Dissertation on

SOC OPTIMIZATION OF A LEAD ACID BATTERY USING BACK PROPAGATION NEURAL NETWORK

A thesis submitted towards partial fulfilment of the requirements for the degree of

<Master of Technology in IT (Courseware Engineering)>

Submitted by

SAIF WASIM

EXAMINATION ROLL NO.: M4CWE22004
UNIVERSITY REGISTRATION NO.: 154486

Under the guidance of

Mr. Joydeep Mukherjee

School of Education Technology
Jadavpur University
Course affiliated to
Faculty of Engineering and Technology
Jadavpur University
Kolkata-700032
India

2022

Course affiliated to
Faculty of Engineering and Technology
Jadavpur University
Kolkata, India

–

CERTIFICATE OF RECOMMENDATION

This is to certify that the thesis entitled **"SOC OPTIMIZATION OF A LEAD ACID BATTERY USING BACK PROPAGATION NEURAL NETWORK"** is a bonafide work carried out by SAIF WASIM under our supervision and guidance for partial fulfillment of the requirements for the degree of Master of Technology in IT (Courseware Engineering) in School of Education Technology , during the academic session 2021-2022.

Mr. Joydeep Mukherjee
SUPERVISOR
Jadavpur University,
Kolkata-700 032

DIRECTOR
School of Education Technology
Jadavpur University,
Kolkata-700 032

Dean- FISLM
Jadavpur University
Kolkata-700032

M.Tech. IT (Courseware Engineering)
Course affiliated to
Faculty of Engineering and Technology
Jadavpur University
Kolkata, India

<u>CERTIFICATE OF APPROVAL</u> **

This foregoing thesis is hereby approved as a credible study of an engineering subject carried out and presented in a manner satisfactory to warranty its acceptance as a prerequisite to the degree for which it has been submitted. It is understood that by this approval the undersigned do not endorse or approve any statement made or opinion expressed or conclusion drawn therein but approve the thesis only for purpose for which it has been submitted.

Committee of final examination for evaluation of Thesis ---

** Only in case the thesis is approved.

DECLARATION OF ORIGINALITY AND COMPLIANCE OF ACADEMIC ETHICS

I hereby declare that this thesis contains literature survey and original research work by the undersigned candidate, as part of his **Master of Technology in IT (Courseware Engineering)** studies.

All information in this document has been obtained and presented in accordance with academic rules and ethical conduct.

I also declare that, as required by this rule and conduct, I have fully cited and referenced all materials and results that are not original to this work.

NAME: Saif Wasim

EXAMINATION ROLL NUMBER: MACWE22004

THESIS TITLE: SOC OPTIMIZATION OF A LEAD ACID BATTERY USINF BP NEURAL NETWORK

SIGNATURE: DATE:

ACKNOWLEDGEMENT

I feel extremely glad in presenting this thesis at School of Education Technology, Jadavpur University, Kolkata-32, in partial fulfilment of the requirements for the degree of Master in IT(Courseware Engineering).

I would like to express my sincere gratitude to my guide, Mr. Joydeep Mukherjee, for his continuous guidance and constant encouragement. He spent precious time for the betterment of my thesis work and suggested many constructive criticism and thought. Without his support and advice, this thesis work would never had been possible.

I would like to express my warm thanks to Prof. Matangini Chattopdhyay, Director of School of educational Technology for her timely encouragement, support and advice. I would also like to thank Dr. Saswati Mukherjee for her constant support during my entire course of work.

I would also like to thank all of my classmates of IT and MMD department who encouraged me to complete all my thesis work successfully. I do wish to thank all of my st and all those who were associated with this thesis contributed in some form or the other.

Finally, I would like to acknowledge and thank my parents for their strong unconditional support, inspiration and encouragement that they have provided me, without which I would be unable to complete this work.

Date:

Saif Wasim
Class Roll No.:002030402004
Exam Roll No.: M4CWE22004
Registration No.: 154486
M. Tech IT(Courseware Engineering):
School of Education Technology,
Jadavpur University,
Kolkata-700032

Contents

1.0 EXECUTIVE SUMMARY

Battery is an important component in our daily lives. Without batteries, all the components that help us to carry day-to-day activities will not work. So, it is important that good quality battery with good charging capabilities is available . Gathering the information about Battery Charging Capacity, State of charge (SOC), is highly important for a battery. SOC is the foremost important parameter to calculate as it tells a lot of valuable information about a battery, like charging capacity, battery charge remaining capacity, etc. For carrying out these functions, a lot of techniques has been used like, Coulomb Counting, Kalman Filter, Hybrid method and there are a lot of methods to carry SOC. But, out of all of these methods, only BP or Back Propagation Neural Network stand out as the winner in this situation because the efficiency of the system because it is almost errorless. The technique involves discharging a Lead- Acid Battery at four different discharge rates and then calculating the terminal voltages and then determining the SOC by Coulomb Counting Method. After that a BP Neural Network is created using MATLAB for further analysis. The input of the function is taken as discharge current and terminal voltage and a practical SOC is taken as an output for further analysing. Then the output with the practical SOC for comparing the results is compared.

2.0 INTRODUCTION

Human lives has been dependent on energy for our day-to-day purposes. Without energy, electricity which is the main component to improve our lives is not avaialble. Electricity is needed for power supplying our cell phones, illuminate our houses, IT industry is dependent on electricity, and much more.

Electricity has been produced largely from fossil fuels like coal petroleum, etc. During the industrial revolution, particularly during 1800s-1900s, a lot of fossil fuels were used which led to enormous air pollution. Researchers have seen a great effect on the environment especially the ozone layer which protects us from harmful radiations.

Due to this, renewable, clean energy like solar, wind, water energy etc is used. Using these renewable energy, battery is used extensively. Charge can be stored for using it in later purposes, because of that, we need to find which battery is best for using . A number of testes are conducted to see which battery is good for use.

Here, testing the SOC matters as it not only gives the battery storage capacity, but also reflects the battery performances. Therefore, SOC is a major matter of concern as it protects the batter from over-charging, and prevent the battery from serious damage. Batteries are of two types:-

1.) Rechargeable
2.) Non-Rechargeable

Rechargeable batteries can be used over and over again by recharging it , whereas the non-rechargeable batteries can not be used after a single use.

The most common rechargeable batteries used worldwide is Lead acid battery because of it's advantages over others. There are many ways to predict the SOC , but here BP Neural Network is used as it has higher accuracy and minimum error.

2.1 OBJECTIVE OF LEAD ACID BATTERY SOC

The main objective of this study is to find the best model which measures the SOC and the battery remaining capacity of a lead acid battery accurately. Also it is analysed the simulated result and show a comparison between practical SOC estimating method and neural network SOC estimating method.

3.0 LITERATURE SURVEY

3.1 BATTERY SOC

Battery performance and SOC are directly linked to each other. Accurate estimation of SOC not only reduces the cost of battery replacement, but also ensures the utilization of battery without harming it. The SOC is required to provide the driver with a precise indication of the remaining range. At present, different types of estimation algorithms are available, but they still have several challenges due to their performance degradation, complex electrochemical reactions, and inaccuracy.

From the research paper [1], it has been found that the SOC estimation of a lead acid battery using backpropagation neural network method shows accurate result with a little error.

A battery is a chemical energy storage source, and this chemical energy cannot be directly accessed. This issue makes the estimation of the SOC of a battery difficult. At present commonly used battery SOC estimation method is based on the internal complex physical and chemical reaction of the established mathematical model, mainly including the open circuit voltage method, discharge test method, impedance method, coulomb counting method, Kalman filtering method, neural network method, etc. The applications of specific SOC estimation methods in battery management system are consequentially different.

There are different methods to estimate the battery SOC. The different methods to estimate the battery SOC, are described in the paper entitled, "The State Of Charge Estimation Methods For Battery: A Review" [4]They are:-

1.) Direct measurements, such as open circuit voltage method, terminal voltage method, impedance spectroscopy method, using physical properties to estimate the battery SOC i.e. voltage, impedance.

2.) Book keeping estimation methods such as coulomb counting method. In this method, it calculates the SOC by integrating the discharge current over time.

3.) Adaptive systems such as BP Neural system. RBF neural network, Fuzzy neural network, Kalman Filter, which are self designing and able to adjust the SOC for different condition automatically.

An estimation method based on improved BP neural network, a combination of genetic algorithm and BP neural network is pointed out in ref. [3]. In this method experiment data has been trained with improved BP neural network and compare this trained data with real values and simulate in Matlab in order to verify the correctness of the algorithm. The ref. [2] described a model based on LM-BP neural network to predict the lead acid battery's remaining capacity, where terminal voltage and discharge current are used as the input and then compare the simulated result with support vector machine (SVM) prediction method.

3.2 PROBLEM STATEMENT

All the methods of measuring SOC of batteries (like Kalman Filter Method, Coulomb Counting Method, Direct Method), give somewhat error in calculating battery SOC.
To overcome this error, we proposed BP Neural Network because it stands errorless and gives maximum accuracy.

3.3 BATTERY

Batteries are of two basic types:

1.) Primary batteries that are discharged once and then discarded and
2.) "Secondary" batteries that are recharged many times.

Here, we are concerned only with the latter type. Another useful distinction is between "consumer" batteries as purchased by individuals, and "industrial" batteries as used in industry and commerce. Most consumer rechargeable batteries are small, single-cell devices (with the notable exception of the automotive battery), whereas industrial batteries tend to be much larger, multi-cell modules.
Battery is a device that converts chemical energy to electrical energy by means of a reaction called redox reaction also known as oxidation-reduction reaction. For rechargeable batteries, the chemical process is reversed. It involves transfer of electrons from one electrode to another. A battery is a group of cells, so the best term to use is "cell".

A cell is an electrochemical device which converts chemical energy to electrical energy. It consists of two electrodes and an electrolyte which acts as a medium for chemical reaction. Whereas, a battery consists of more than one cell.[5]

3.4 COMPONENTS OF A CELL AND BATTERY

A cell has three major component: -

1.) Anode- This is the negative electrode which give up electrons to the external circuit and gets oxidized during the electrochemical reaction.
2.) Cathode- This is the positive electrode which accepts electrons and gets reduced.
3.) Electrolyte- This may be a liquid, semi-liquid or solid paste which acts as the medium for the flow of electrons. The electrolytes are nothing but water with dissolved salts, alkalis to carry out ionic activities.

    Some batteries have solid electrolyte too. This solid electrolyte has an added advantage i.e., the battery can be taken anywhere.

In a physical system, anode is selected on the basis of the below parameters: -

*Efficiency as a reducing agent
*High coulombic output (Ah/ g)
*Good conductivity
*Stability
*Ease of fabrication
*Low cost

A cathode is selected on the basis of:-

*Must be an efficient oxidizing agent
*Be stable when in contact with the electrolyte
*Have a useful working voltage

Electrolyte is selected such that it has a good ionic conductivity but not electrical conductivity as this would cause internal short-circuit.
There are other important characteristics that should be taken in mind i.e., non-reactivity, low cost etc. The anode and cathode physically remain separated or isolated in a cell to prevent internal short-circuiting. They are separated by a separator , which is permeable to the electrolyte in order to maintain the electrical conductivity. There are various shapes and sizes of a cell, like cylindrical , spherical , flat, etc.

3.5 CLASSIFICATION OF CELLS AND BATTERIES

Cells or batteries can be classifies into two groups:-

1.) Primary:- These are the cells that after charging , cannot be used and hence are discarded. Also, they take a long time to charge.
2.) Secondary:-These cells can be used again after recharging back to their original form. This is done by reversing the process i.e., passing current through the opposite direction.

Also a cell can be classified as a dry or wet based on the electrolyte. A dry cell has a paste electrolyte and a wet cell has a liquid electrolyte.[1]

3.6 CELL OPERATION

3.6.1 Discharge:

During discharge period, when the cell is connected to the external load , electrons flow from anode to cathode resulting in oxidation. By the flowing of the anions and cations to the anode and cathode respectively, the circuit is formed. Below the diagram shows the diagram of the flow of anions to cations.

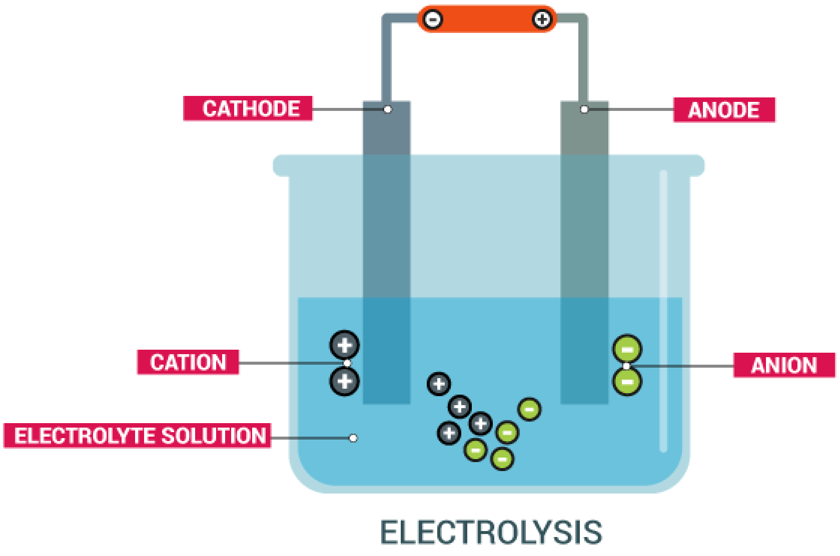


Figure 3.1 Flow of anions and cations during discharge process [18]

3.6.2 Charge:

Recharge of a rechargeable cell or battery is a result of the electrons flowing from cathode to anode i.e., the current flow in reverse direction that of discharge and this results in the battery composition deposit again in its original position.[8]

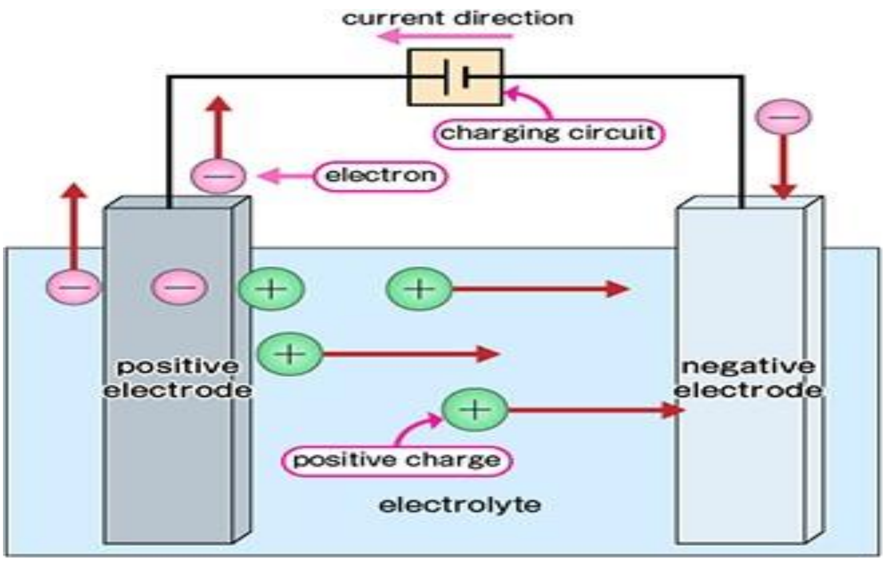


Figure 3.2 Flow of electrons in a rechargeable cell [19]

3.7 Battery Specifications

3.7.1 Basics:

- *Cell*: A cell is a unit of a battery. It is also called module which consists of several cells, connected in either series or parallel or in both combinations as per the requirements. A battery pack is assembled by connecting modules together, again either in series or parallel as per the requirements.
- *Battery categories:* Batteries can be classified into power and energy. Batteries can be high power or high energy, but not both. Some other categories can be high durability, which indicates higher battery life at the expense of power and energy.
- *C rates and E rates:* Batteries can be also classifies into discharge current, which is often expressed as a C-rate in order to normalize against battery capacity. A C-rate is a measure of the discharge rate relative to its maximum capacity. A 1C rate means, the current will discharge the battery in 1 hour. If the same battery is discharged at a lower C-rate, say at C/10 rate, then it might be expected that the battery would run longer (theoretically 10 hours) before fully discharged. Similarly, an E-rate describes the power that discharges the entire battery in 1 hour.
- *Physical shapes and sizes:* Shapes of batteries are mainly cylindrical and sometimes non-cylindrical. There are different sizes of batteries such as pencil(AA), Remote control battery(AAA), button size, etc.

3.7.2 Battery conditions:

These are the most important factors that determine the present condition of a battery:

- *State of charge (%SOC):* This indicates the battery remaining capacity of a battery or a cell .
- *Depth of discharge (%DOD):* The percentage of battery capacity that has been discharged expressed as a percentage of maximum capacity. A discharge to at least 80% DOD is considered as deep discharge which is actually bad for a battery and immediately needs to be charged.
- *Open circuit voltage:* It is the battery terminal voltage at no load condition. It depends on the battery state of charge and it increases with SOC.

3.7.3 Battery technical specifications:

These types of specifications are mentioned on battery technical specification sheets.

- *Nominal Voltage*: The reference voltage of the battery, also says as ‘normal voltage’. It measures in Volt.[8]
- *Cut-off Voltage*: The maximum allowable voltage which defines the ‘empty’ state of the battery.
- *Nominal Capacity*: Battery capacity is the total ampere-hours available when the battery is discharged at a certain discharge current. It is calculated by multiplying the discharge current (in Ampere) with discharge time (in Hour). The unit of capacity is ampere-hour (Ah).
- *Nominal Energy*: Like nominal capacity, it is also calculated by multiplying the discharge power (in Watt) with discharge time (in Hour). Therefore, the unit of nominal energy is watt-hour (Wh).
- *Cycle Life:* It represents the number of discharge-charge cycles, before failing the battery to meet specific performance criteria. The cycle life is low if the DOD is high.
- *Specific Energy:* Specific energy, the nominal energy per unit mass, is a characteristic of the battery. It determines the battery weight and measures in Wh/kg.
- *Maximum continuous discharge current:* It is the maximum current at which the battery can be discharged continuously without damaging the battery.
- *Charge Voltage:* It is the voltage which is applied to a depleted battery during charging process.
- *Float Voltage*: When the battery is fully in charged condition, the float voltage is that voltage, which is applied to the battery for compensating the self discharge of the battery.
- *Internal Resistance*: The resistance within the battery which is normally small, generally different for charging and discharging.[9]

3.8 Lead Acid Battery

The Lead acid battery is one of the most important and the widely used battery. It is used in the industries, electric vehicles, used as energy storage devices etc. It is a completely rechargeable battery and can be used many times.[12]

By far, the most common type of storage battery is the ubiquitous lead-acid battery, which was invented by Planté in 1859 and greatly improved by Faure in 1881. Over 90% of the world market for medium-to-large rechargeable batteries is still met by lead-acid; most of the rest is satisfied by nickel-cadmium. For small-sized batteries, traditional nickel-cadmium cells are being rapidly replaced by nickel metal-hydride and lithium-ion cells on account of their superior performance. Since its invention, the lead-acid battery has undergone many developments, most of which have involved modifications to the materials or design, rather than to the underlying chemistry. The electrode reactions of the cell are unusual in that the electrolyte (sulfuric acid) is also one of the reactants.[2]

Types of Lead -Acid Batteries:-

1.) Traditional Flat – Plate Batteries
2.) Low- Maintenance Flat Batteries
3.) Tubular-Plate Batteries
4.) Valve-Regulated Batteries[6]

The lead-acid battery finds wide use in a variety of applications, ranging from automotive starting/lightning/ ignition (SLI) and industrial batteries for motion power, to large stationary batteries for standby-power, including small sealed cells for portable application. The unique combination of low cost, chemical and physical stability, and rechargeability are unmatched by other rechargeable systems. Its specific energy is 35-50 Wh kg-1 and peak power is 150-400 W kg-1.[17]

3.8.1 Advantages:-

1.)Popular low cost secondary battery.

