial X^2} \right) d\xi$$

Based on variational theory we get the value for Lagrange
multiplier,
$\lambda = -1$

This gives the equation as,

$$\theta^*_1(X,Fo) = \theta^*_o(X,Fo) - \int_0^{Fo} \left(\frac{(\partial \theta^*_o(X,Fo)}{\partial \xi} \right) - \left(\frac{(\partial^2 \theta^*_o(X,Fo)}{\partial X^2} \right) d\xi \qquad \textbf{(1)}$$

We have assumed initial condition as $\theta^*(X,0) = 1 - X + \sin(m\pi X)$

$$\frac{\partial \theta^*_o(X,Fo)}{\partial \xi} = 0$$

$$\frac{\partial^2 \theta^*_o(X,Fo)}{\partial X^2} = -(m\pi)^2 \sin(m\pi X)$$

Putting the values in eqn.(1) we get,

$\theta_1^*(X,Fo) = 1 - X + \sin(m\pi X)(1 - (m\pi)^2.Fo)$

Now 2nd iteration i.e. for n=1 we get the value for $\theta^*(X,Fo)$ as,

$\theta_2^*(X,Fo) = 1 - X + \sin(m\pi X)(1 - (m\pi)^2 Fo + \frac{1}{2}*((m\pi)^2 Fo)^2)$

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Similarly, after successive 'n' iterations, we get

$\theta^*(X,Fo)=1-X+\sin(m\pi X)(1 - (m\pi)^2Fo + \frac{1}{2}*((m\pi)^2Fo)^2 - \frac{1}{6}*((m\pi)^2Fo)^3 + \ldots..)$ ….(2)

Eqn. (2) shows a Taylor's expansion for e^{-x}

We get the equation (2) when we estimate the value of $\theta^*(X,Fo)$ for 'n' iterations,

So according to VIM algorithm,

$$\theta^*(X,Fo) = \lim_{n\to\infty} \theta_n^*(X,Fo)$$

The value of limit converges $\theta_n^*(X,Fo)$ to,

$$\theta^*(X,Fo) = 1 - X + sin(m\pi X) * e^{-(m\pi)^2.Fo}$$

or this above equation can be written as,

$$\theta^*(X,Fo) = \theta_1^* + (\theta^*_2 - \theta^*_1)X + \sin(m\pi X)e^{-(m\pi)^2.Fo}$$

7 VALIDATION

The given non -dimensional 1D heat conduction is given as,

$$\left(\frac{\partial^2\theta^*(X,Fo)}{\partial X^2}\right)-\frac{\partial\theta^*(X,Fo)}{\partial Fo}=0 \qquad \text{for } 0 \le X \le 1,\ Fo > 0$$

- **For 1st case when Temperature at ends is equal to zero.**

  i.e $T_1 = T(0,t) = 0$

  $\Rightarrow \theta_1^* = \theta^*(0,Fo) = 0$

  $T_2 = T(1,t) = 0$

  $\Rightarrow \theta_2^* = \theta^*(1,Fo) = 0$

To validate the result, we also solve the same dimensionless one-dimensional heat conduction equation by separation of variables method,

Assuming the solution is of the form,

$\theta^*(X,Fo) = f(X).Y(Fo)$ …….(1)

Where f is a function of 'X' and Y is a function of 'Fo'

$fY' = f''Y$

$\Rightarrow \left(\frac{f''}{f}\right) = \left(\frac{Y'}{Y}\right) = k(\text{constant})$ ……(2)

We can see that L.H.S is a function of 'X' and R.H.S is a function of 'Fo'. Since 'X' and 'Fo' are independent variables then (2) is true only if both L.H.S and R.H.S is equal to the constant 'k'.

$$\Rightarrow \left(\frac{d^2 f}{dX^2}\right) - kf = 0 \qquad \text{.......(3)}$$

Also,

$$\Rightarrow \left(\frac{dY}{dF_o}\right) - k.Y = 0 \qquad \text{.....(4)}$$

Since $\theta^*(X,Fo) = f(X).Y(Fo)$

Using first boundary condition, $\theta^*(0,Fo) = 0$, $Y(Fo) \neq 0$

We get $f(0) = 0$ ……(5)

Similarly using 2nd boundary condition, we get

$f(1) = 0$ ……(6)

Solving (3) and (4), three cases arise,

- When k is positive i.e. $k > 0$ and $k = p^2$

  $f = c_1.\, e^{pX} + c_2 e^{-pX}$ and $Y = c_3.\, e^{-Fo.p^2}$

- When k is negative i.e. $k = -p^2$

  $f = c_4 \cos pX + c_5 \sin pX$ and $Y = c_6.\, e^{-Fo.p^2}$

- When k is zero, $k = 0$

  $f = c_7 X + c_8$, $Y = c_9$

Comparing the cases with the physical conditions of one-dimensional heat conduction equation, best condition is the second case where $k = -p^2$

So, the equation of the form of separation of variables can be written as,

⇨ $\theta^*(X,Fo) = (c_4\cos pX + c_5\sin pX) * c_6.\ e^{-Fo.p^2}$

We know that $f(X) = (c_4\cos pX + c_5\sin pX)$

Using equation (5),

⇨ $c_4 = 0$

⇨ $f(X) = c_5\sin pX$

Using equation (6),

⇨ $c_5\sin p = 0$

For non-trivial solution, $p = n\pi$

⇨ $k = -n^2\pi^2$

And f(X) can be written as,

$f(X) = c_5\sin(n\pi X)$

We know that for every eigen value there is unique eigen function

$f_n(X) = (c_5)_n \sin(n\pi X)$ ..……(7)

Similarly, for eqn. (4)

$Y(Fo) = c_6.\ e^{-Fo.p^2}$

The eigen function we obtain,

$Y_n(Fo) = (c_6)_n\ e^{-Fo.n^2\pi^2}$ ………..(8)

So, Using eqn.(7) and (8) we get,

$\theta_n^*(X,Fo) = f_n(X).Y_n(Fo)$

⇨ $\theta_n^*(X,Fo) = (c_5)_n \sin(n\pi X).\ (c_6)_n\ e^{-Fo.n^2\pi^2}$

⇨ $\theta_n{}^*(X,Fo) = B_n \,.\, \sin(n\pi X).\ e^{-Fo.n^2\pi^2}$

Where $B_n = (c_5)_n.\ (c_6)_n$

$$\theta^*(X,Fo) = \sum_{n=1}^{\infty} \theta_n^*\left(X, Fo\right)$$

$$= \sum B_n.\ sin\left(n\pi X\right).e^{-Fo.n^2\pi^2}$$

Here, $B_n = 2\int_0^1 \theta^*\left(X,0\right).\sin(n\pi X)dX$

The initial conditions are $\theta^*(X,0) = \sin(m\pi X)$

$$B_n = 2\int_0^1 \sin(m\pi X).\sin(n\pi X)dX$$

$$= \frac{(m-n)sin(\pi(n+m)) + (m+n)sin(\pi(n-m))}{\pi(n^2-m^2)}$$

$$= \frac{\sin(\pi(m-n))}{\pi(m-n)} - \frac{\sin(\pi(m+n))}{\pi(m+n)}$$

Simplifying the above expression, we get

$$B_n = \frac{2n\sin m\pi\cos n\pi - 2m\cos m\pi \sin n\pi}{\pi(n^2-m^2)}$$

Here we can see that, the $\sin n\pi$ will be zero as n =1,2,3..

