B PHARMACY FIRST YEAR FIRST SEMESTER EXAMINATION 2019 MATHEMATICS- IM

TIME: THREE HOURS

FULL MARKS: 100

USE SEPARATE ANSWERSCRIPTS FOR SEPARATE PARTS.

Part-I

Attempt any five questions.

1. i) Without expanding the determinant prove that $\begin{vmatrix} 0 & -a & -b \\ a & 0 & c \\ b & -c & 0 \end{vmatrix} = 0.$

ii) If
$$A = \begin{pmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{pmatrix}$$
, show that $A^2 - 10A + 16I_3 = 0$. Hence obtain A^{-1} .

4+6

2. Obtain the row reduced echelon matrix which is row equivalent to the matrix

$$\begin{bmatrix} 0 & -3 & -6 & 4 & 9 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & -1 \\ 1 & 4 & 5 & -9 & -7 \end{bmatrix}$$
. Find the rank of the given matrix. 5+5

3. Solve the following differential equations.

4+6

i)
$$(D^4 - 2D^3 + 5D^2 - 8D + 4)y = \cosh x + \sin x$$
.

ii)
$$\frac{dy}{dx} + x\sin 2y = x^3 \cos^2 y.$$

4. State the Rolle's theorem and illustrate the geometrical interpretation. Using Mean Value Theorem prove that $\sqrt{101}$ lies between 10 and 10.05.

6+4

5. State Leibnitz's theorem. If
$$y = \cos(m\sin^{-1}x)$$
, then prove that $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$.

6. i) Evaluate $\lim_{x\to 0} \frac{\sin x - x + \frac{x^3}{6}}{x^5}$.

ii) Find the maximum value of
$$\left(\frac{1}{x^2}\right)^{2x^2}$$
, x>0.

7. .i) Evaluate
$$\int_0^\infty x^{\frac{3}{2}} (1+2x)^{-5} dx$$
.

ii)
$$f(x,y) = \cos\left(\frac{4}{x}\right)e^{x^2y-5y^3}$$
. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

Part-2 Attempt question no.8 and any THREE from the rest

- 8. i) State Spearman's formula for rank co-relation co-efficient.
- ii) Write down the probability density function (p.d.f) for chi-square(χ^2) variate and also its mean & standard deviation.
 - iii) What is the density function of a standard normal variate? 2+2+1
- 9. Draw Histogram, Frequency Polygon & less than Ogive for the following data.

Wages	50-59	60-69	70-79	80-89	90-99	100-109	110-119
No. Of Employees	8	10	16	14	10	5	2
Limpioyees							

- 10. i) Define Mean, Median & Mode in a set of observations. What is the approximate relation between Mean, Median & Mode.
- ii) Prove that if $x_1 \& x_2$ are two positive values of a variate, their geometrical mean (G.M) is equal to the geometrical mean(G.M) of their arithmetic mean(A.M) & harmonic mean(H.M).
 - iii) State Baye's Theorem for conditional probabilities.
- iv) Find the mean and variance of Poisson distribution.

4+4+2+5

- 11. i) Two boxes contain respectively 4 white, 2 black balls & 1 white, 3 black balls. One ball is transferred from first box into second, and then one ball is drawn from the later. It turns out to be black. What is the conditional probability that the transferred ball was white?
- ii) The mean weight of 500 male students at a certain college is 151 lbs. & standard deviation 15 lbs. Assuming the weights are normally distributed, find how many students weigh
 - between 120 & 155 lbs.
 - More than 155 lbs.

[Given that $\varphi(0.27) = 0.6064$, $\varphi(2.07) = 0.9808$, $\varphi(t)$ is the area under standard normal curve to the left of the ordinate at t].

- iii) From an urn containing N_1 white & N_2 black balls, balls are successively drawn without replacement. What is the probability that 'i' black balls will precede the 1st white ball?

 5+6+4
- 12. i) State the classical definition of probability and write down two defects of it.

ii) Given
$$f(x) = \begin{cases} \frac{1}{4}, -2 \le x \le 2\\ 0 \text{ elsewhere} \end{cases}$$

Check if f(x) is a probability density function; Also find $P\{(2x+3)>5\}$.

iii) Find the modal wage from the following:

Wages	50.00- 59.99	60.00-69.99	70.00- 79.99	80.00- 89.99	90.00- 99.99	100.00- 109.99	110.00- 119.99
No. of Employees	8	10	16	14	10	5	2