2.)Offers low internal impedance and can deliver very high current. It is available in wide range of sizes and capacity and in worldwide.

3.)Has indefinite self life if the battery stored without electrolyte and available in maintenance free designs.

4.)SOC estimation of a lead acid battery is quite easy.[7]

3.8.2 Disadvantages:-

1.)Very heavy and bulky.

2.)It has relatively low cycle life (300 to 500 cycles) and limited energy density (typically 30-40 Wh/kg).

3.) Not suitable for fast charging and has danger from overheating during charging. It must be stored in a charge condition, once the electrolyte has been introduced. It suffers from sulphation, partial short and buckling.

3.8.3 Structure:

Acid battery is a normally a combination of lead acid cells. A simple lead acid cell consists of container, plate, separator, post terminal and electrolyte which shown in below figure.

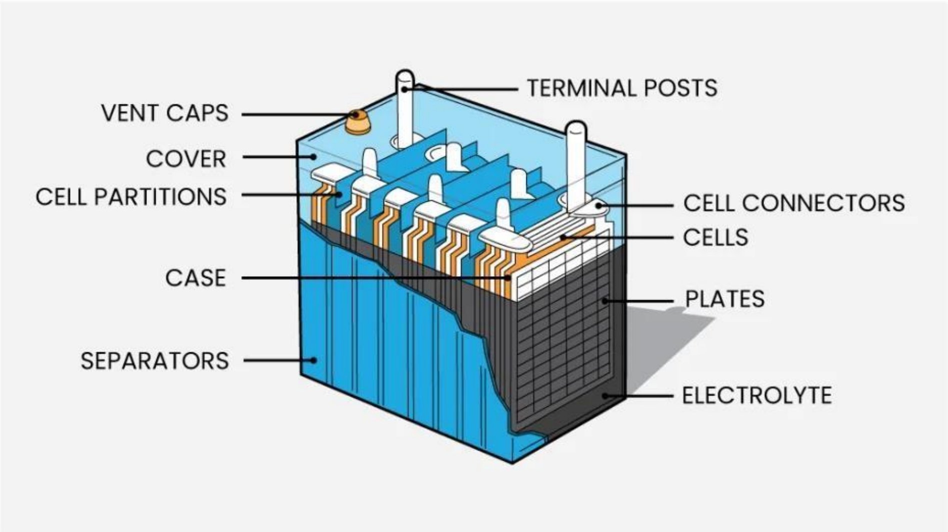


Figure 3.3 Structure of a lead acid battery [20]

- *Container:* - The container is made of hard rubber, glass or celluloid, so as to accommodate the active plate's separators and the electrolytes. The plates rest on ribs provided at the bottom of the container and the space between ribs is known as Sediment Chamber.

- *Plates:* There are two types of plate present in a lead acid cell, one is positive plate and another is negative plate.

  i) Positive Plate: Positive plates are of two types-

  Plante plate or formed plates are prepared by the process of repeated charging and discharging. They are made of pure lead at the beginning which changes to lead per oxide (PbO_2) after charge.

  Pasted or faure plates are made of rectangular lead grid into which the active material i.e. lead per oxide (PbO_2) is filled in the form of a paste.

  ii) Negative Plate: Negative plates are made of rectangular lead grid, and the active material is spongy lead (Pb) which is in the form of a paste.

- *Separators:* These are made of thin sheets of chemically treated porous wood or rubber. They are used to avoid short circuit between the positive and negative plate.
- *Post terminal:* It is a small pole extended upward from each group of welded plates from the plate connector which forms the post terminal.
- *Electrolyte:* The electrolyte is a liquid or semi-solid paste medium which is used in a lead acid cell is dilute sulphuric acid (H_2SO_4). The specific gravity of electrolyte is 1.24 to 1.28. It varies according to manufacturer's specifications.

3.8.4 Working Principle:

Lead acid batteries are composed of Lead per oxide (PbO_2) as cathode, sponge metallic lead (Pb) as anode and dilute sulphuric acid (H_2SO_4) as electrolyte. H_2SO_4 is a good electrolyte for a lead acid battery. The voltage of a fully charged cell is 2.1V to 2.6V and the voltage falls 1.8V to 1.6V after discharge. As the cell discharges to a certain limit, both electrodes are converted to lead sulfate. The process is reversal i.e., it reverses on charge.[11]

Anode Equation:

$Pb(aq) + H_3O^+(aq) + H_2O(l) \leftrightarrow PbSo_4(s) + H_3O(aq) + 2e^-$ $\varepsilon^0 = -0.356v$

Cathode Equation:

$PbO_2(s) + 3H_3O^+(aq) + HSO_4^-(aq) + 2e^- \leftrightarrow PbSO_4(s) + 5H_2O(l)$ $\varepsilon^0 = 1.685v$[5]

3.8.5 Characteristics:

- *General Performance Characteristics:* The general performance characteristics of the lead-acid cell, during charge and discharge is shown in the below diagram. Here, as the cell gets discharged, the voltage decreases due to depletion of material, internal resistance losses, and polarization. If we take the discharge current is constant, the voltage under load decreases smoothly to the cut-off voltage and the specific gravity decreases proportionally to the Ampere-hours discharged.[5]

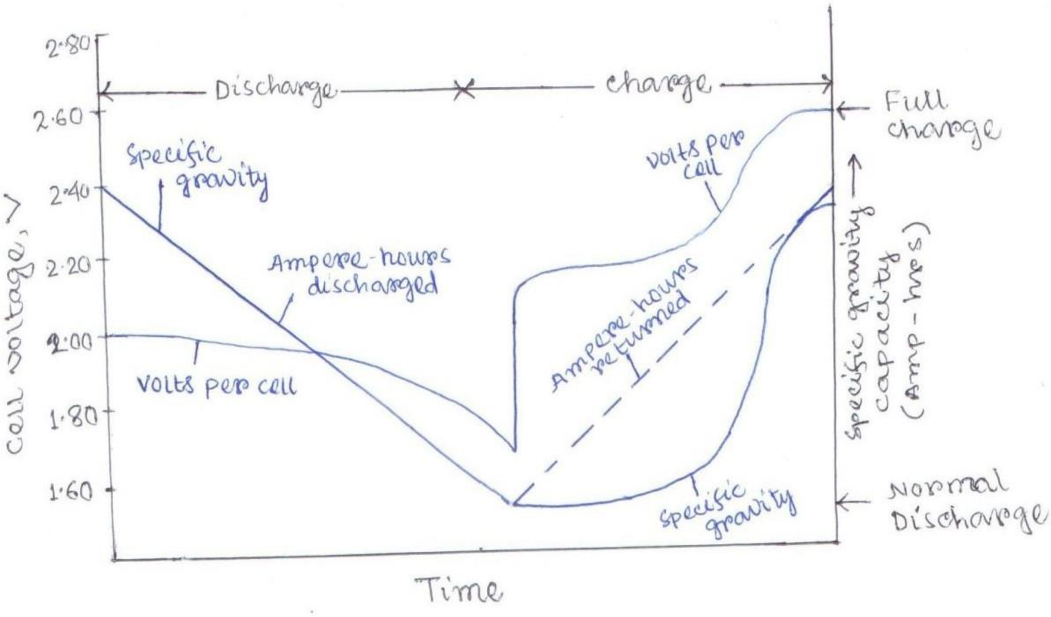


Figure 3.4 Typical voltage and specific gravity characteristics of lead-acid cell at constant rate discharge and charge. [21]

- *Open circuit voltage characteristics:* The open-circuit voltage of a battery system is normally a function of temperature and electrolyte concentration expressed in the form of Nernst equation for the lead-acid cell

$$E = 2.047 + \frac{RT}{F} ln\left(\frac{\alpha H2SO4}{\alpha H2O}\right)$$

Since the concentration of the electrolyte varies, the relative activities of H_2SO4 and H_2O in the Nernst equation also changes. A graph of the open-circuit voltage versus electrolyte concentration at 25 is shown. Here, we can see that the plot is fairly linear above 1.10 specific gravity, but also shows strong deviations at lower concentrations. Therefore, the open-circuit voltage is also affected by the temperature. The temperature coefficient of the open-circuit voltage of the lead-acid battery is shown in the graph below. Whenever the dE/dT is positive, such as above 0.5 Molar H_2SO_4, , the reversible potential of the system increases with increasing temperature. But, below 0.5 M, the temperature coefficient is negative. Most lead-acid batteries operate above 2 Molar H_2SO_4 (1.120 specific gravity) and have a thermal coefficient of about +0.2 mV/ . [14]

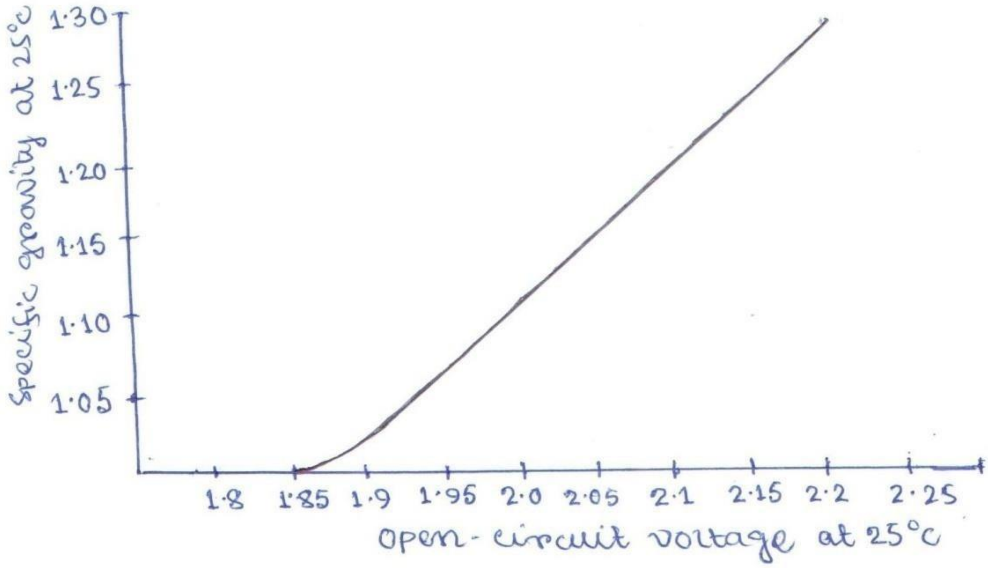


Figure 3.5 Open-circuit voltage of lead-acid cell as a function of electrolyte specific gravity. [22]

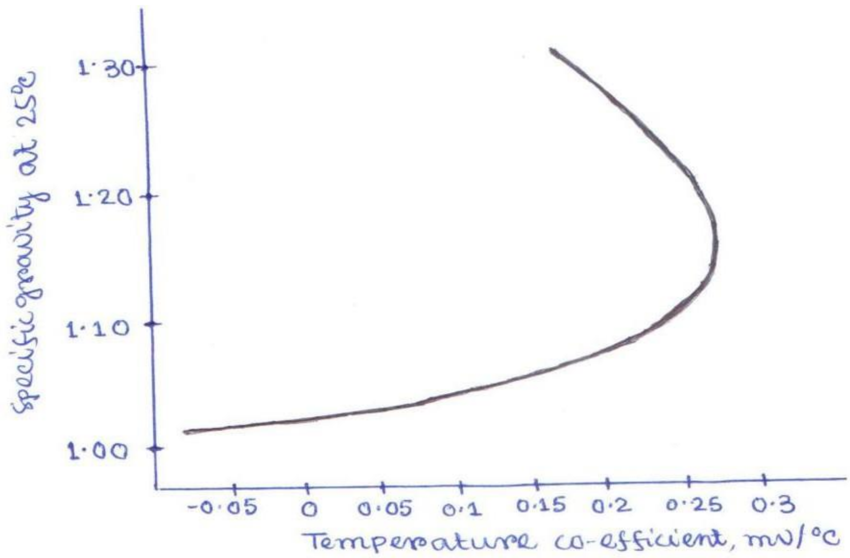


Figure 3.6 Temperature coefficient of open-circuit voltage of lead-acid cell as a function of electrolyte specific gravity. [23]

3.8.6 Charging Methods

Various methods are used to charge a lead-acid battery. First is constant-voltage charging, the conventional method for lead-acid batteries and is also preferred for VRLA batteries. However, we can also use constant current, taper current, and others.[15]

- *Constant Voltage Charging*: Constant-voltage chargers maintain nearly the same voltage input to the battery throughout the charging process, regardless of the battery's SOC. These type of chargers provide a high initial current to the battery because of greater potential difference between the battery and charger. A constant-voltage charger may charge as much as 70% of the previous discharge in the first 30 minutes or even less. This proves useful in many battery applications involving multiple discharge scenarios. Voltage of the battery increases as the battery gets charged.

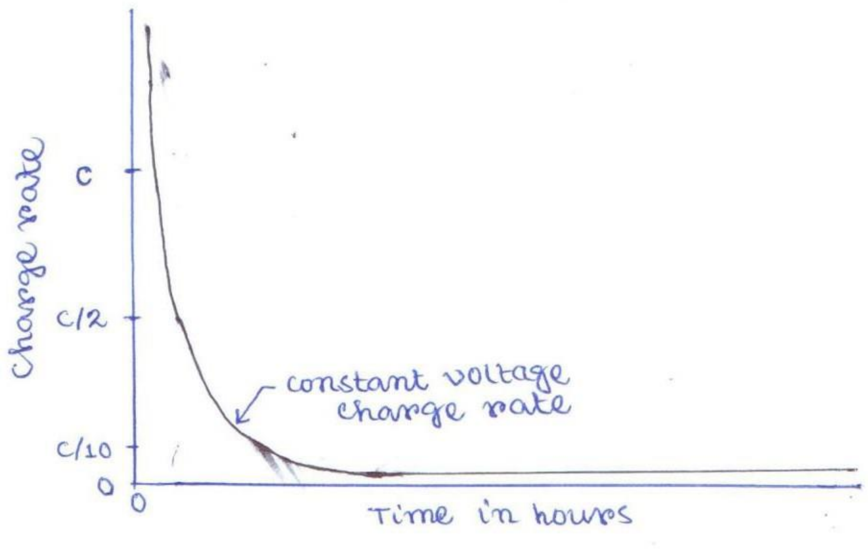


Figure 3.7 Charge rate versus time for a typical constant voltage charger [24]

- *Constant Current Charging:* Constant-current charging simply means that the charger supplies a relatively uniform current, regardless of the battery state of charge or temperature. Here, a more expensive and sophisticated charger is used. This method is used for recovering batteries that have suffered from extensive storage without charging. These also has the capability of overcharging a battery if there is not some voltage limiting function.
- *Trickle Charging:* A trickle charge is a continuous constant-current charge , but it is used at a low (about **C**/100) rate because these chargers are used to maintain the battery in fully charged condition. It is also used for charging battery for losses from self-discharge as well as the energy discharged during intermittent use of the battery. Trickle charging is also used widely for portable tools and equipment such as flashlights and battery powered screwdrivers.[16]

- *Float Charging:* Float charging is rarely used because it is used when the battery rarely gets discharged. A typical model where float charging is used consists of the float charger, battery and the load in parallel. During normal operation, the load draws the power from the charger. When the supply to the charger is interrupted, the battery steps in. Float charging of a battery involves charging the battery at a reduced voltage. This reduced voltage reduces the possibility of overcharging. The Float charger ensures that the battery is always in the charged condition and is therefore considered "floating". The Float charger starts by applying a charging voltage to the battery. As the battery gets charged, its charging current reduces gradually. The float charger senses the reduction in charging current and reduces the charging voltage. If the battery gets drained, the float charger will again increase the charging voltage and process continues. Float chargers can be connected indefinitely to the batteries.

- *Boost Charging:* It involves a high current for a short period of time to charge the battery. Normally it is used if the battery has been discharged heavily. It enables quick charging of batteries that are depleted. For example, a 4V lead acid battery which has been discharged, first it will be boost charged with a charging voltage of around 4.35-4.4 volts. However, as the battery voltage rises, the charger will switch over to the float charging mode with a float voltage of 4.25 V.

- *Taper Charging:* This is an inexpensive way to charge a sealed lead acid battery. Here, constant voltage or constant current is applied to the battery through a combination of resistance, diode and transformer. This type of chagrining won't damage the battery if left on charge for too long (even when left on the battery permanently), and they don't change their charging characteristics if the line voltage changes.

3.8.7 Defects:

- *Sulphation:* If a battery is left in a discharged condition for prolonged time, it causes sulphation and offers high internal resistance. The sulphation can be removed by recharging the cell for a longer period at a low rate which is called a trickle charge.
- *Buckling:* The bending of electrodes due to discharging, overcharging, improper electrolyte and improper temperature is known as buckling.
- *Partial short:* The sediments falling from the plates short-circuiting the positive and negative electrodes cause overheating of particular cell during both charging and discharging process. To avoid this such a cell may be replaced with a new cell.

3.8.8 Care and Maintenance:

Lead-acid batteries or any rechargeable batteries can function for longer if properly maintained. So regular maintenance of a lead acid battery is necessary to use for a long time in best condition. They are:-

- The battery should not be discharged beyond the minimum value of voltage.
- Avoid over discharging the battery.
- Maintain appropriate level of the electrolyte.
- Keep the battery clean or free from corrosion.
- Proper charger should be used.
- Overheating should be avoided.
- If the battery has been kept in an unused condition for a long time, then the battery should be put on a trickle charge.