This implies,

$$B_n = \frac{-2n\sin m\pi\cos n\pi}{\pi(n^2-m^2)}$$

$\theta^*(X,Fo) = \sum B_n.\ sin\left(n\pi X\right).e^{-Fo.n^2\pi^2}$ for m =1 and n =1,2,3,…. With X =0.3 comes out to be,

$$\theta^*(X,Fo) = \sum \frac{2n\sin m\pi \cos n\pi}{\pi(m^2 - n^2)} . \sin(n\pi X).e^{-Fo.n^2\pi^2}$$

Here we take m =1.0 and X = 0.3 and plot θ *(X,Fo) v/s X

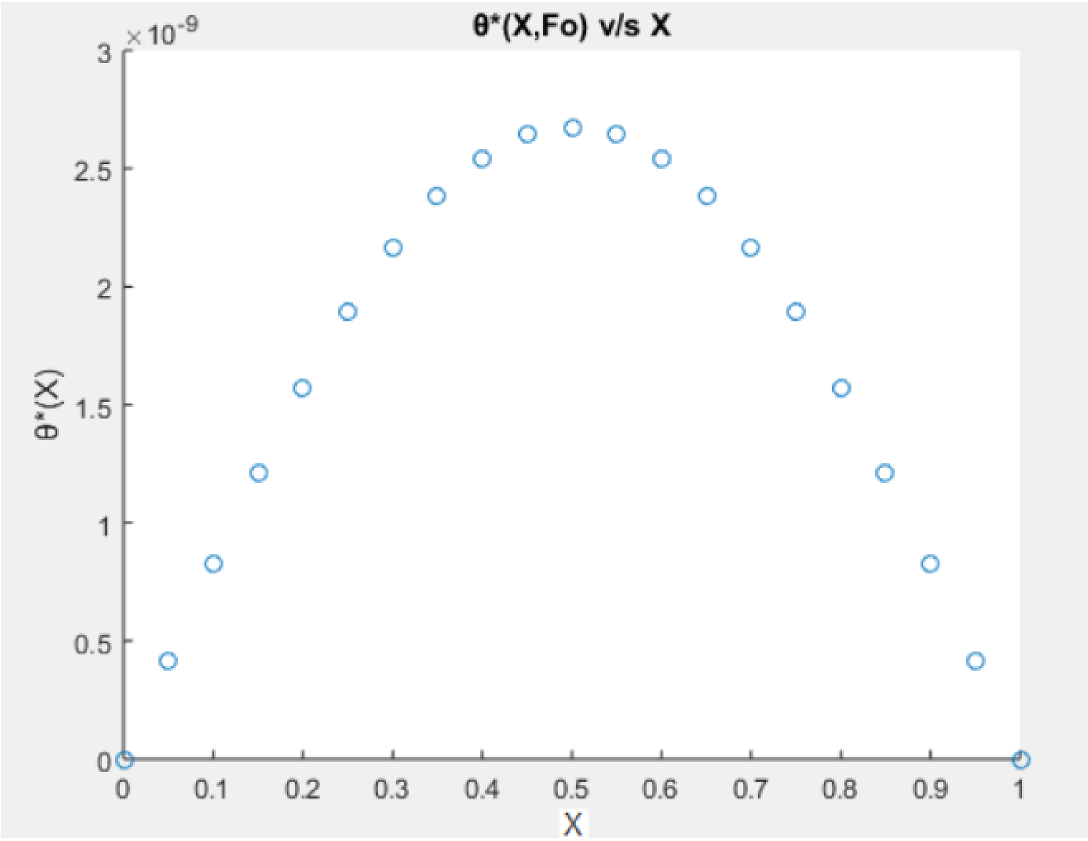

Figure 3 q̇(x) v/s X obtained from Separation of variables method

As we can see the values for $\theta^*(X,Fo)$ obtained from VIM of the same form as that obtained from separation of variables method for this condition.

- **Temperature at extreme ends is constant and equal to T_1 and T_2 ($T_1 > T_2$).**

Now we consider the 2nd Dirichlet boundary condition.

Writing the boundary conditions in dimensionless form,

$\theta_1^* = \theta^*(0,Fo) = 1$

$\theta_2^* = \theta^*(1,Fo) = 0$

In this case, the temperature distribution has two parts i.e steady part and transient part,

$$\theta^*(X,Fo) = \theta^*_{steady}(X,Fo) + \theta^*_{trans}(X,Fo)$$

$\theta^*_{steady}(X,Fo)$ and $\theta^*_{trans}(X,Fo)$ is given as,

$$\theta^*_{steady}(X,Fo) = \theta_1^* + (\theta_2^* - \theta_1^*)X$$

$$\theta^*_{trans}(X,Fo) = \sum b_n . \sin(n\pi X)$$

In VIM analysis we took the initial conditions as

$$\theta^*(X,0) = \theta_1^* - (\theta_1^* - \theta_2^*) X + \sin(m\pi X)$$

$\theta^*_{trans}(X,Fo)$ is evaluated as above from separation of variables method.

So, $\theta^*(X,Fo) = \theta_1^* + (\theta_2^* - \theta_1^*)X + \sum_{n=1}^{\infty} b_n . \sin(n\pi X) . e^{-Fo.p^2}$

Where $B_n = 2\int_0^1 \theta^*(X,0).\sin(n\pi X)dX$

$$\theta^*(X,0) = 1 - X + \sin(m\pi X)$$

$$B_n = \frac{2(((\pi m cos(\pi m)-1)n^2+m^2)sin(\pi n)-\pi sin(\pi m)n^3cos(\pi n)+\pi n^3-\pi m^2 n)}{\pi^2 n^2\cdot(n^2-m^2)}$$

Since sin(nπ) will be zero,

This implies,

$$B_n = \frac{-2sin(\pi m)n^3cos(\pi n)+\pi n^3-\pi m^2 n)}{\pi n^2\cdot(n^2-m^2)}$$

$$\theta^*(X,Fo) = \theta_1^* + (\theta_2^* - \theta_1^*)X + \sum_{n=1}^{\infty} b_n.\ sin(n\pi X).\ e^{-Fo.p^2}$$

$$= \theta_1^* + (\theta_2^* - \theta_1^*)X + \sum_{n=1}^{\infty} \frac{-2sin(\pi m)n^3cos(\pi n)+\pi n^3-\pi m^2 n)}{\pi n^2\cdot(n^2-m^2)}.\ sin(n\pi X).\ e^{-(n\pi)\wedge 2.Fo}$$

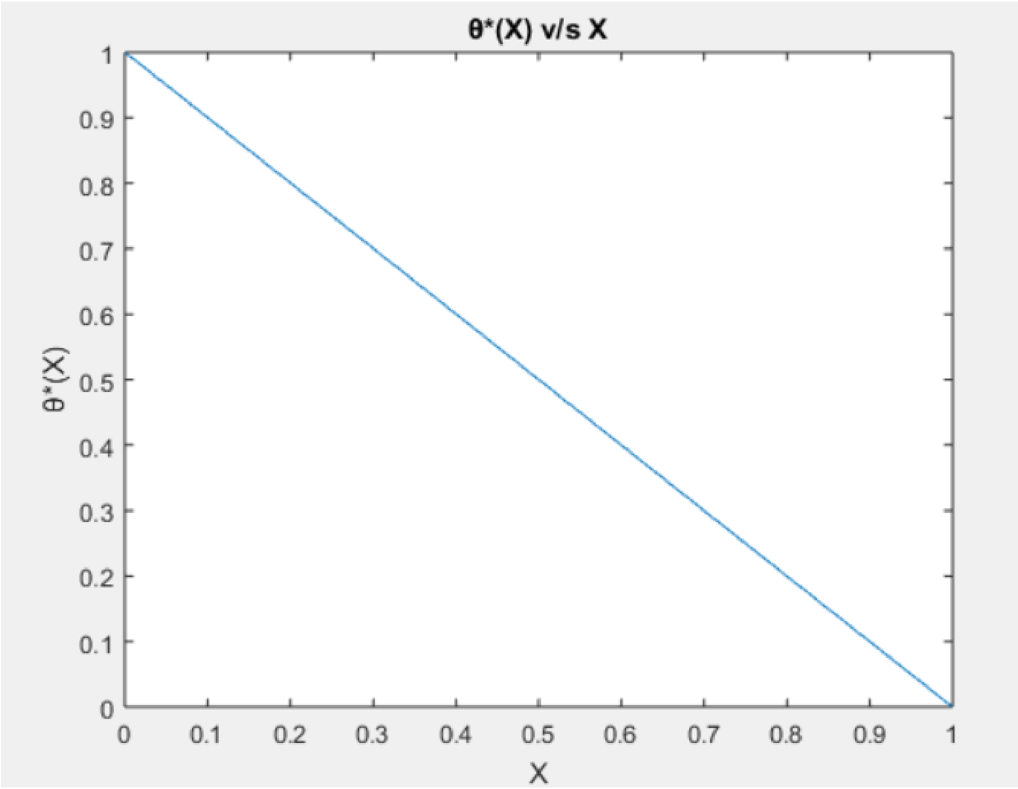

Figure 4 θ*(X) v/s X obtained from separation of variables for m=1.0

The above expression for $\theta^*(X,Fo)$ obtained is of same form as that obtained by Variational iterative method for the given initial and boundary conditions.

8. RESULTS AND DISCUSSION

Consider the one-dimensional conduction equation in dimensionless form

$$\left(\frac{\partial^2\theta^*(X,Fo)}{\partial X^2}\right)-\frac{\partial\theta^*(X,Fo)}{\partial Fo}=0 \qquad \text{for } 0 \le X \le 1,\ Fo > 0$$

From the Variational Iterative method we see that the value of $\theta^*(X,Fo)$ comes as follows for the condition

$$\theta^*(X,Fo) = sin(m\pi X)*e^{-(m\pi)^2.Fo}$$

The temperature distribution V/S time is shown at a section in the cylinder wall at X =0.3 (random value) between $0 \le X \le 1$. Value of m taken is 1.0.

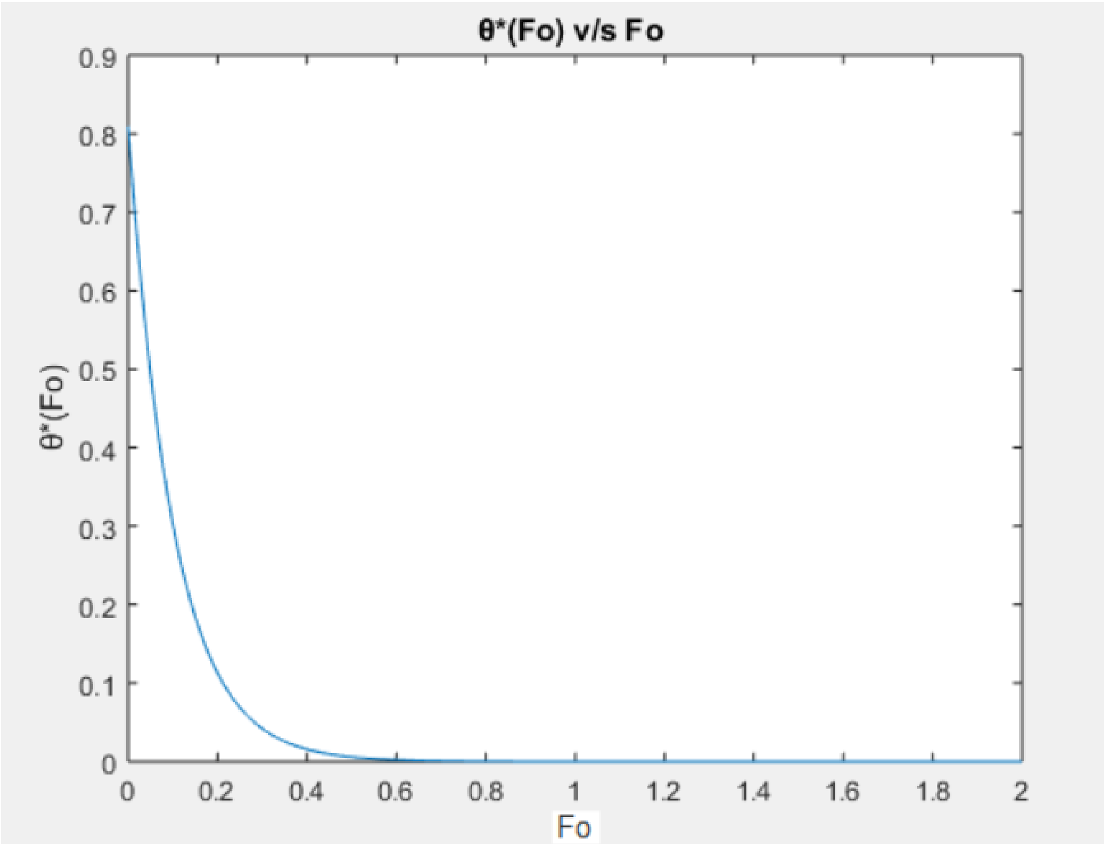

Figure 5 **$\theta^*(F_o)$ v/s F_o for cylinder wall at a section of cylinder wall(X=0.3)**

We can see that at for a particular point on the cylinder wall the temperature decreases exponentially with time passing for a constant value of thermal diffusivity and other thermophysical properties. Also, the variable ‘m’ is considered as constant.