- Make sure that, while charging, the positive terminal of the charger is connected to the to the positive terminal of the battery, and the negative terminal of the charger to the negative terminal of the battery.[13]

3.8.9 Applications

The lead-acid battery is manufactured in a variety of sizes and designs, ranging from less than 1Ah to over 10,000 Ah. Different types of lead-acid batteries are available and these are used in various application.

The lead acid batteries are used in automotive industry, electric vehicles, hybrid electric vehicles, industrial trucks. These are also used in submarines, aircraft for supplying the power. Stationary type lead acid batteries are used in uninterruptible power systems, load levelling, signalling, etc. It is also used in emergency lighting, portable tools, appliances, radio, TV, alarm systems, etc.[12]

CHAPTER 1

STATE OF CHARGE ESTIMATION

1.1 **NECESSAITY OF SOC**

The battery SOC which describes the remaining battery capacity is an important parameter for a battery. Accurate SOC estimation can not only protect battery, prevent over discharge, and improve the battery life but also allow the application to make rational control strategies to save energy.

1.2 **Definition**

The SOC of a battery is defined as the ratio of its current capacity (t) to the nominal capacity (Q_n). The nominal capacity is given by the manufacturer and it represents the maximum amount of charge that can be stored in the battery. The SOC can be defined as follows:

$$\mathrm{SOC}(t) = \frac{Q(t)}{Qn}$$

The units of SOC are given as percentage points i.e., (0% = empty; 100% = full).

1.3 **Determination of SOC:**

Battery is an electrochemical device in which chemical energy is converted into electrical energy. So, it is difficult to estimate the SOC of a battery as chemical energy of the battery cannot be measured directly. But with the help of different parameters of battery, such as impedance, terminal voltage, it is possible to determine the SOC accurately and with greater efficiency. *The methods, commonly used for estimating the battery SOC are based on the internal complex physical and chemical reaction of the established mathematical model, such as open circuit voltage method, discharge test method, impedance method, coulomb counting method, Kalman Filtering Method, Neural Network Method, etc.*

1.)Direct Measurement:

We all know that SOC cannot be measured directly, but it is possible only from direct measurement variables like terminal voltage , impedance, etc.

(i) *Open circuit voltage method*: The open-circuit voltage is the battery voltage when no current is flowing in or out of the battery. The open-circuit voltage is a function of State-of-Charge, OCV = *f*(SOC), and the function *f* is expected to remain the same during the life-time of the battery, i.e. it is not dependent on the age of the battery. The OCV method based on the OCV of batteries is proportional to the SOC when they are disconnected from the loads for a period longer than two hours. There is approximately a linear relationship between the SOC of the lead-acid battery and its open circuit voltage (OCV). The relationship between the OCV and SOC is not same for all the batteries. This is because the conventional OCV-SOC is different among batteries. There is a problem in that the relationship of the OCV-SOC should be measured to estimate accurately the SOC.

(ii) *Terminal Voltage Method*: This method is based on the measurement of terminal voltage when the battery is discharged. The terminal voltage of battery is approximately linear proportional to the SOC. But in this case, the error estimated is large, because at the end of discharge of battery, the terminal voltage is suddenly dropped. During discharge, battery temperature increases, which also affects the battery terminal voltage.

(iii) *Impedance Method*: During the cell charge-discharge cycles, the composition of the active chemicals in the cell changes as well, because the chemicals are converted between the charged and discharged states, this will be reflected in changes to the cell impedance also. Thus, measurements of cell internal impedance can also be used to determine SOC. However, these are not widely used because there are difficulties in measuring the impedance.

(iv) *Impedance Spectroscopy Method*: SOC can also be estimated with impedance spectroscopy using the Spectro complex modelling method. This opens applications in automotive manufacturing where some batteries are discharged longer than others during testing and debugging and need charging before transit. Measuring SOC by impedance spectroscopy can also be used for load levelling systems where a battery is continuously under charge and discharge.

(v) *Chemical Method*: Batteries having only liquid electrolyte is used in this method for finding SOC. Here, specific gravity or pH is used for determining the SOC. Hydrometers, which measure the specific gravity of a battery, are used to calculate the SOC of a battery. During charging, the lead sulphate is converted into lead and the concentration of sulphuric acid increases, which also increasing the specific gravity of electrolyte. During discharging, the sulphuric acid is again dissociated and removed from electrolyte. As SOC of the battery decreases, the specific gravity of electrolyte also decreases. They are directly proportional to each other.

The below table provides the BCI (Battery Council International) readings of starter batteries. While BCI specifies the specific gravity of a fully charged starter battery at 1.265, battery manufacturers may go for 1.280 and higher. Increasing the specific gravity will move the SOC readings upwards on the look-up table. A higher specific gravity will improve battery performance but shorten battery life because of increased corrosion activity. Besides charge level and acid density, a low fluid level will also change the specific gravity. When water evaporates, the specific gravity reading rises because of higher concentration. The battery can also be overfilled, which lowers the number. When adding water, we allow it for some time for mixing, before taking the specific gravity measurement.

Table: BCI standard for SOC estimation of a starter battery with antimony (Readings are taken at 26°C (78°F) after a 24h rest)

Approximate state-of charge	Average specific gravity	Open circuit voltage			
		2V	6V	8V	12V
100%	1.265	2.10	6.32	8.43	12.65
75%	1.225	2.08	6.22	8.30	12.45
50%	1.190	2.04	6.12	8.16	12.24
25%	1.155	2.01	6.03	8.04	12.06
0%	1.120	1.98	5.95	7.72	11.89

2.)Coulomb Counting Method: This method calculates the SOC by measuring the battery current and integrating it with time. Since no measurement can be perfect, this method suffers from long-term drift and lack of a reference point. Therefore, the SOC must be re-calibrated on a regular basis, such as by resetting the SOC to 100% when a charger determines that the battery is fully charged.

3.)Kalman Filter Method: To overcome the shortcomings of the Voltage method and the Current integration method, Kalman filter can be used. The battery of which the state of charge is to be determined can be modelled with an electrical model; the Kalman filter will then predict the over-voltage, due to the current, and in combination with coulomb counting, make an accurate estimation of the state of charge. The strength of a Kalman filter is that it is able to adjust it’s trust of the battery voltage and coulomb counting in real time.

4.)BP Neural Network Method:

This is by far the most import and efficient way to estimate SOC of a battery. It is applied in SOC estimation due to their good ability of nonlinear mapping, self-organization, and self-learning. The BP neural network contains an input layer, one or more hidden layers and an output layer. The number of neurons of input layer and output layer depend on the numbers of input and output of the network. Usually, in BP neural network a set of inputs and target outputs are set to get the output in BP neural network. After training, the network produces the best output based on the target outputs in respect of given inputs. In case of battery SOC estimation process, the variables, such as terminal voltage, discharge current, discharge time, impedance, temperature etc. are used as the inputs of the BP neural network. The below figure shows a typical architecture of the SOC estimating BP neural network.

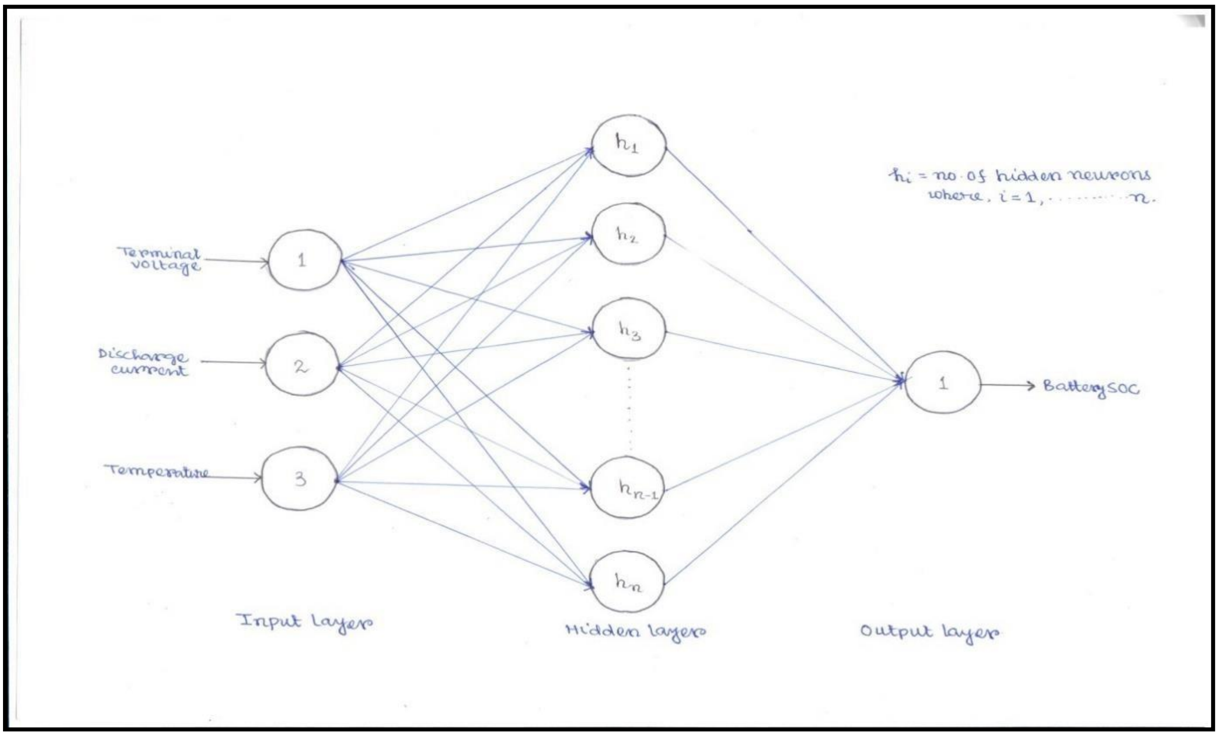


Figure 1.1 Architectural model of a BP Neural Network [25]

CHAPTER 2

BP OR BACK PROPAGATION NEURAL NETWORK

Neural networks, which are simplified models of the biological neuron system, is a massively parallel distributed processing system made up of highly interconnected computing elements, called neuron. These neurons have the ability to learn and thereby acquire knowledge and make it available for use. This learning process known as training and the ability to solve a problem using knowledge acquire is called inference.

2.1 Biological neurons and artificial neurons:

Neurons, the basic elements of human brain, are connected together and form a complex network. Neuron is a small cell that receives electro-chemical signals from various sources and responds to other neurons by transmitting impulses. A neuron comprises a nucleous, called soma. There are long irregular shaped filaments, called dendrites which are attached to the soma which acts as the input channels of neurons. Another type of link attached to the soma is

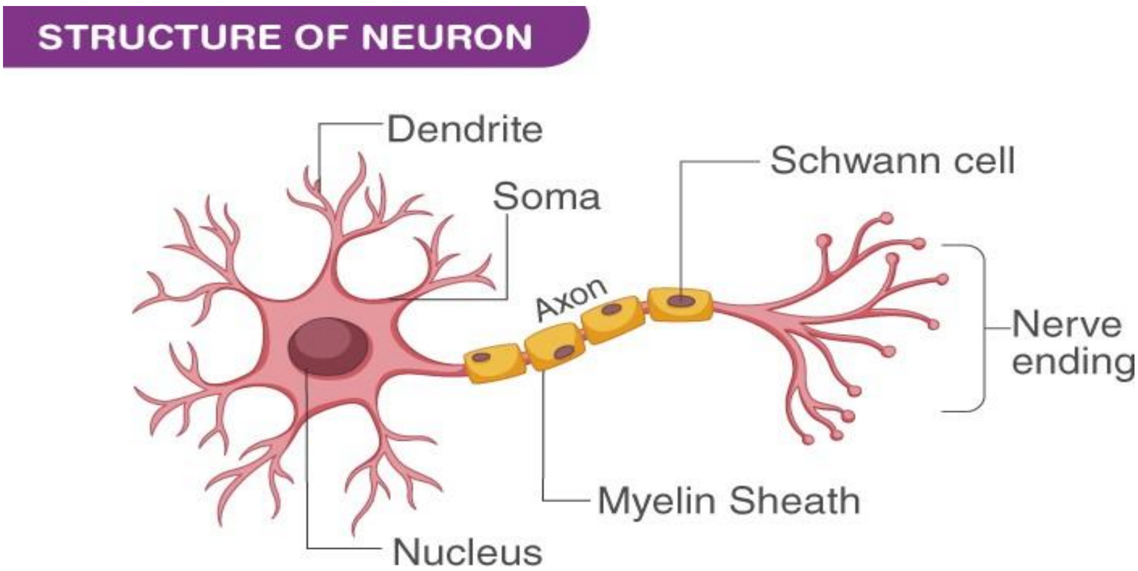


Figure 2.1 Structure of a biological neuron [26]

the axon. The axon serves as output cells. These are nonlinear threshold devices which are connected with dendrite links of another neurons through a specialized contact called synapse. If the cumulative input received by soma raise the internal electric potential of the cell, then the neuron fires by propagating the action potential down the axon to excite or inhibit other neurons. Here, neurons sum the inputs that they receive and have turned out to be a reasonable approximation.

A simple model of an artificial neuron has been shown. Here we can see, just like the biological neuron, artificial neuron receives the inputs through the input channel, sum up them and then passed through the thresholding device as output.

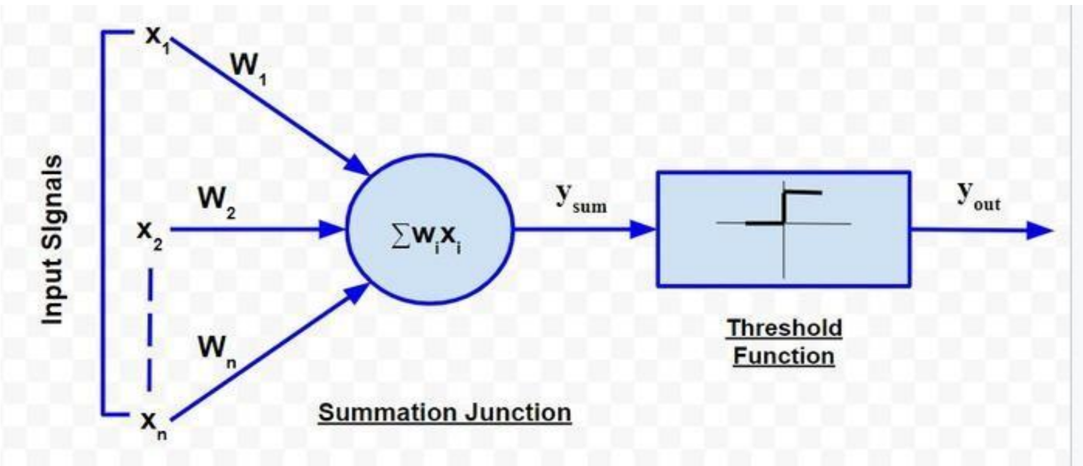


Figure 2.2 Simple model of an artificial neuron [27]

The output is such that the sum is greater than the threshold value. Here, we can clearly see that each and every component is similar to the biological neuron. Therefore, the name artificial neuron.

Here, the total input received by Soma of the artificial neural network is

$I=\sum w_i x_i$; where i= 1 to n;

Where x_i is the inputs to artificial neuron and w_i is the weights attached to the input links.

If the sum passed through a linear device then the final output becomes,

$$y = \Phi(I)$$

$$y = \Phi(\sum w_i x_i)$$

Φ is called Activation Function or Transfer function.

2.2 Different types of neural network:

- **Single layer feed forward network:**

  Single layer feed forward network comprises an input layer and an output layer. The input neurons receive the input signals and the output neurons receive the output signals. The input neurons and output neurons are connected together through synaptic links, which carrying the weight from every input to the output neuron but not vice-versa.

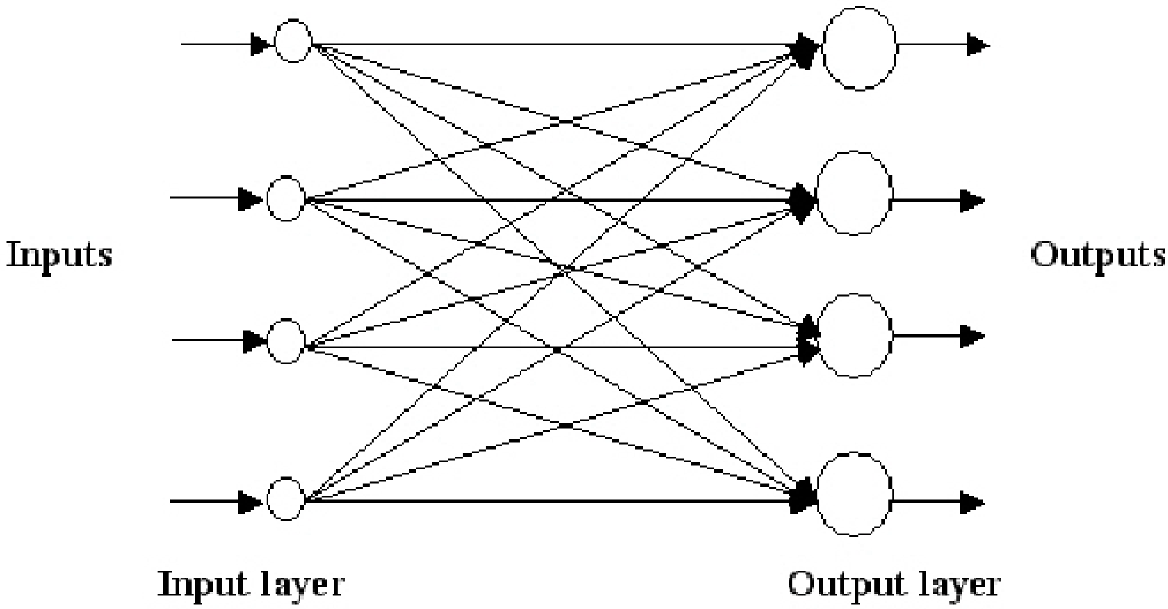


Figure 2.3 Single layer feed forward network [28]

- **Multilayer feed forward network:**

  In this network, there is an another layer known as hidden layer. The hidden layer aids performing useful intermediary computations before directing the input to the output layer. A multilayer feed forward network has a configuration l-m-n, has 'l' number of input neurons in input layer, 'm' number of hidden neuron in hidden layer and 'n' number of output neurons in output layer which is shown in the below figure.