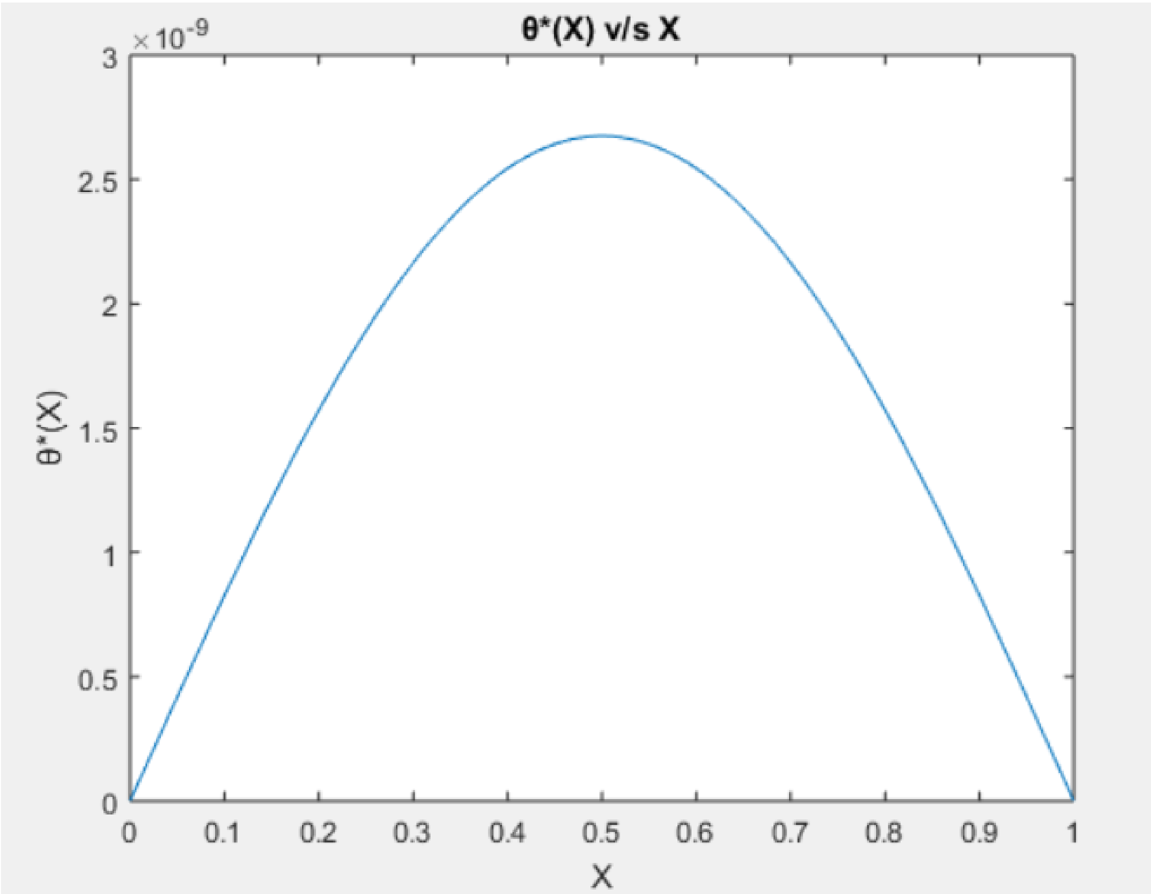

Figure 6 **θ*(X) v/s X at Fo = 2.0**

With constant m =1.0 we can see that the nature of temperature distribution w.r.t X i.e. distance from one extreme end is sinusoidal in nature.

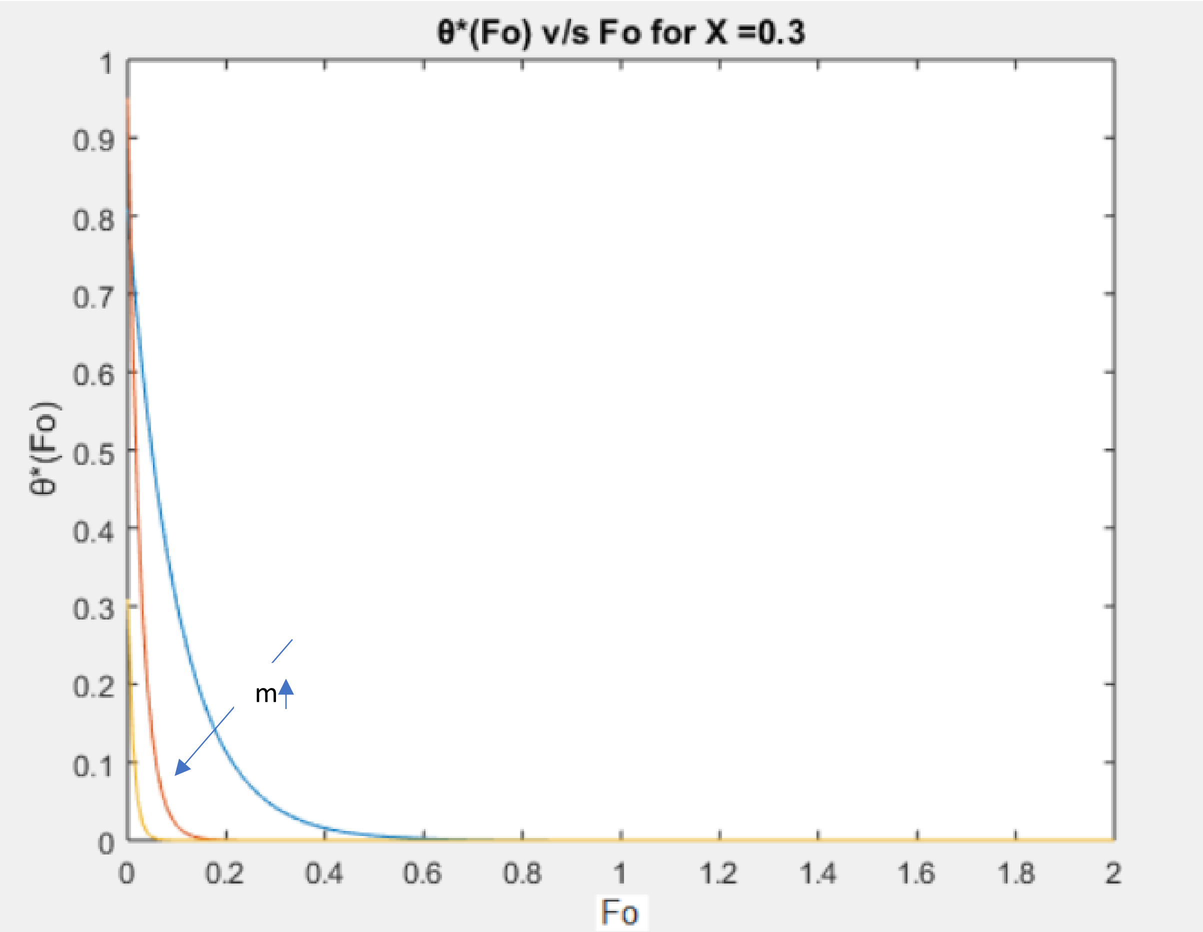

Figure 7 $\theta^*(F_o)$ v/s F_o for X =0.3

In fig.5 we can see the variation in temperature distribution w.r.t change in the value of ‘m’. As value of ‘m’ increases the rate of decrease of temperature decreases.