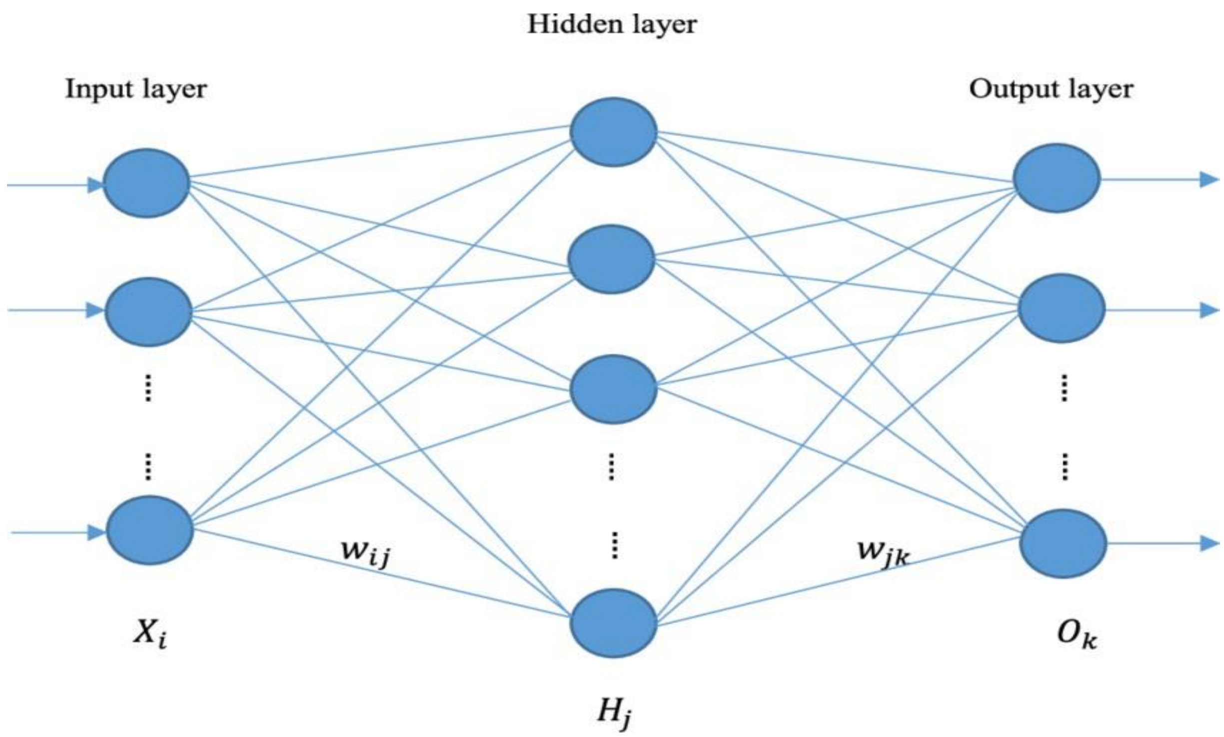


Figure 2.4 Multilayer feed forward neural network (l-m-n configuration) [29]

- **Recurrent Network:**

  In this type of network, the output is fed back to the input or hidden layer for further training process to minimize the error.

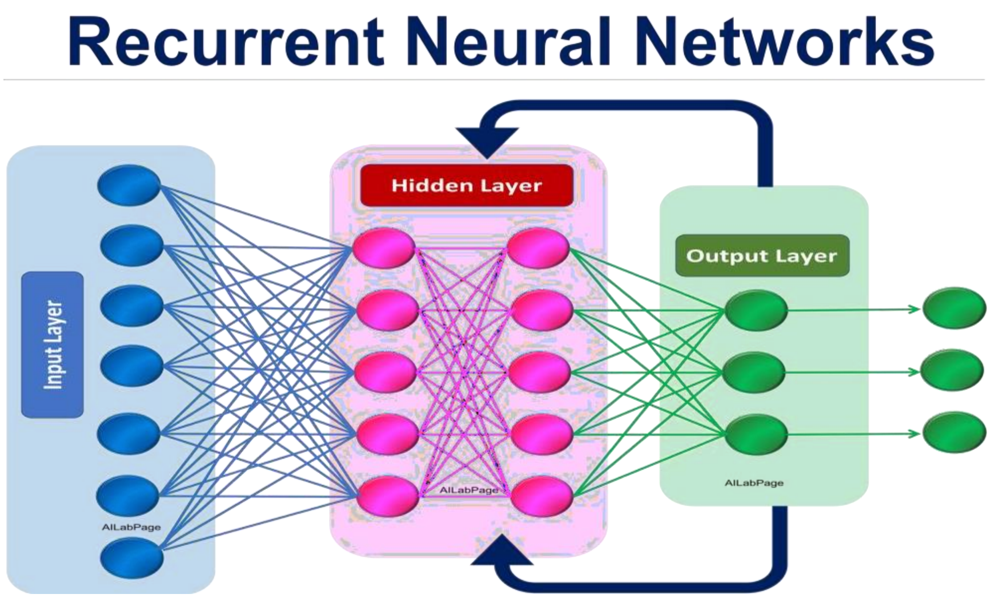


Figure 2.5 A recurrent neural network [30]

2.3 Neural network learning methods:

There are three types of learning methods i.e., supervised learning, unsupervised learning and reinforced learning.

- **Supervised learning:**
  It is based on error based learning. Here, we use a pair of input-output pattern to train the network. A particular output known as the target output is present for a particular input. After that we change the input based on the target output by training i.e., changing the network parameters to improve the performance based on this error.
- **Unsupervised learning:**
  In this learning method, the system learns by itself. This is because, in this case, a set of input is present without the corresponding output values.
- **Reinforced learning:**
  Similar to the unsupervised learning, no target output is present. Although it assumes that a teacher is present during training, it indicates that the computed output is correct or incorrect. This information helps the network in its learning process.

2.4 Backpropagation neural network

Backpropagation is responsible for the most efficient and errorless calculation of SOC. It is a systematic method of training multilayer feed forward artificial neural networks. It is built on high mathematical foundation and has very good application potential. In this neural network, supervised learning method is used. Here, error is produced and based on this error, the backpropagation neural networks adjust its weight for better performance. In backpropagation neural network this error is calculated by gradient descent method.

2.5 Backpropagation learning:

Consider a multilayer feed forward network network as shown in the figure below which has one linear input layer having 'j' number input neurons, one sigmoid hidden layer having 'k' number hidden neurons and one linear output layer having 'l' number output neurons. Notation 'I' and 'O' denote corresponding input and output respectively. The activation function mainly chosen as a non-linear function to show the non-linear behaviour in respect of biological neuron.

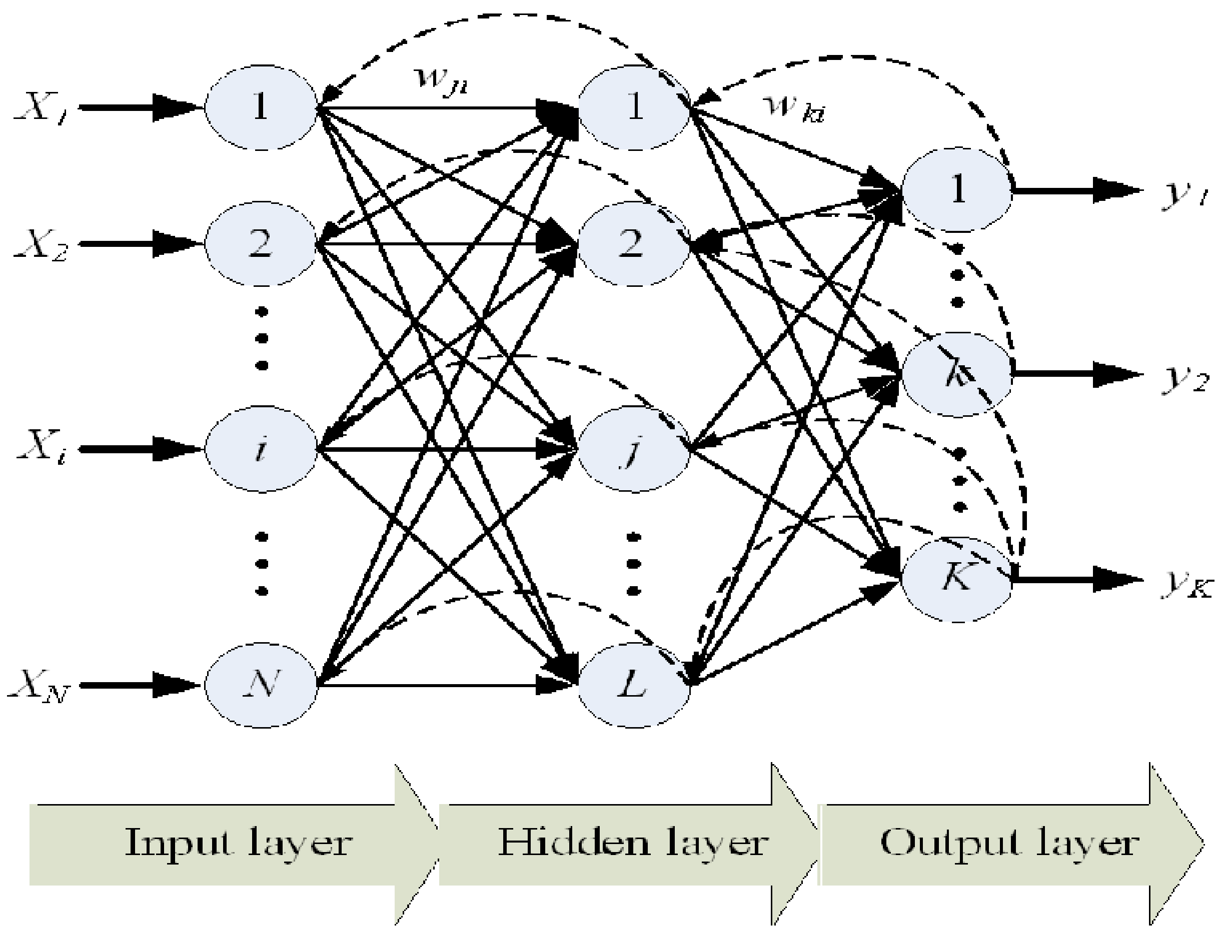


Figure 2.6 Multilayer feed forward backpropagation network [31]

- **Computation of Input Layer:**

  The output of input layer,

  $\{O\}_I = \{I\}_I$

  j × 1 j × 1

  The input of hidden neuron,

  $I_{Hp} = V_{1p}\, O_{I1} + V_{2p}\, O_{I2} + \ldots\ldots\ldots + V_{jp}\, O_{Ij}$

  Where p=1,2,3,… ....,k

  So the input to the hidden layer,

  $\{I\}_H = \{V\}^T \{O\}_I$

  k× 1 k×l j× 1

- **Computation of Hidden Layer:**

  The output of the hidden layer,

  $O_{Hp} = (1/1+e^{-\lambda I_{Hp}})$

The input to the output neuron,

$I_{Oq} = W_{1q} O_{H1} + W_{2q} O_{H2} + \ldots\ldots + W_{kq} O_{hk}$

Where q= 1,2,3,… .... ,l

So the input to the output neurons,

$$\underset{l\times 1}{\{I\}_O} = \underset{l\times k}{\{W\}^T} \underset{k\times 1}{\{O\}_H}$$

- **Computation of Output Layer:**

  The output of the network,

  $$\underset{l\times 1}{\{O\}_O} = \underset{l\times 1}{\{I\}_H}$$

- **Calculation of Error:**

  Considering any r^{th} output neuron and for the training example we have calculated the output 'O' for which the target output 'T' is given. The error norm in output for the r^{th} output neuron is given by

  $$E^1_r = 1/2\, e_r^2 = ½\,(T - O)^2$$

  The Euclidean norm of error E^1 for the first training pattern is given by

  $$E^1 = 1/2 \sum_{r=1}^{l} (\mathrm{Tor} - \mathrm{Oor})^2$$

  By using the same technique for all training pattern, the outcome

  $$E\,(V,W) = \sum_{i=1}^{l} \dot{E}^{i}\ (V, W, I)$$

  Where E is the error function depending on the k(j + l) weights of [W] and [V].

- **Training of network:**

  Training means "the learning process". Here, the backpropagation learning follows the supervised learning method, where an external reference signal (target output) is present during learning process and it generates an error signal by comparing the reference with the obtained response. Based on this error, the synaptic weights of network are adjusted to improve the system performance. The training of backpropagation neural network incorporates the following steps-

  - Initializing the synaptic weights.
  - Applying each input pattern to the input neurons and propagate it in forward direction i.e. from input layer to hidden layers and from hidden layers to output layer.
  - Calculating the error by comparing the output network with the target output and then calculating the error signal.

- The error signals are fed back to the input layer from the output layer in order to adjust the synaptic weights.
- Process continues till the error is low or minimum.

After training the backpropagation neural network, it can be tested on a set of unknown inputs to see how well it performs in untrained pattern. This process is known as ‘inference’. Therefore, the basic backpropagation algorithm loop structure isas follows:

Initialize the weights

Repeat

For each training pattern

Train on that pattern

End

Until the is acceptably low

2.6 Application:

Backpropagation is used in various types of problems like classification, function estimation, time-series prediction, etc.

4.0 PROPOSED APPROACH

4.1 EXPERIMENT

An experiment is performed using a 7AH lead acid battery and a lead acid battery is used because of it's wide applications. A 12V, 7AH lead acid battery is used for this experimental setup. The battery has been discharged at the discharge rate 10C, 5C, 2.5C, and 1C, i.e. the load currents are 0.7A, 1.4A, 2.8A, and 7A respectively.

For each load, the experimental setup has been taken. Then the following steps has been conducted. They are as follows:

- First the battery is connected with load at different discharge rates. Then the DC voltmeter and DC Ammeter is connected to measure the discharge current, terminal voltage, load current.
- Then the open circuit voltage (V_{OCV}) before discharging the battery.
- After that the battery discharging process is started. Then the battery terminal voltage is measured at each one minute interval. The discharge current has been fixed constant for entire procedure.
- After the discharge process has been completed the battery is disconnected.

4.2 APPARATUS

- ***BATTERY***

  Brand: Exide, Model: EXTREME 12XR7B-B, Model No: 12XR7B-B,

  Specification: Capacity: 7AH, Voltage: 12 V

  Experimental Requirement: To obtain the required data, this battery has been used. It is used for discharging at different rates and measure the terminal voltage for each discharge rate.

Figure 4.1 A 12 V Lead Acid Battery [32]

- ***DIGITAL MULTIMETER***

  Brand: METRAVI, Model: Metrasafe-13

  Specifications:

  DC Voltage: 0 - 400mV/4V/40V/400V/1000V, Accuracy: ± 0.5%,

  DC Current: 0 – 400uA/4000uA/40mA/400mA/20A, Accuracy: ± 1.0%,

  Resistance: 0 - 400Ω/4kΩ/40KΩ/400kΩ/4MΩ/40MΩ, Accuracy: ± 1.0%,

  Requirement in this experiment: Multimeter has been used to measure the terminal voltage and open circuit voltage of the battery. Also used to measure the resistance.

Figure 4.2 A Digital Multimeter [33]

- ***DIGITAL VOLTMETER***

  Brand: CABS Electra, Model: CE0102V, Serial No: 022553,

  Specifications:

  Auxiliary Supply: 230V, 50Hz

  Outputs: (0-1000) V DC

Requirement in this experiment: The terminal voltage of a battery is calculated using this instrument.

Figure 4.3 A Digital DC Voltmeter [34]

- ***DIGITAL DC AMMETER***

  Brand: CABS Electra, Model: CE0102A, Serial No: 022604,

  Specification:

  Auxiliary Supply: 230V, 50Hz

  Output: (0-20) A DC

  Requirement in this experiment: The discharge current or load current is measured using this instrument.

Figure 4.4 A Digital DC Ammeter [35]

- **Rheostat:**

  Specification: 3A, 20Ω; 5A, 100Ω; 6A, 50Ω

  Requirement in this experiment: Also known as variable resistance, it is used for discharging the battery.

Figure 4.4 Rheostat [36]

5.0 EXPERIMENTATION AND RESULTS

The data obtained from conducting the experiment(shown in Figure 1), has been used to analyse and select a model network that provides the best result. The entire procedure involves two steps:

- **Step-I:**

➢ Our first step is to calculate the SOC of the battery by Coulomb Counting Methods.

$$\%SOC = \frac{Battery\ remaining\ capacity}{Battery\ nominal\ capacity} \times 100\%$$

➢ After that, the BP neural network is created using MATLAB neural network toolbox. The terminal voltage and discharge current as two inputs of neural network is taken. The percentage of SOC (%SOC) has been used as the target output of the neural network.

➢ After that the network is trained and retained to get the best performance.

➢ Then comparison is made between practical results and simulated results. Also, the percentage of error (%Error) is calculated between target output and simulated output.

$$\%Error = \frac{Practical\ SOC(\%) - Simulated\ SOC(\%)}{Practical\ SOC(\%)} \times 100\%$$

Here, a two layer feed-forward network is used, containing one hidden layer and one output layer, has been used to train the network. The activated function of hidden layer is hyperbolic tangent sigmoid function and the activated function of output layer is linear transfer function. Neural Network Fitting Tool GUI (nftool) with Levenberg-Marquardt back propagation algorithm in MATLAB Neural Network Toolbox (Version 7.10.0.499 (R2010a)) has been used to train and retrain the networks.

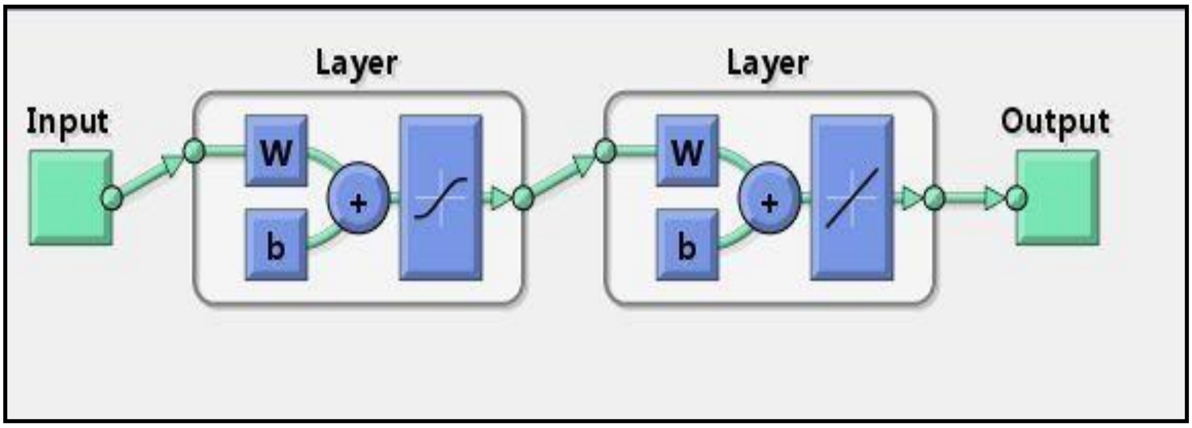


Figure 5.1 A Schematic diagram of a two layer feed forward neural network [37]

During training the data, the input has been selected randomly and is divided into three sets. First is the ‘Training’ set, a percentage of samples used to optimize the network, second is the ‘Validation’ set, a percentage of samples used to stop training, and third is the ‘Testing’ set, a percentage of samples used to evaluate network performance. In this experiment the ‘Training’ set, ‘Validation’ set and ‘Testing’ set are 70%, 15%, and 15% of total samples respectively. After the network is trained, it evaluates the performance using MSE (Mean Squared Error is the average squared difference between outputs and targets. Lower values are better. Zero means no error) and Regression analysis (Regression, R Values measure the correlation between outputs and targets. An R value of 1 means a close relationship, 0 a random relationship).