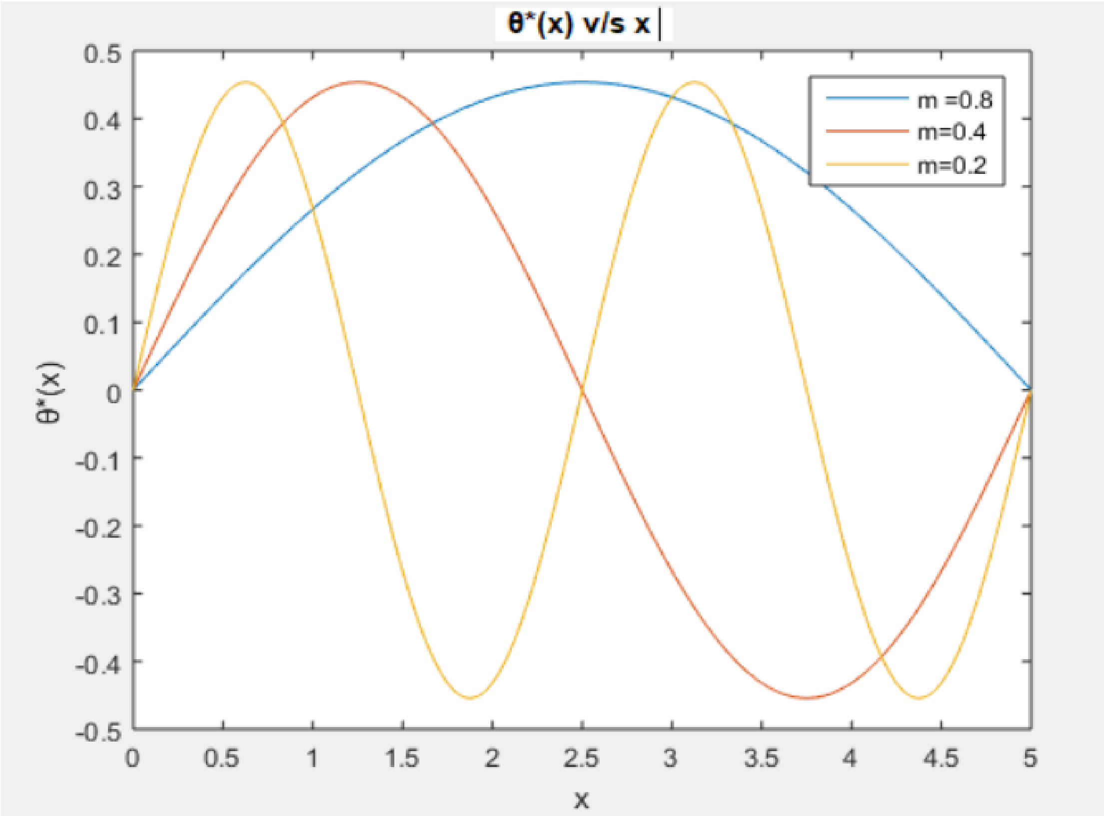

Figure 8 θ*(x) v/s x at Fo = 2 showing effect of m on θ* variation

Here we can see the variation of temperature as value of constant 'm' varies. The sinusoidal curve achieve more steadiness with X as 'm' value increases

Now, to find the heat flux ‘q̇’ we use the fourier’s law of heat conduction,

$$\dot{q} = -K.\frac{\partial T}{\partial x} = \frac{-K.T_o}{L}.\frac{\partial \theta^*(X,Fo)}{\partial X}$$

We have $\theta^*(X,Fo) = \sin(m\pi X) * e^{-(m\pi)^2.Fo}$

So, $\frac{\partial \theta^*(X,Fo)}{\partial X} = m\pi.\cos(m\pi X).\, e^{-(m\pi)^2.Fo}$

$$\dot{q}(X,Fo) = \frac{-K.T_o}{L}.m\pi.\cos(m\pi X).\, e^{-(m\pi)^2.Fo}$$

Now as we can see that q̇ is a function of X and Fo.

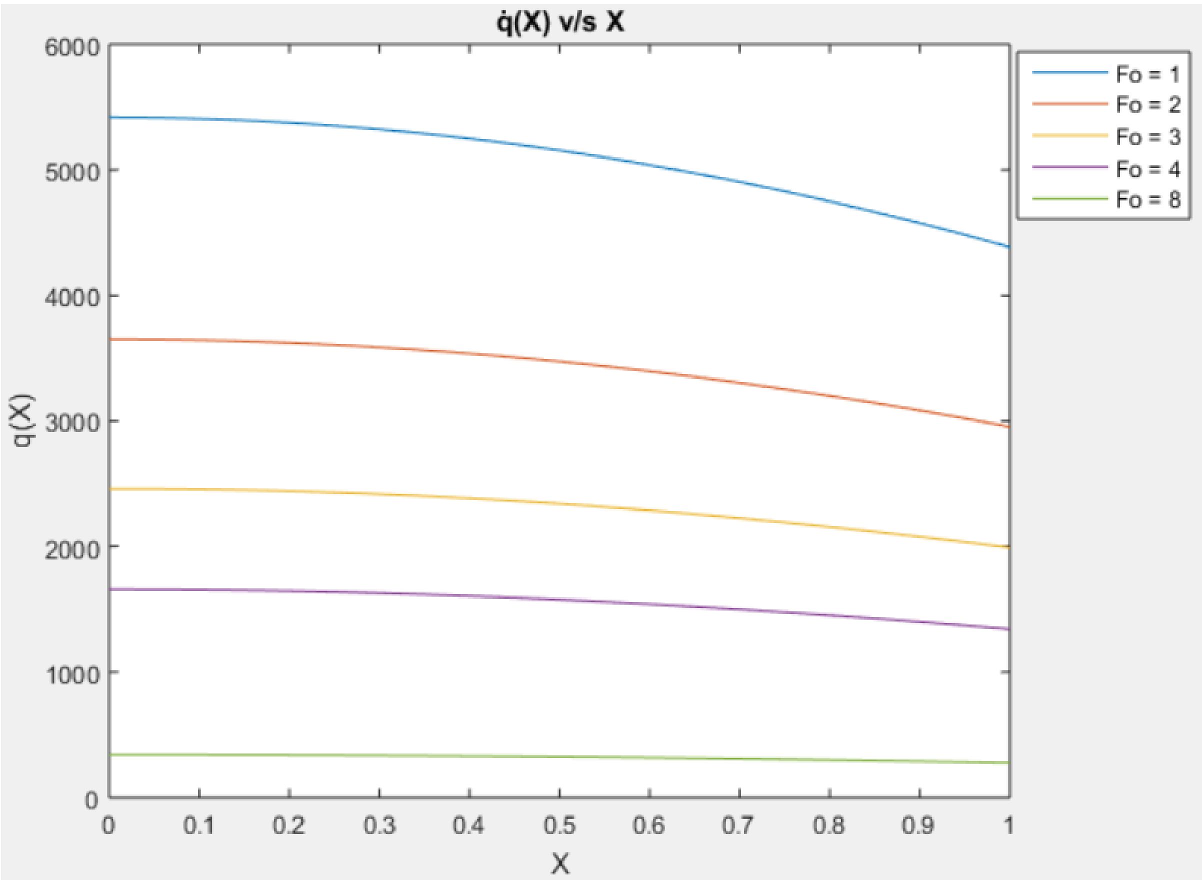

Figure 9 Heat flux v/s X for K =40W/mK, L =1m and T_o = 300K

As value of Fo increases the q̇(X) decreases and steady. Now to calculate total heat transfer from the cylinder wall,

$$q = \int_0^{Fo} \int_0^1 \frac{-K.T_o}{L}.m\pi.cos(m\pi X).e^{-(m\pi)^2.Fo}\, dXdFo$$

$$= \frac{-K.T_o}{L}.m\pi.\int_0^{Fo} e^{-(m\pi)^2.Fo}.\int_0^1 \cos(m\pi X)\, dXdFo$$