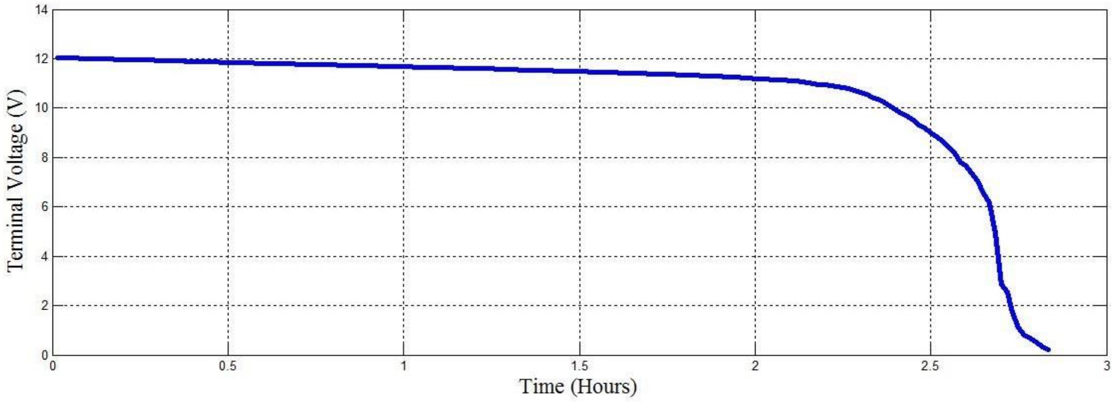


Figure 5.2 Time versus Terminal Voltage for Discharge Current 0.7 A[Stimulated from MATLAB]

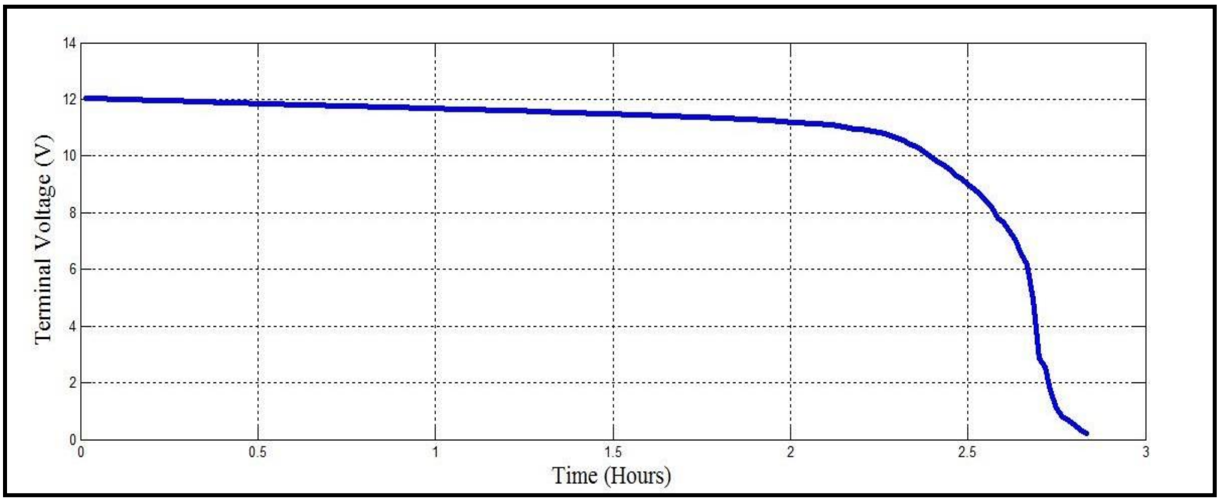


Figure 5.3 Time versus Terminal Voltage for Discharge Current 1.4 A[Stimulated from MATLAB]

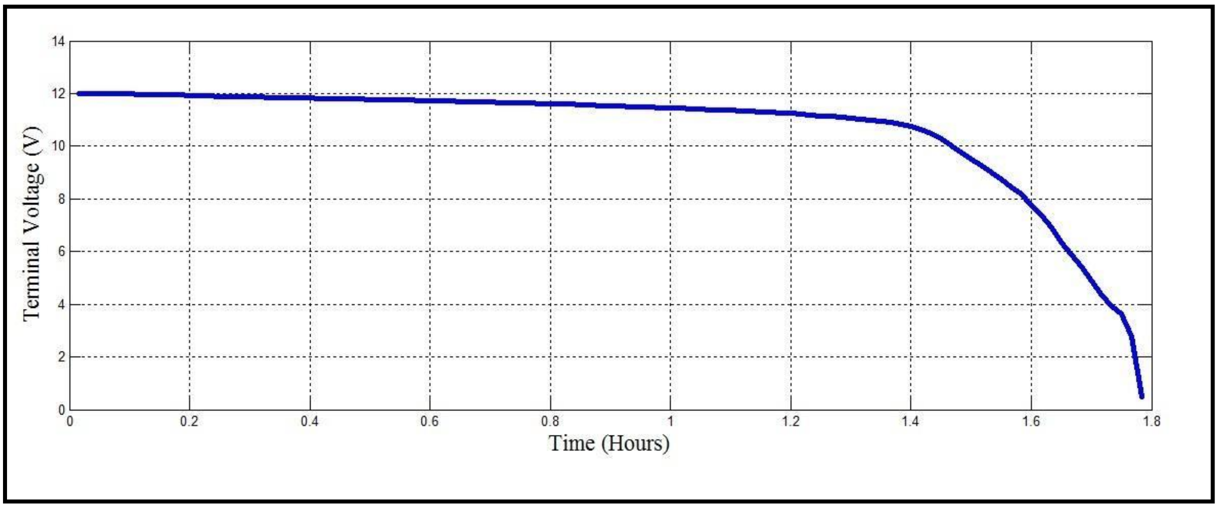


Figure 5.4 Time versus Terminal Voltage for Discharge Current 2.8 A[Stimulated from MATLAB]

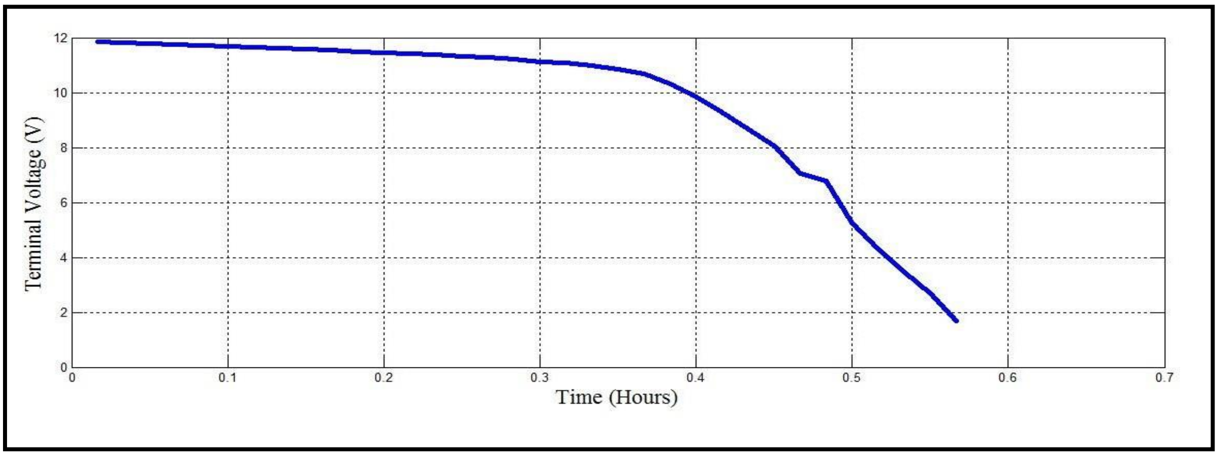


Figure 5.5 Time versus Terminal Voltage for Discharge Current 7.0 A[Stimulated from MATLAB]

6.0 COMPARATIVE ANALYSIS

The data obtained from experiment, the data set of Input and Target Output have been used to train a two layer feed forward backpropagation neural network considering different hidden neurons in hidden layer i.e. 3 neurons, 4 neurons, 5 neurons, 10 neurons, 15 neurons, 20 neurons and 21 neurons. After that, a comparison between practical SOC and Simulated SOC for different hidden neurons and calculate the percentage of error is made. The same procedure is applied for the data set of Input and Target Output (170 samples), which obtained from experiment for 1.4A discharge current. A model network will be chosen using these two data sets, because there are sufficient input-output samples to train the network and evaluate its performance. Figure 6.1 and Figure 6.4 show the plot between terminal voltage and different SOC including practical SOC for 0.7A discharge current and 1.4A discharge current respectively.

The errors of network for different neurons have been plotted in respect of terminal voltage which shown in Figure 6.2 and Figure 6.3.

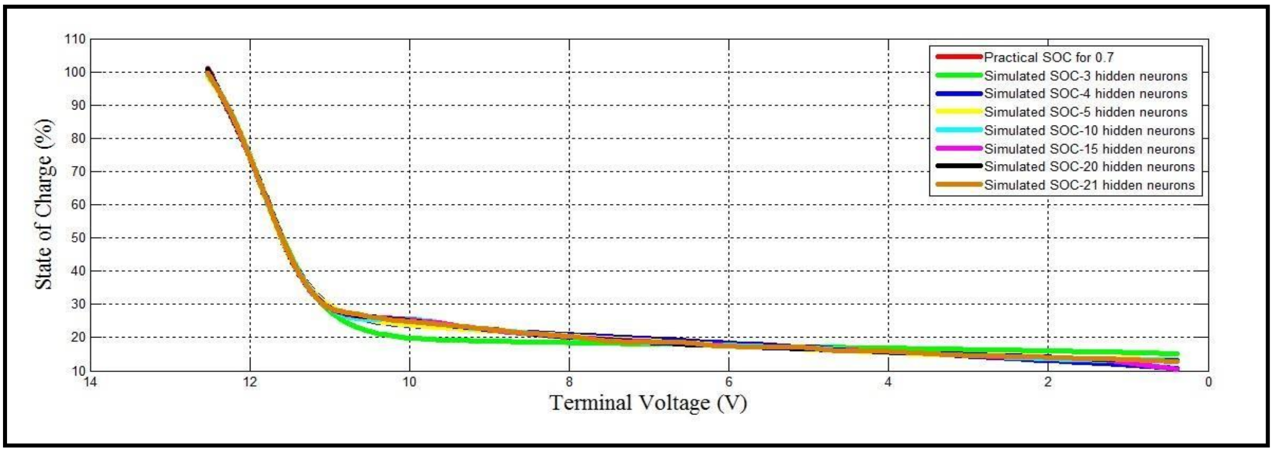


Figure 6.1 Terminal Voltage versus Practical SOC and Simulated SOC of different hidden neuron for 0.7 A Discharge Current [Stimulated from MATLAB]

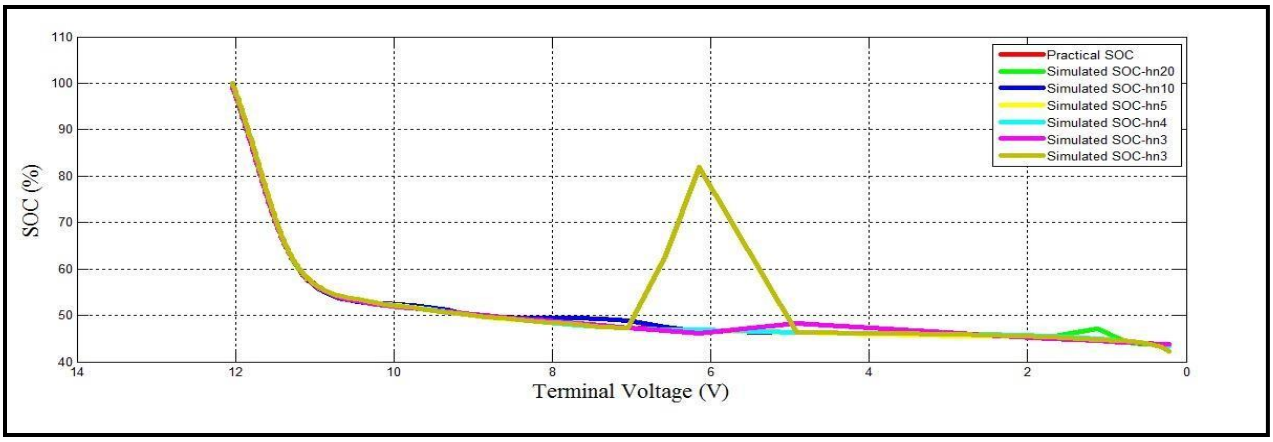


Figure 6.2 Terminal Voltage versus Practical SOC and Simulated SOC of different hidden neuron for 1.4 A Discharge Current [Stimulated from MATLAB]

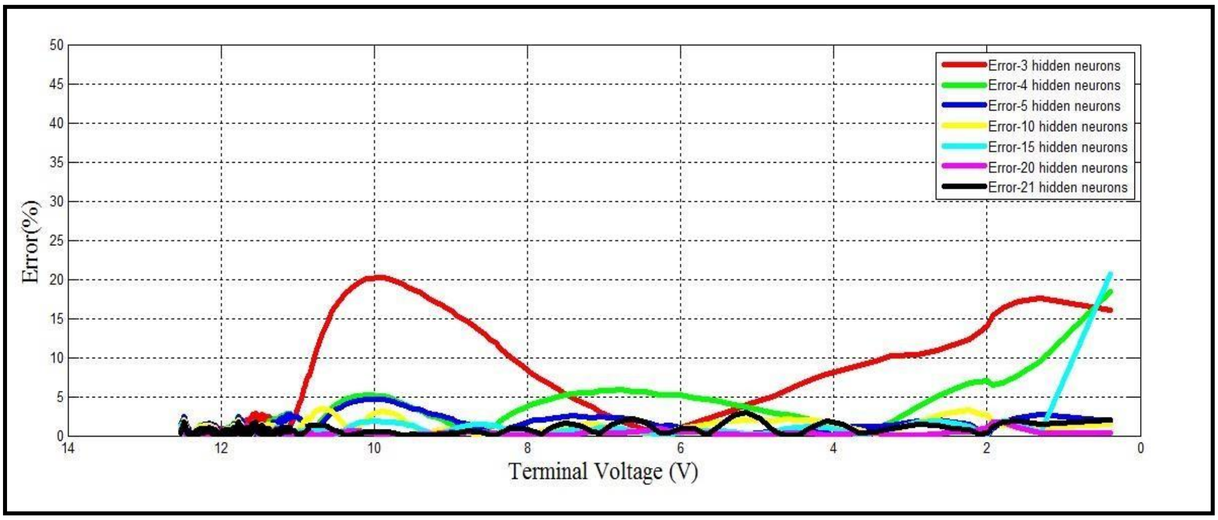


Figure 6.3 Terminal Voltage versus Error of different hidden neuron for 0.7 A Discharge Current [Stimulated from MATLAB]

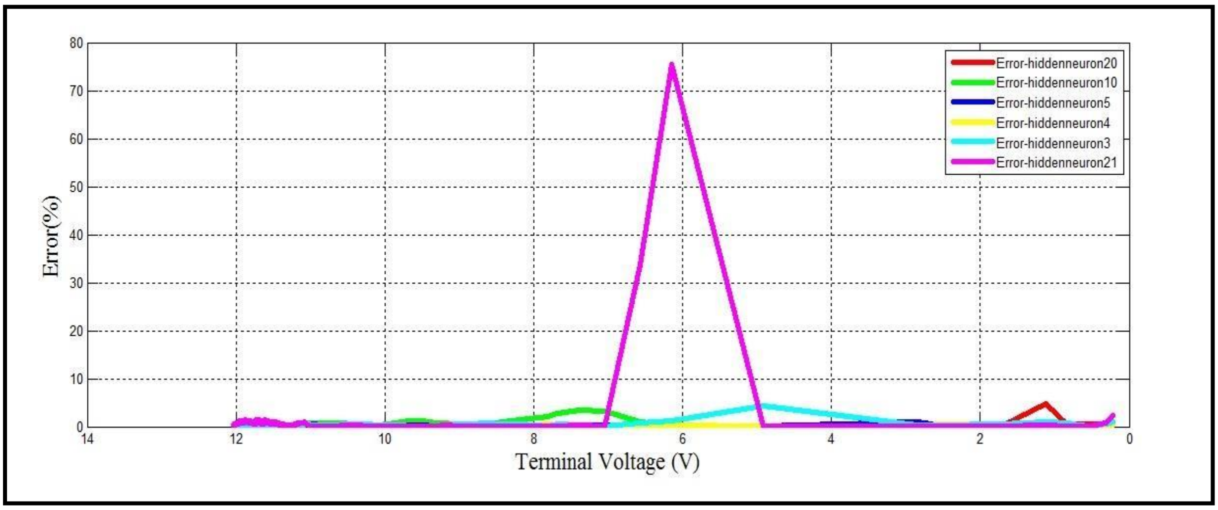


Figure 6.4 Terminal Voltage versus Error of different hidden neuron for 1.4 A Discharge Current [Stimulated from MATLAB]

From the above, it is clear from the Figure 6.1, Figure 6.2, Figure 6.3 and Figure 6.4 that the simulated SOC of network containing 20 hidden neurons shows a very close relation with practical SOC and the error is lower in this case. The results are that the average percentage of errors are 0.444% and 0.2032% for 0.7 A discharge current and 1.4 A discharge current respectively. The regression plot and performance plot for both cases are shown in Figure 6.5, Figure 6.6, Figure 6.7 and Figure 6.8. The overall regression values are 0.99976 for discharge current 0.7 A and 0.99913 for discharge current 1.4 A which are obtained from neural network containing 20 hidden neurons. The final mean square errors are also lower in both cases.