$$= \frac{KT_o}{L.m^2.\pi^2}.\sin(m\pi).(e^{-(m\pi)^2 Fo} - 1)$$

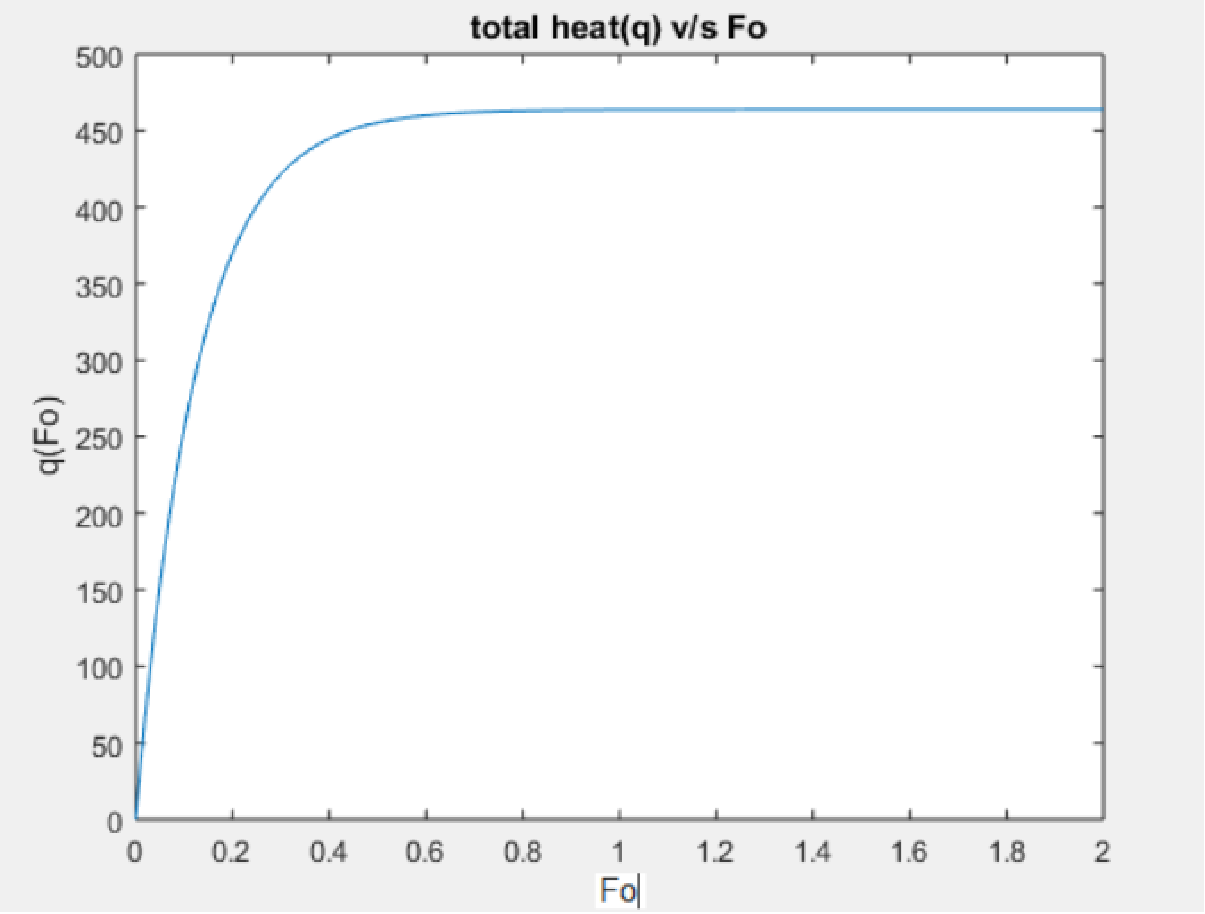

Figure 10 Total heat transfer(q) v/s Fo for Fo > 0 and K =40W/mK, L =1m and T_o = 300K

- **For temperature at ends equal to T_1 and T_2 ($T_1>T_2$)**

For 2nd Dirichlet boundary condition,

The cylinder wall is exposed to two different temperatures T_1 and T_2 we can obtain the value of temperature distribution by using Variational iterative method,

We write the various conditions in dimensionless form, Also, here $T_1>T_2$ which is greater than ambient temperature T_∞

$\theta^* = \frac{T - T_2}{T_1 - T_2}$, X = x/L

$\theta_1^* = \theta^*(0,Fo) = 1$

$\theta_2^* = \theta^*(1,Fo) = 0$

Taking initial conditions as,

⇨ $\theta^*(X,0) = \theta_1^* + (\theta_2^* - \theta_1^*)X + \sin(m\pi X)$

We obtain the temperature distribution as given below,

$$\theta^*(X,Fo) = \theta_1^* + \left(\theta_2^* - \theta_1^*\right) X + \sin(m\pi X) e^{-(m\pi)^2 Fo}$$

By using VIM method, we generated the result for Test value of m taken as equal to 1.0 and at a section of X = 0.3