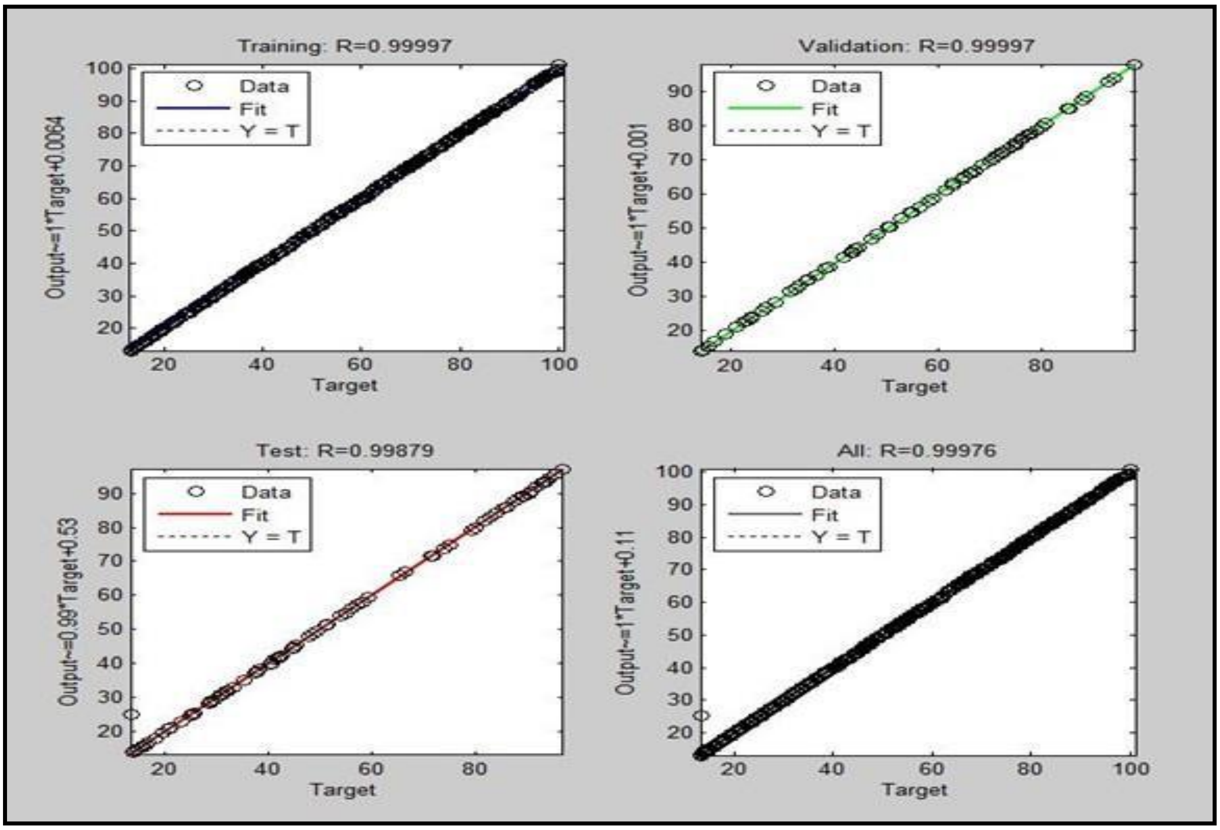


Figure 6.5 Regression Plot for Discharge Current 0.7 A[Stimulated from MATLAB]

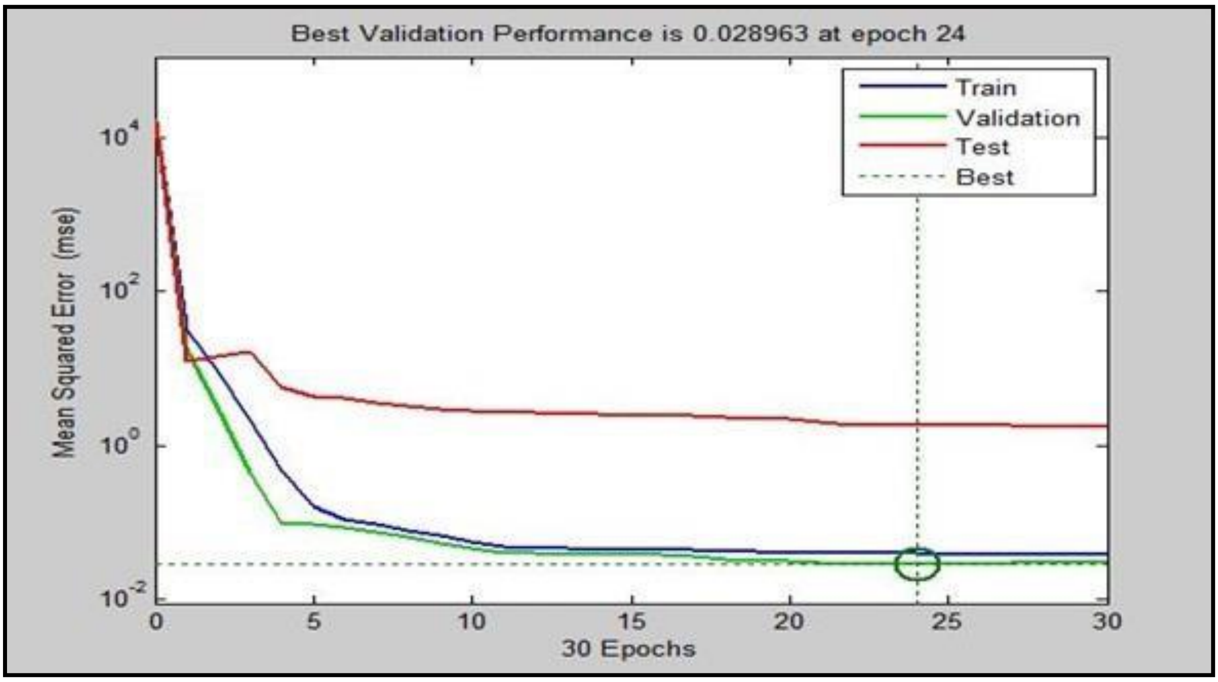


Figure 6.6 Performance Plot for Discharge Current 0.7 A [Stimulated from MATLAB]

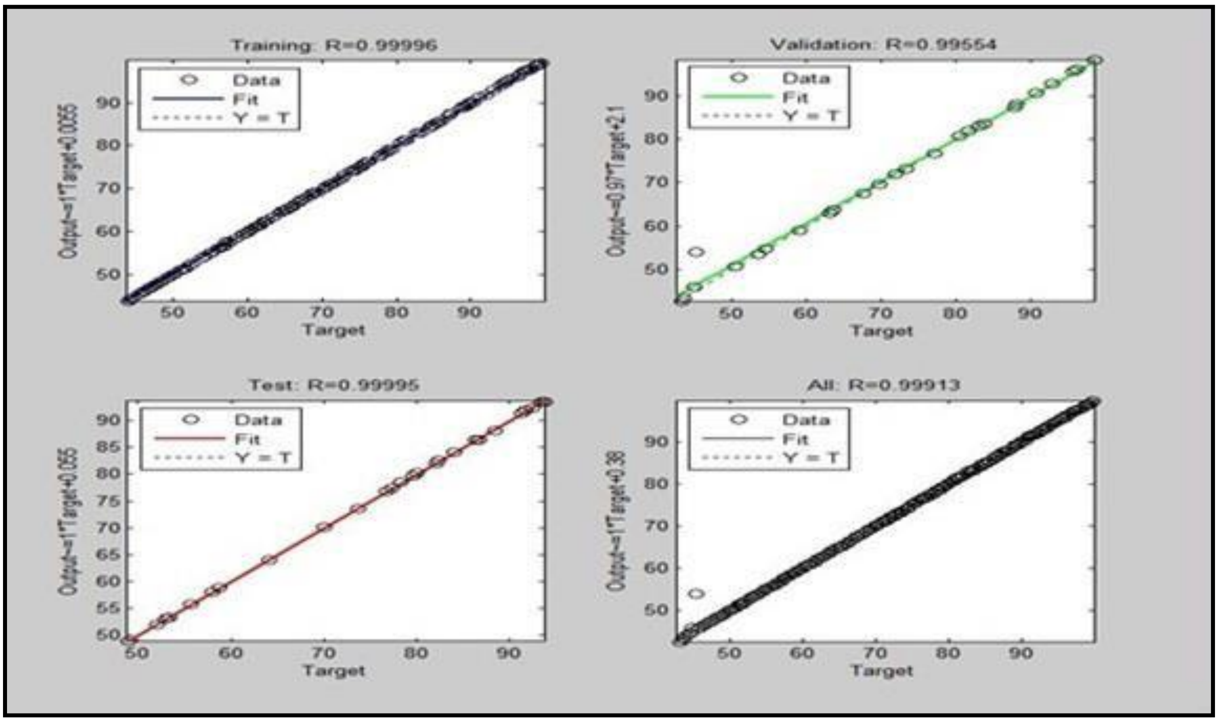


Figure 6.7 Regression Plot for Discharge Current 1.4 A [Stimulated from MATLAB]

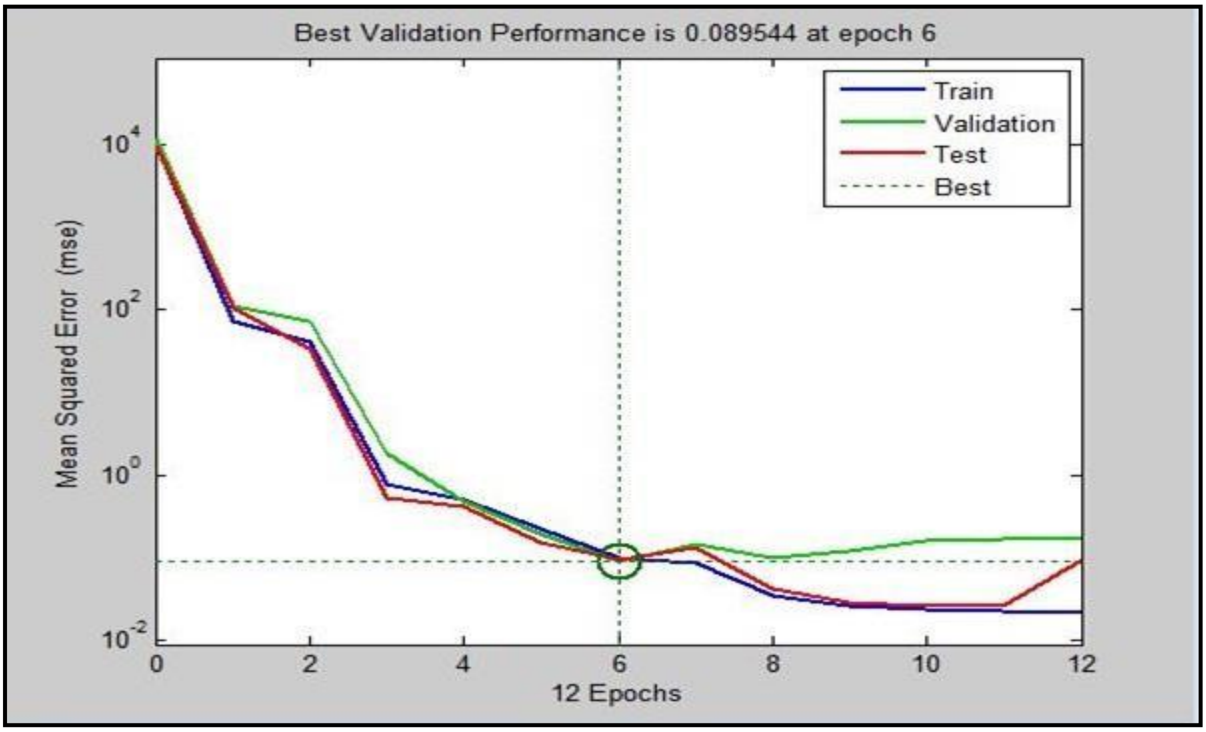


Figure 6.8 Performance Plot for Discharge Current 1.4 A [Stimulated from MATLAB]

Conclusion: Therefore, a conclusion is drawn that two layer feed forward neural network, having 20 hidden neurons in hidden layer shows the optimum performance.

- **Step-II:**

The data sets(input-output of 832 samples) obtained from different discharge rates have been used to train the network and prepare neural network for interference. Figure 6.10 and Figure 6.11 shows the Regression Plot and Performance Plot of the trained network, which also indicates that the trained network is best

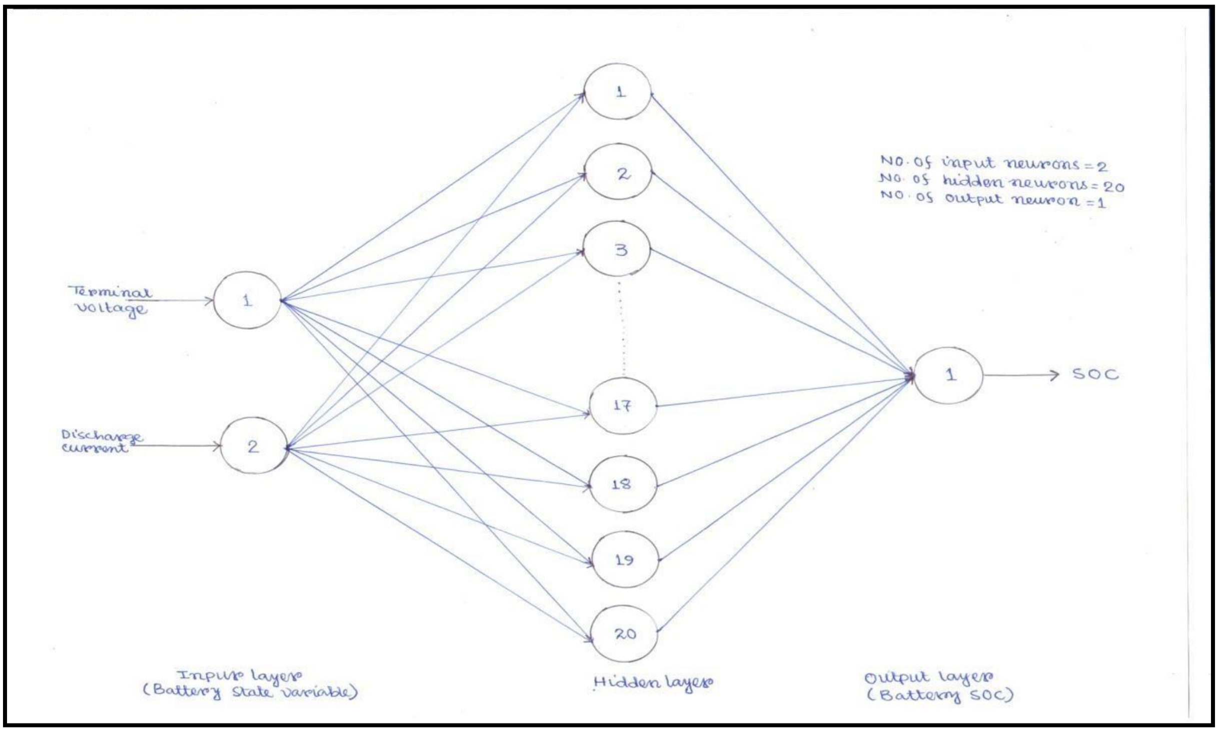


Figure 6.9 Architecture of model backpropagation neural network [Self-Drawn]

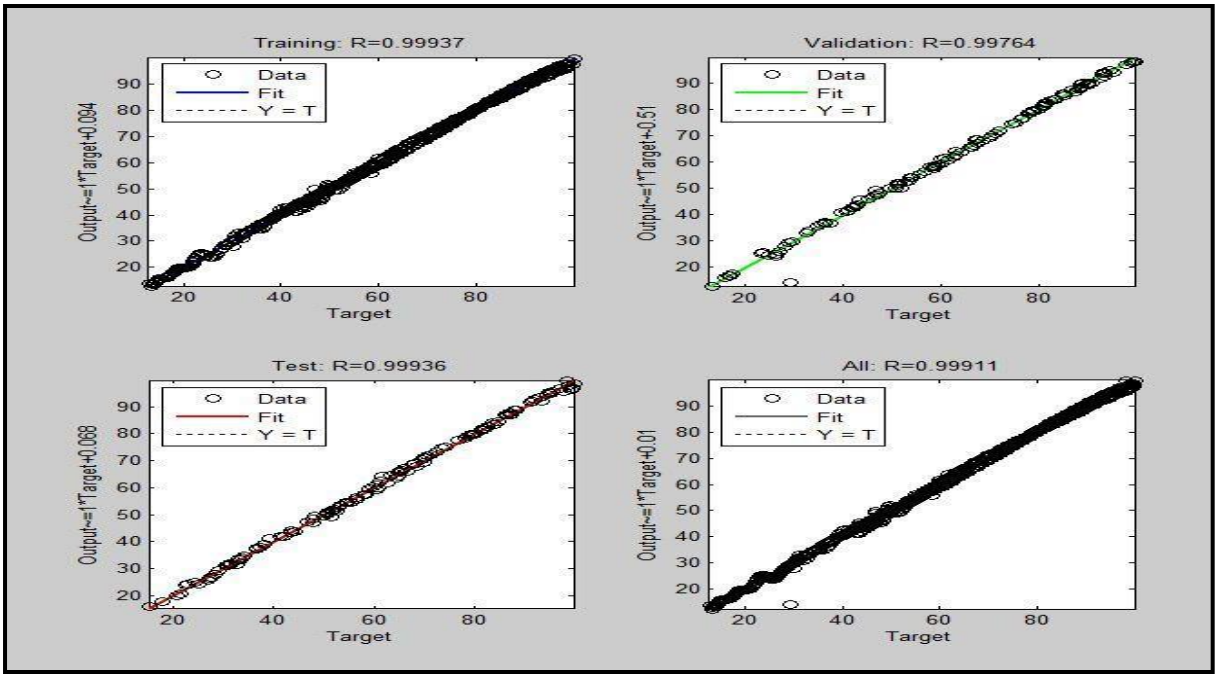


Figure 6.10 Regression Plot of model network [Stimulated from MATLAB]

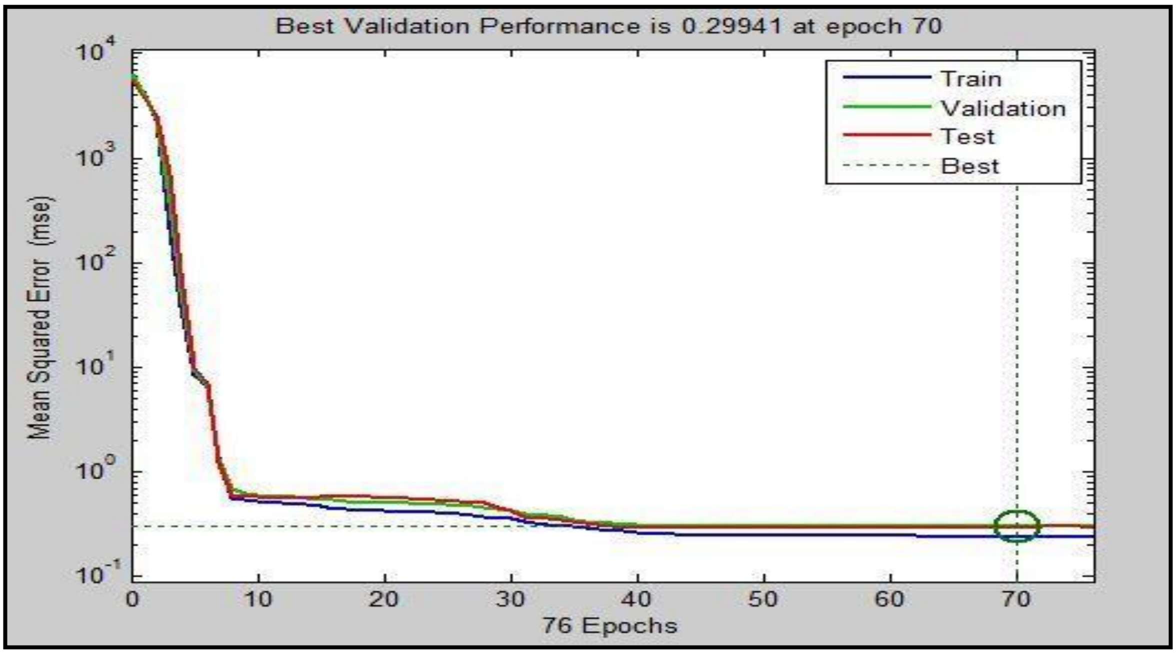


Figure 6.11 Performance Plot of model network [Stimulated from MATLAB]

for inference. Then each input data set of different discharge current has been used to simulate the model network, to know how accurate it predicts the battery state of charge. Figures 6.12 to 6.19 show the simulated SOC, practical SOC and errors in respect of terminal voltage and time.

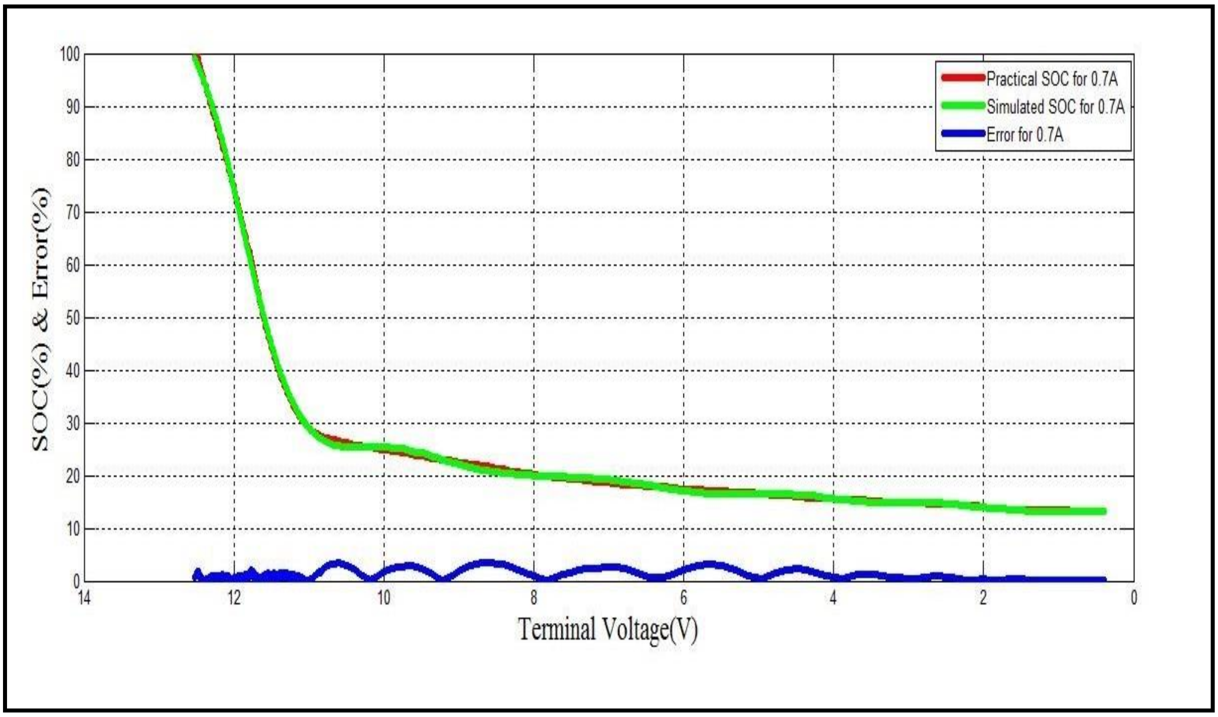


Figure 6.12 Plot between Terminal Voltage versus SOC & Error for 0.7 A [Stimulated from MATLAB]

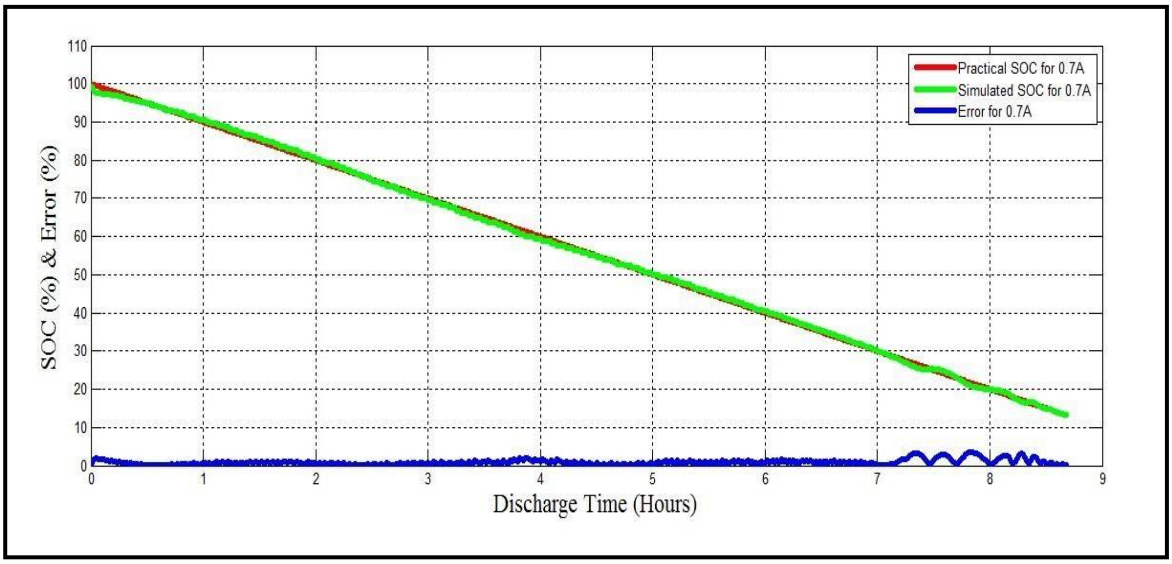


Figure 6.13 Plot between Time versus SOC & Error for 0.7 A [Stimulated from MATLAB]

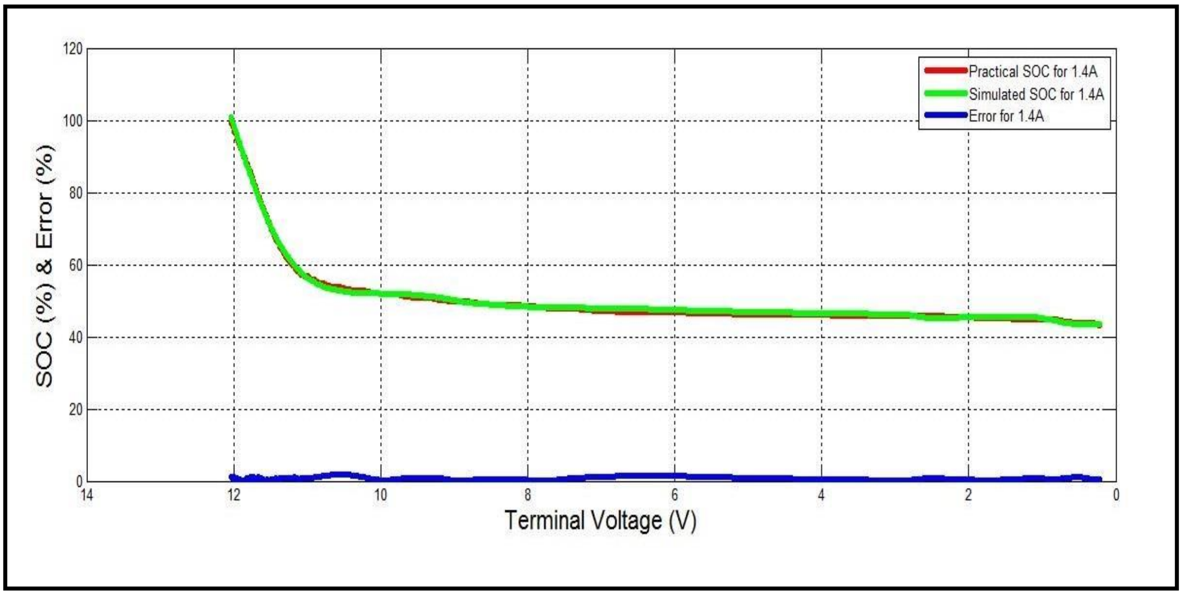


Figure 6.14 Plot between Terminal Voltage versus SOC & Error for 1.4 A[Stimulated from MATLAB]

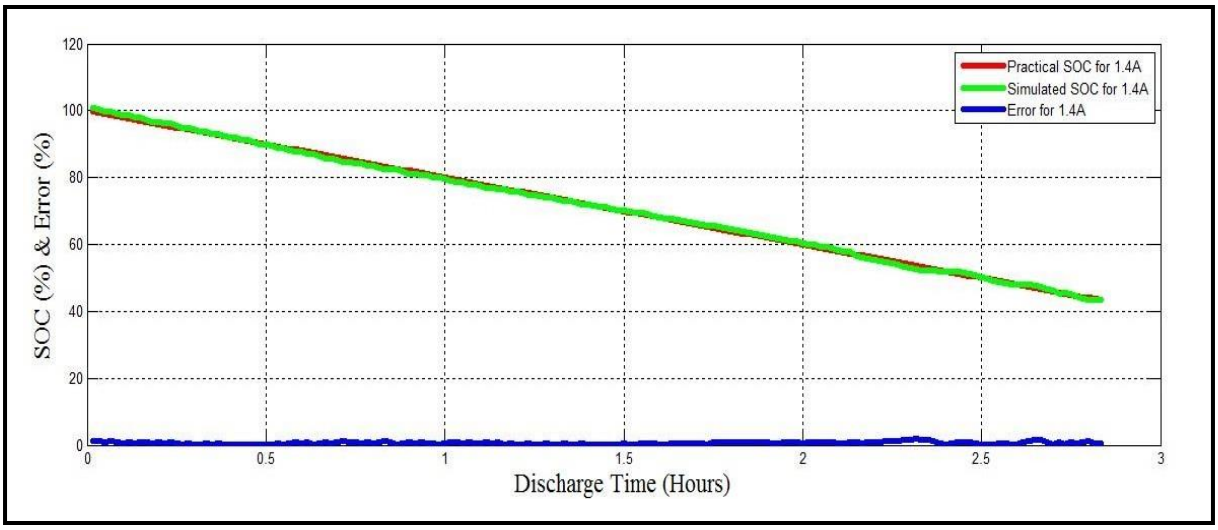


Figure 6.15 Plot between Time versus SOC & Error for 1.4 A[Stimulated from MATLAB]

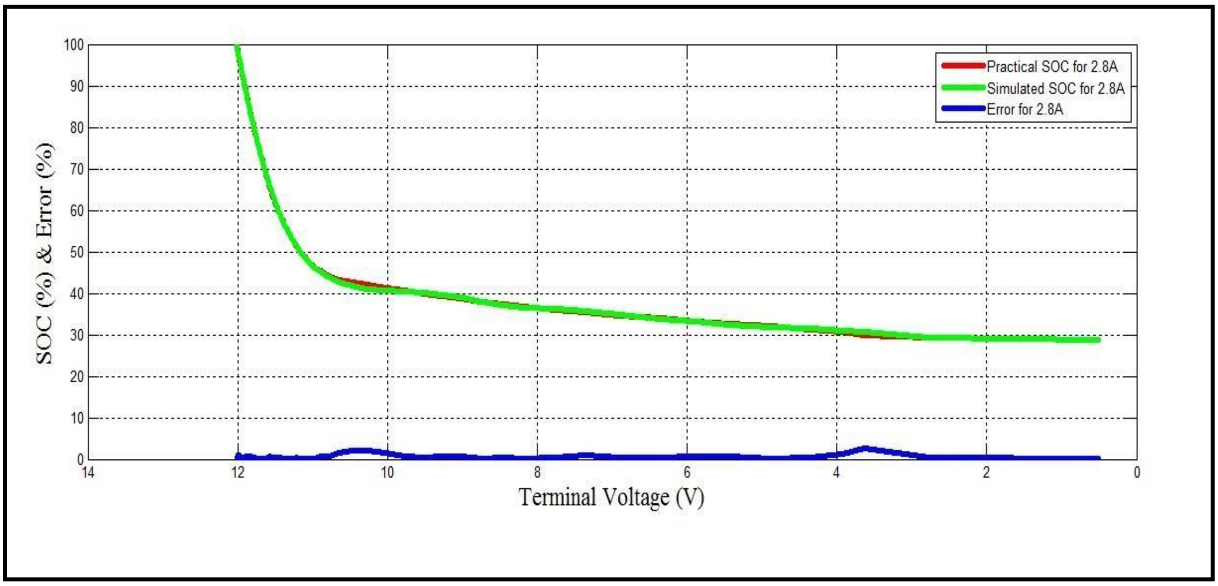


Figure 6.16 Plot between Terminal Voltage versus SOC & Error for 2.8 A[Stimulated from MATLAB]

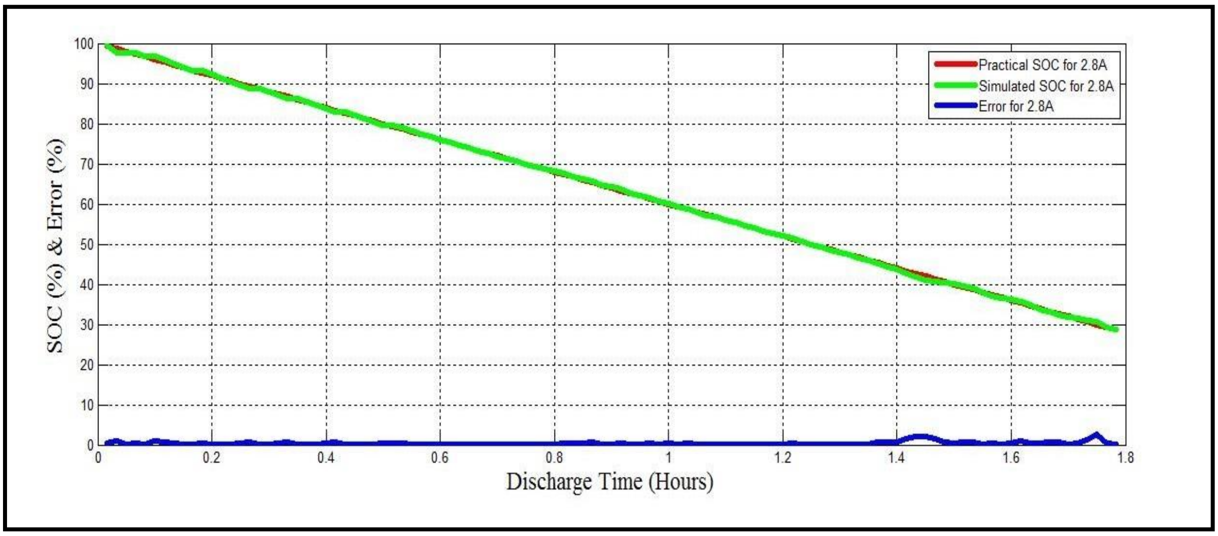


Figure 6.17 Plot between Time versus SOC & Error for 2.8 A [Stimulated from MATLAB]

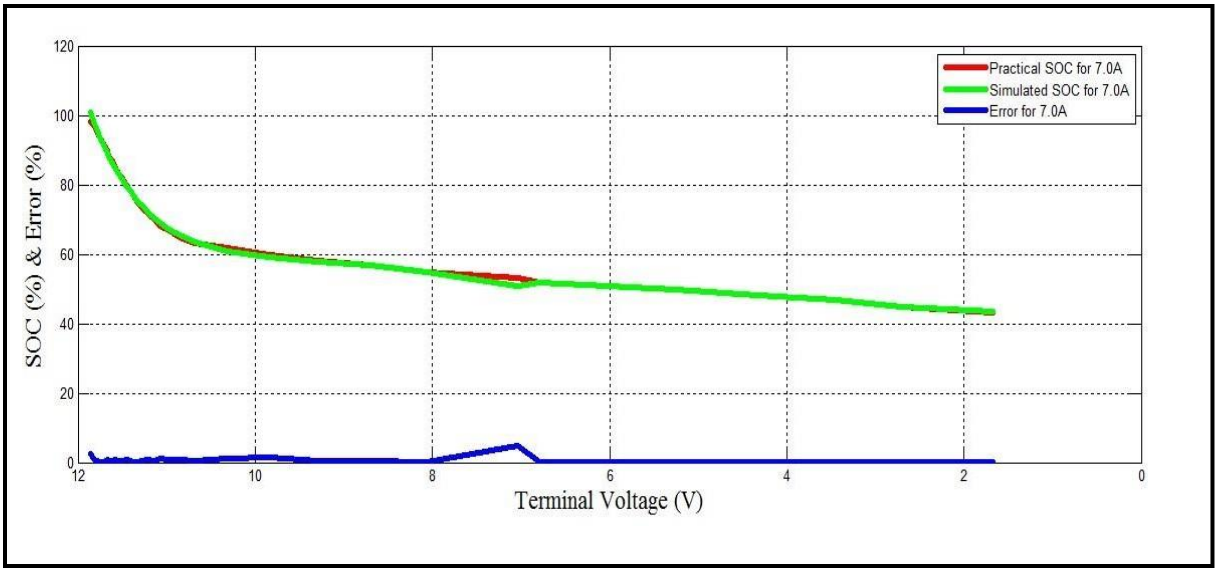


Figure 6.18 Plot between Terminal Voltage versus SOC & Error for 7.0 A [Stimulated from MATLAB]

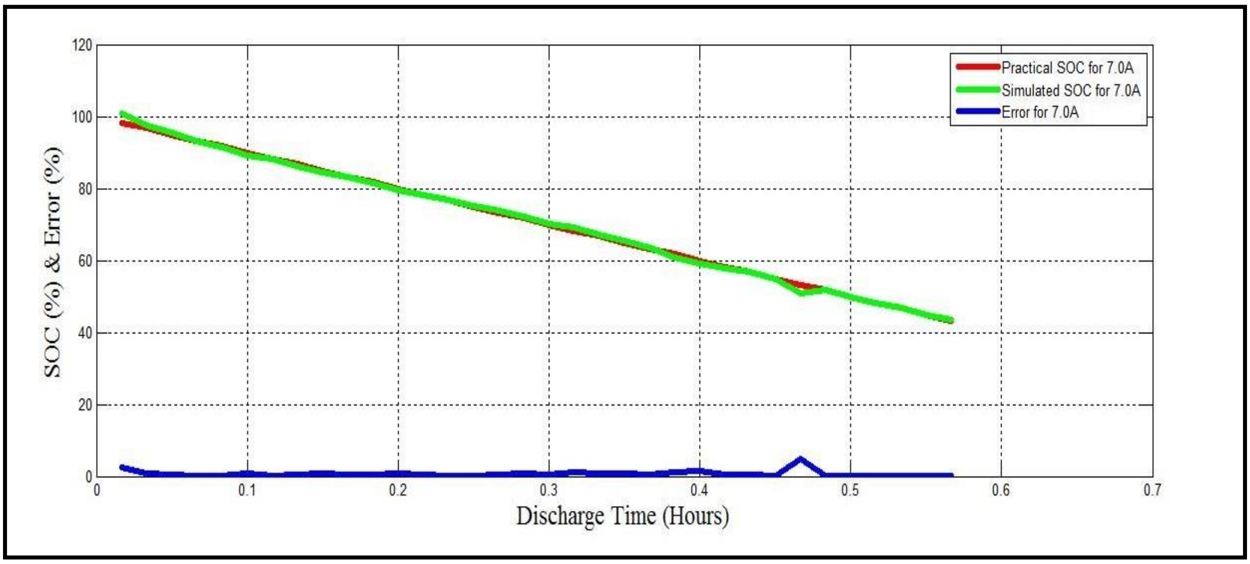


Figure 6.19 Plot between Time versus SOC & Error for 7.0 A [Stimulated from MATLAB]

Conclusion: The average percentage errors are 0.37%, 0.68%, 0.63%, and 0.88% for discharge current 0.7 A, 1.4 A, 2.8 A, and 7.0 A. So it is concluded that this backpropagation neural network model predicts the battery SOC very accurately with little error.