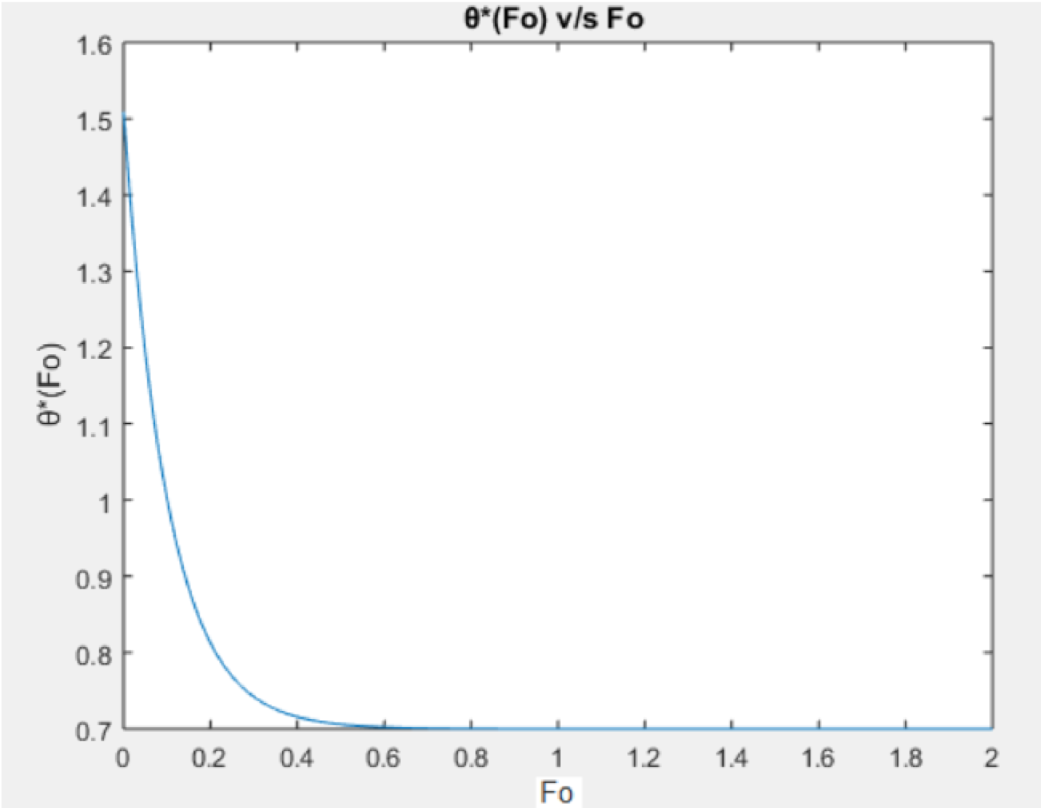

Figure 11 **θ*(Fo) v/s Fo in cylinder wall with time at a section (X=0.3)**

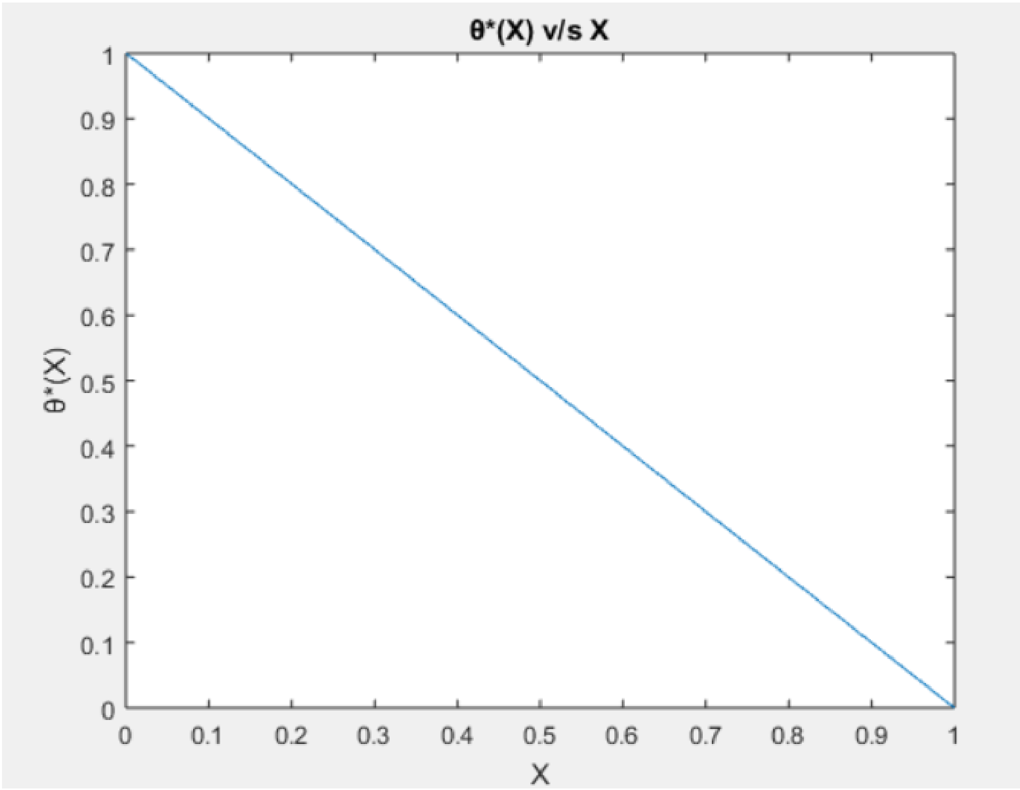

Figure 12 **θ*(X) v/s X at F =2.0**

As we can see the above figure here, the variation of temperature with X is linear in nature.

Now, to find the heat flux '$\dot{q}$' we use the Fourier's law of heat conduction,

$$\dot{q} = -K.\frac{\partial T}{\partial x} = \frac{-K.(T_1 - T_2)}{L} \cdot \frac{\partial \theta^*(X, Fo)}{\partial X}$$

We have $\theta^*(X,F_o) = \theta_1^* + \left(\theta_2^* - \theta_1^*\right) X + \sin(m\pi X) e^{-(m\pi)^2 Fo}$

$$\frac{\partial \theta^*(X, Fo)}{\partial X} = \left(\theta_2^* - \theta_1^*\right) + m\pi.\cos(m\pi X)\ e^{-(m\pi)^2 .Fo}$$

$$\dot{q} = \frac{-K*(T_1 - T_2)}{L} .\left(\left(\theta_2^* - \theta_1^*\right) + m\pi.\cos(m\pi X)\ e^{-(m\pi)^2 .Fo}\right)$$

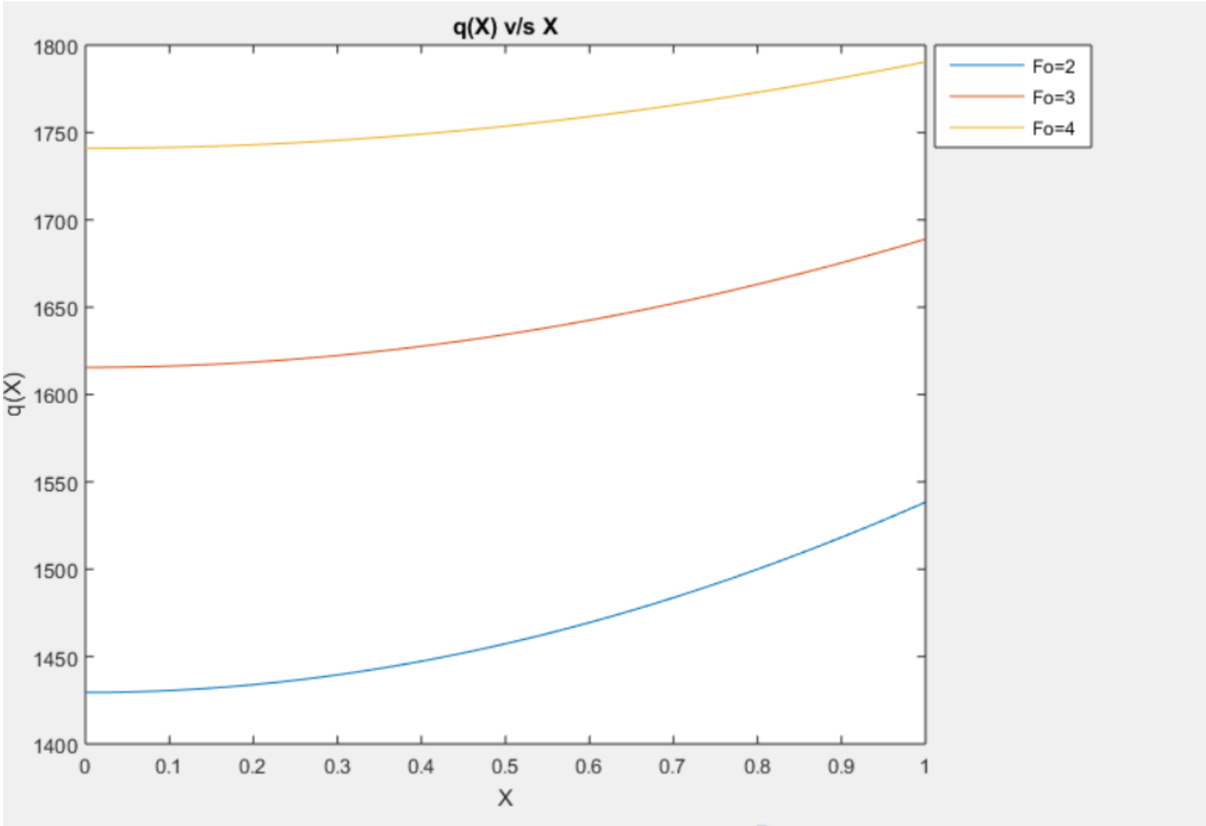

Figure 13 $\dot{q}$(X) v/s X

As we can see from the above graph that as Fo increases the heat flux increases and achieves steadiness with increase in time.

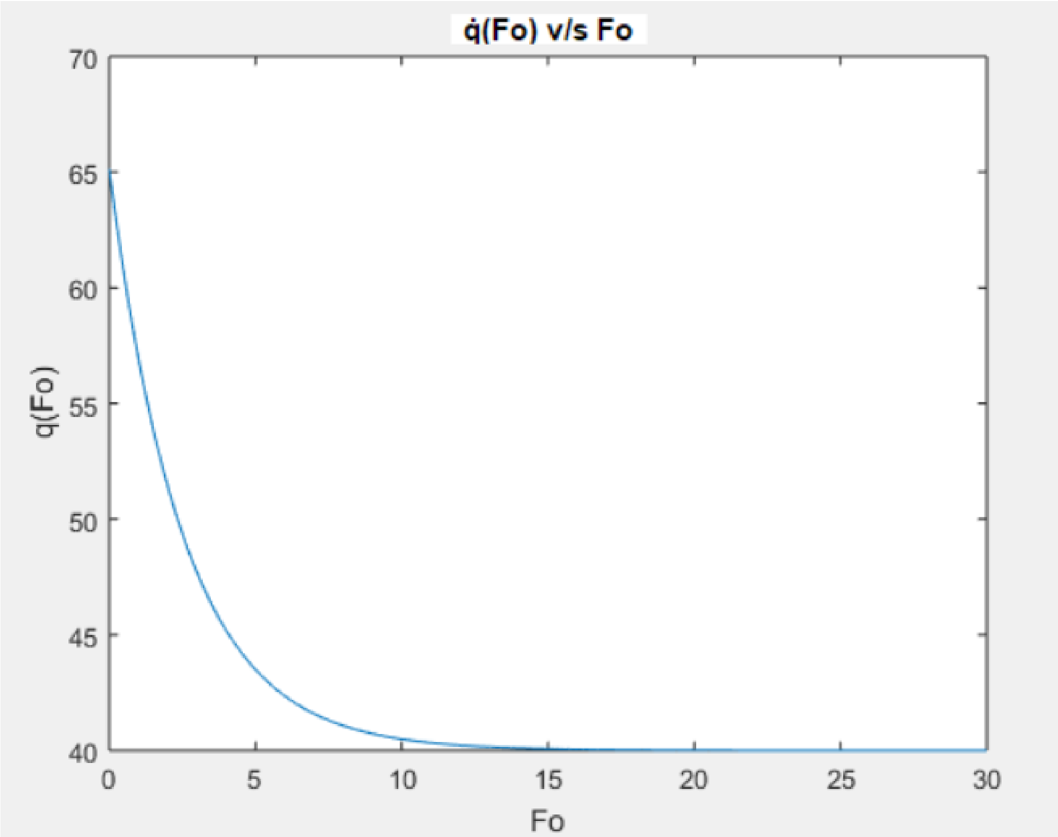

Figure 14 **Heat flux variation versus Fo(q̇(Fo) v/s Fo at X =0.3**

Now to calculate total heat transfer from the cylinder wall,

$$q = \int_0^{Fo} \int_0^1 \frac{-K*(T_1 - T_2)}{L} . \left(\left(\theta_2^* - \theta_1^*\right) + m\pi.\cos(m\pi X)\ e^{-(m\pi)^2.Fo}\right) dXdFo$$

$$= \frac{-K*(T_1 - T_2)}{L} \int_0^{Fo} \int_0^1 \left(\left(\theta_2^* - \theta_1^*\right) + m\pi.\cos(m\pi X)\ e^{-(m\pi)^2.Fo}\right) dXdFo$$

$$= \frac{-K*(T_1 - T_2)}{L} \int_0^{Fo} \int_0^1 \left(\left(\theta_2^* - \theta_1^*\right) + m\pi.\cos(m\pi X)\ e^{-(m\pi)^2.Fo}\right) dXdFo$$

$$= \frac{K*(T_1 - T_2)}{L} \left(Fo + \frac{\sin(m\pi)}{m^2.\pi^2} . (e^{-(m\pi)^2 Fo} - 1) \right)$$

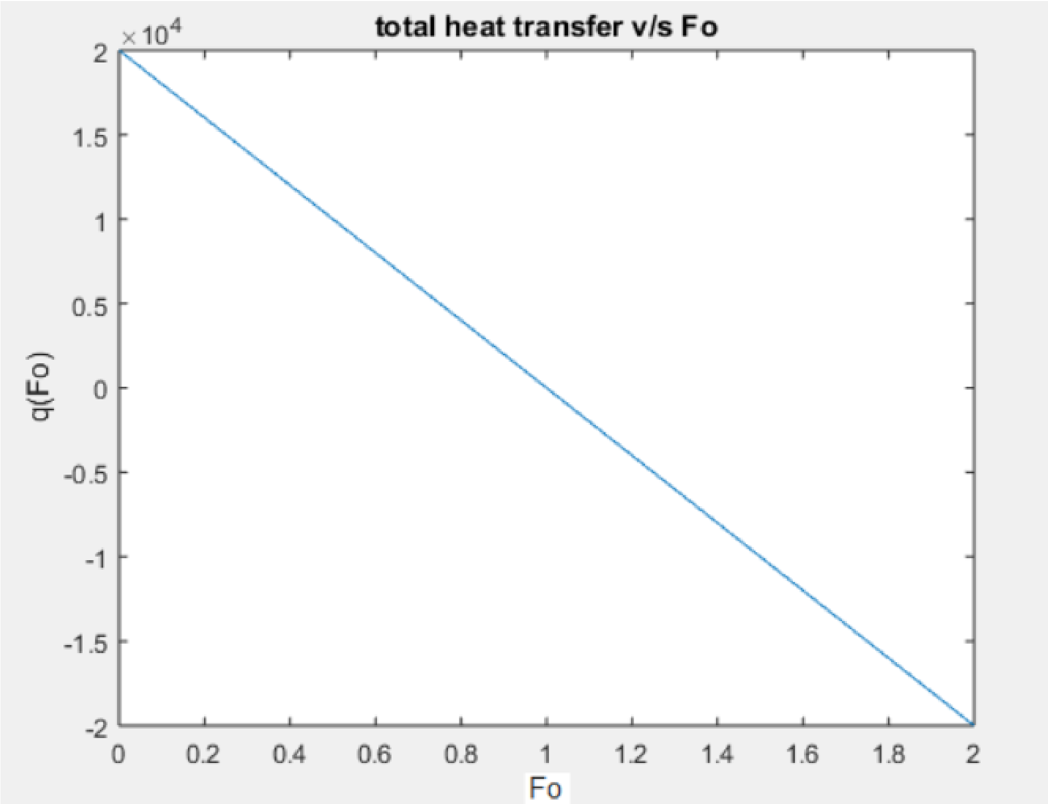

Fig.13 Total heat transfer from the cylinder v/s Fo

In the above graph, T_1 = 350K and T_2 = 300K and m = 1 and K =40 W/mK

Due to the nature of temperature distribution the Total heat transfer graph follows a linear profile of heat transfer.

9. CONCLUSION

Through this thesis work we saw that how variational iteration method is used to do thermal analysis of cylinder wall of the Internal combustion engine subjected to different boundary conditions i.e. Dirichlet and Neumann. We presented the analysis of several cases beginning from the cylinder wall insulated at the extreme ends to the cylinder wall subjected to two different temperatures. Apart from that we also analysed the temperature distribution when the inner side of wall is subjected to uniform heat flux while the other side is subjected to the ambient conditions. All this analysis done for the partial differential equation for the one-dimensional heat conduction equation, the variables are made dimensionless so as to eliminate any unnecessary coefficients that can affect the exact solution. For the temperature distribution we validated the results with separation of variables method. Also, we came to know that how separation of variables method has some shortcomings when the partial differential equation becomes inhomogeneous, as the method becomes quite complex to get the exact solution.

- For variational iteration method we can say that it is a reliable and accurate method as compared to existing techniques. It gives rapid convergent successive approximations without any restrictions that may cause any change in physical behaviour of problem.
- This method can give successive correct approximations because of using correction functional. Also, for non-linear problems if the solution exists, through rapid convergence of approximations it becomes easy to carry out the computational work. Apart from that we see that there are some drawbacks of this method also which are rectified by HVIM and MVIM method.
- Sometimes choosing the initial approximation becomes difficult as one cannot guess the initial approximation by just observing the differential equation of the given physical problem. So, to overcome that difficulty, in

some cases one needs to solve the equation by other methods such as separation of variables method to get he exact solution in order to have an idea about the initial approximation.

- The other major problem with VIM method for thermal analysis of cylinder wall is that one cannot take initial condition as a polynomial equation because the iterations do not generate any new value and keeps on generating the same value till n^{th} iteration. Also, the solution that is generated do not adhere to the physical problem. This problem was later solved by introduction of MVIM method [23][24].

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