7.0 CONCLUSION

One of the important parameter to optimize battery is State of Charge(SOC). This thesis work has presented a model to optimize the state of charge estimation of a lead acid battery based on the analysis using BP neural network. The discharge current and the terminal voltage have been used as inputs. These data are collected by a practical experiment using a lead acid battery for four different discharge rates. The SOC determined by Coulomb Counting Method has been used as the target of the training network. Then the model has been simulated based on BP neural network in neural network toolbox in MATLAB for each discharge condition the simulated results have been compared with practical results.

The outcome is that the SOC estimation of a lead acid battery using BP neural network predicts almost accurately compared with practical results. It has also been observed that the SOC estimation using BP neural network shows accurate measurement with a small average percentage error.

So, it can be concluded that the SOC prediction using BP Neural network is almost errorless. Also small time is required to train the network, which is appropriate for many practical applications.

8.0 FUTURE SCOPE

This method can be implemented in other types of batteries too. But there are drawbacks also. They are:

1.) Slow convergence speed
2.)Poor generalization.

Further work is required to implement the improved BP neural network, such as fuzzy neural network, genetic algorithm based neural network, which overcomes the drawbacks of BP neural network.
During train the network in MATLAB the LM-BP algorithm has been used to simulate the network. Other types of algorithm can also be used to stimulate the network.

To obtain the target output, other method other than Coloumb Counting Method such as open circuit voltage method, Kalman filter method, etc. can also be used.
Here two parameters have been used. We may extend it using three parameter. Further work can be extend to apply this to practical battery.

9.0 REFERENCE

[1] F. Husnayain, A.R. Utomo, PS. Priambodo, “State of Charge Estimation for a Lead-Acid Battery Using Backpropagation Neural Network Method”, IEEE conference, November 2014, pp 274-278.

[2] X. Yang, F. Jiang, X. Wu, “Prediction of lead-acid storage battery’s remeaning capacity based on LM-BP neural network”, IEEE conference, 25-27 May,2013, pp 1908-1912.

[3] Chao Dang, Guanlan Wang, “Estimation of power battery SOC based on improved BP neural network”, IEEE International conference, 2014, pp 2022-2027.

[4] Wen Yean Chang, “The State Of Charge Estimation Methods For Battery: A Review”, Hindawi Publishing Corporation ISRN Applied Mathematics Volume 2013, Article ID 953792, 7 pages.

[5] CÉSAR A.C. SEQUEIRA, MÁRIO R. PEDRO, “LEAD-ACID BATTERY STORAGE”, June 2,2014

[6] Sefu Kitaronka, “Lead-Acid Battery”, 18 January,2022

[7]https://batteryuniversity.com/article/can-the-lead-acid-battery-compete-in-modern-times

[8] Linden David, Reddy B. Thomas, Handbook of Batteries, McGraw-Hill, New York, Third Edition.

[9] ‘A Guide to Understanding Battery Specifications’, MIT Electric Vehicle Team, December 2008

[10] David G. Enos, “Lead-Acid Batteries and Advanced Lead-Carbon Batteries”,2014

[11] Andrew D. Ballantyne,Jason P. Hallett,D. Jason Riley,Nilay Shah and David J. Payne, “Lead acid battery recycling for the twenty-first century”,16 May 2018

[12] https://en.wikipedia.org/wiki/Lead%E2%80%93acid_battery#:~:text=They%20are%20used%20in%20back,micro%20hybrid%20vehicles%2C%20and%20motorcycles.

[13] https://www.trojanbattery.com/tech-support/battery-maintenance/#:~:text=Batteries%20should%20be%20charged%20after,away%20from%20a%20charging%20battery.

[14] Naveen Kumar Elumalai, Ashraf Uddin, “Open Circuit Voltage Of Organic Solar Cells”,2016

[15] Bhangu B.S., Bentley P., Stone D.A., Bingham C.M., “Nonlinear observers for

predicting state-of-charge and state-of-health of lead-acid batteries for hybrid-electric vehicles", IEEE Transactions on Vehicular Technology, Volume: 54, Year: May, 2005, pp 783 – 794.

[16]] J.A.K.S. Jayasinghe, K.K.K.D. Nadishan, "Neural Network Based State of Charge (SOC) Estimation of Electric Vehicle Batteries", International Journal of Scientific and Research Publications, Volume 4, Issue 12, December 2014

[17] Sefu Kitaronka, "Lead Acid Battery", January 2022

[18] https://www.google.com/search?q=flow+of+anions+and+cations+during+discharge+process&sxsrf=ALiCzsalBJgTF71GXdYy4skw7DSrUiiksw:1661359662027&source=lnms&tbm=isch&sa=X&ved=2ahUKEwj_8v6D99_5AhXmALcAHURdAlYQ_AUoAXoECAEQAw&biw=1920&bih=929&dpr=1#imgrc=mjQwEZxPDZ069M

[19] https://sinovoltaics.com/learning-center/storage/lead-acid-batteries/

[20]https://www.google.com/search?q=flow+of+electrons+in+a+rechargeable+cell&sxsrf=ALiCzsZbgc37IqunGUqHOAeO8-WPCibRsg:1661360159313&source=lnms&tbm=isch&sa=X&ved=2ahUKEwjV6Y7x-N_5AhUo7nMBHeLoBuAQ_AUoAXoECAEQAw&biw=1920&bih=872&dpr=1#imgrc=EgaEMe8EqyDTEM

[21]https://www.google.com/search?q=Typical+voltage+and+specific+gravity+characteristics+of+lead-acid+cell+at+constant+rate+discharge+and+charge.&sxsrf=ALiCzsYPvRO5Cz9gkg1bPrwwWQytdrQOlQ:1661360268059&source=lnms&tbm=isch&sa=X&ved=2ahUKEwjsjvyk-d_5AhWQlNgFHe8YCsQQ_AUoAXoECAEQAw&biw=1920&bih=872&dpr=1#imgrc=MEA0ZWlG-AuUaM

[22] https://www.google.com/search?q=Open-circuit+voltage+of+lead-acid+cell+as+a+function+of+electrolyte+specific+gravity.&sxsrf=ALiCzsYLuBm7U2GUjQofaCQ06PG6xWGWUA:1661360356172&source=lnms&tbm=isch&sa=X&ved=2ahUKEwjFlv7O-d_5AhUtFrcAHZE1D5sQ_AUoAXoECAEQAw&biw=1920&bih=872&dpr=1#imgrc=XhiuEk3wp2kbkM

[23] https://www.google.com/search?q=Temperature+coefficient+of+open-circuit+voltage+of+lead-acid+cell+as+a+function+of+electrolyte+specific+gravity.&sxsrf=ALiCzsY5Gh6OKAtPjGSx0UmLwYd2aE3deg:1661360442202&source=lnms&tbm=isch&sa=X&ved=2ahUKEwjK_

oD4-
d_5AhVlldgFHajmDBcQ_AUoAXoECAEQAw&biw=1920&bih=872&dpr=1#imgrc=q2yG53SJRc8dzM
[24]https://www.google.com/search?q=Charge+rate+versus+time+for+a+typical+constant+voltage+charger&sxsrf=ALiCzsaB1wxFYM9o4iYCAWABAN_kUnicpg:1661360503805&source=lnms&tbm=isch&sa=X&ved=2ahUKEwik8rCV-t_5AhVn33MBHc2ZAaQQ_AUoAXoECAEQAw&biw=1920&bih=872&dpr=1#imgrc=9OhxYv4T0qaoPM
[25]https://www.researchgate.net/figure/A-simple-BP-neural-network-architecture_fig1_251196332
[26]https://www.google.com/search?q=biological+neuron+structure&sxsrf=ALiCzsb7-TWenuk3wVsZ_6SmiQogpXrBZQ:1661360683974&source=lnms&tbm=isch&sa=X&ved=2ahUKEwifyqXr-t_5AhUtFLcAHQp-CPAQ_AUoAXoECAEQAw&biw=1920&bih=872&dpr=1#imgrc=Sfc15T2QXeEdYM
[27]https://www.google.com/search?q=artificial+neuron+model&sxsrf=ALiCzsbsj5vQsnF9UEYEfEEJkeWyV7Fxew:1661360745318&source=lnms&tbm=isch&sa=X&ved=2ahUKEwih1cWI-9_5AhUryHMBHXuQBwgQ_AUoAnoECAEQBA&biw=1920&bih=872&dpr=1#imgrc=NQN08PUMKsmfNM
[28] https://www.google.com/search?q=single+layer+feed+forward+network&tbm=isch&ved=2ahUKEwj-lsSJ-9_5AhVmj9gFHTxNBlEQ2-cCegQIABAA&oq=single+layer+feed+forward+network&gs_lcp=CgNpbWcQAzIECCMQJzIFCAAQgAQyBggAEB4QCDIECAAQGDIECAAQGDoHCCMQ6gIQJzoECAAQQzoICAAQsQMQgwE6CggAELEDEIMBEEM6CAgAEIAEELEDOgcIABCxAxBDOgsIABCABBCxAxCDAVC5BVjuSGD9SmgBcAB4BIABkgGIAZMokgEEMC4zOZgBAKABAaoBC2d3cy13aXotaW1nsAEKwAEB&sclient=img&ei=a1oGY761F-ae4t4PvJqZiAU&bih=872&biw=1920#imgrc=4nIaf3Taw-JONM

[29] https://www.google.com/search?q=multilayr+feed+forqard+neural+network&tbm=isch&ved=2ahUKEwj_k4Gm-9_5AhXtjtgFHSlGAvYQ2-cCegQIABAA&oq=multilayr+feed+forqard+neural+network&gs_lcp=CgNpbWcQAzoECCMQJzoFCAAQgAQ6BggAEB4QCDoECAAQGDoHCCMQ6gIQJzoECAAQQzoICAAQgAQQsQM6CwgAEIAEELEDEIMBOgcIABCxAxBDOggIABCxAxCDAToGCAAQChAYUNEEWIBLYMJNaAFwAHgEgAGJAYgBsS2SAQQwLjQ3mAEAoAEBqgELZ3dzLXdpei1

pbWewAQrAAQE&sclient=img&ei=p1oGY7-kBu2d4t4PqYyJsA8&bih=872&biw=1920#imgrc=uWGG1MUHW9OgYM

[30] https://www.google.com/search?q=recurrent+neural+network&tbm=isch&ved=2ahUKEwjQxIrH-9_5AhUzj9gFHaHoDgYQ2-cCegQIABAA&oq=recurrent+neural+network&gs_lcp=CgNpbWcQAzIECCMQJzIFCAAQgAQyBQgAEIAEMgUIABCABDIFCAAQgAQyBQgAEIAEMgUIABCABDIFCAAQgAQyBQgAEIAEMgUIABCABDoHCAAQsQMQQzoGCAAQHhAHUJscWME0YPg6aABwAHgAgAGHAYgBwhGSAQQwLjE4mAEAoAEBqgELZ3dzLXdpei1pbWfAAQE&sclient=img&ei=7FoGY9CeHLOe4t4PodG7MA&bih=872&biw=1920#imgrc=LqiijbTFYW9YjM

[31]https://www.google.com/search?q=multi+layer+feed+forward+backpropagation+network&tbm=isch&ved=2ahUKEwjbkPvm-9_5AhW_k9gFHRdgCxwQ2-cCegQIABAA&oq=multi+layer+feed+forward+backpropagation+network&gs_lcp=CgNpbWcQAzoECCMQJzoFCAAQgAQ6BwgjEOoCECc6BAgAEEM6CAgAEIAEELEDOgcIABCxAxBDOgsIABCABBCxAxCDAToECAAQGFCOA1ijWGCbW2gBcAB4BIABkAGIAeg0kgEEMC41NJgBAKABAaoBC2d3cy13aXotaW1nsAEKwAEB&sclient=img&ei=L1sGY5u9E7-n4t4Pl8Ct4AE&bih=872&biw=1920#imgrc=_dClf_Kyi4MgnM

[32]https://www.google.com/search?q=12+v+lead+acid+battery+images&tbm=isch&ved=2ahUKEwi1nrTk_N_5AhWBieYKHZAXBrwQ2-cCegQIABAA&oq=12+v+lead+acid+battery+images&gs_lcp=CgNpbWcQAzoECCMQJzoGCAAQChAYOgQIABAeUOcCWNULYPoMaABwAHgAgAF8iAHCB5IBAzAuOJgBAKABAaoBC2d3cy13aXotaW1nwAEB&sclient=img&ei=NlwGY7WrGIGTmgeQr5jgCw&bih=872&biw=1920#imgrc=7bjWcJaauc0VnM

[33]https://www.google.com/search?q=digital+multimeter&tbm=isch&ved=2ahUKEwiR-aOI_d_5AhVBjNgFHRU8B_0Q2-cCegQIABAA&oq=digital+multi&gs_lcp=CgNpbWcQARgAMggIABCABBCxAzIFCAAQgAQyBQgAEIAEMgUIABCABDIFCAAQgAQyBQgAEIAEMgUIABCABDIFCAAQgAQyBQgAEIAEMgUIABCABDoECCMQJzoLCAAQgAQQsQMQgwE6CAgAELEDEIMBOgQIABBDOgoIABCxAxCDARBDOgcIABCxAxBDOgYIABAeEAVQtxBYmSxgxTxoAXAAeACAAYwBiAHwDpIBBDAuMTWYAQCgAQGqAQtnd3Mtd2l6LWltZ8ABAQ&sclient=img&ei=gVwGY9G0JsGY4t4Plfic6A8&bih=872&biw=1920#imgrc=38p9TXBwtk5luM

[34]https://www.google.com/search?q=digital+dc+voltmeter&tbm=isch&ved=2ahUKEwjV-NWx_d_5AhW4jtgFHbUkDvEQ2-cCegQIABAA&oq=digital+dc+voltmeter&gs_lcp=CgNpbWcQAzIFCAAQgAQyBQgAEIAEMgUIABCABDIFCAAQgAQyBggAEB4QBTIGCAAQHhAIMgYIABAeEAgyBggAEB4QCDIGCAAQHhAIMgYIABAeEAg6BAgjECc6BAgAEEM6CwgAEIAEELEDEIMBOggIABCABBCxAzoICAAQsQMQgwFQqAVYpyZg8CdoAHAAeACAAaYBiAHjF5IBBDAuMjOYAQCgAQGqAQtnd3Mtd2l6LWltZ8ABAQ&sclient=img&ei=2FwGY9WsGrid4t4Ptcm4iA8&bih=872&biw=1920#imgrc=1rfl0enY0EML_M

[35]https://www.google.com/search?q=digital+dc+ammeter&tbm=isch&ved=2ahUKEwj0ovqd_t_5AhUTK7cAHTqoCUYQ2-cCegQIABAA&oq=digital+dc+ammeter&gs_lcp=CgNpbWcQAzIFCAAQgAQyBggAEB4QCDIECAAQGDIECAAQGDoECCMQJzoGCAAQHhAHOgYIABAeEAU6CAgAEIAEELEDOggIABCxAxCDAVCmBljvHWCMH2gCcAB4AIABxQGIAdwQkgEEMC4xN5gBAKABAaoBC2d3cy13aXotaW1nwAEB&sclient=img&ei=u10GY7TZH5PW3LUPutCmsAQ&bih=872&biw=1920#imgrc=2mFiC1YTdAybdM

[36]https://www.google.com/search?q=rheostat&tbm=isch&ved=2ahUKEwiDhazJ_t_5AhVIyKACHbi3CfYQ2-cCegQIABAA&oq=rheostat&gs_lcp=CgNpbWcQAzIHCAAQsQMQQzIECAAQQzIICAAQgAQQsQMyBAgAEEMyBQgAEIAEMgUIABCABDIFCAAQgAQyBQgAEIAEMgUIABCABDIFCAAQgAQ6BAgjECc6BggAEB4QCDoECAAQGDoHCCMQ6gIQJzoICAAQsQMQgwE6CwgAEIAEELEDEIMBUKsFWN4eYIciaAFwAHgEgAGFAYgBqhaSAQQwLjI0mAEAoAEBqgELZ3dzLXdpei1pbWewAQrAAQE&sclient=img&ei=Fl4GY4OiH8iQg8UPuO-msA8&bih=872&biw=1920#imgrc=FY-zbw3ypKYOtM

10.0 APPENDIX

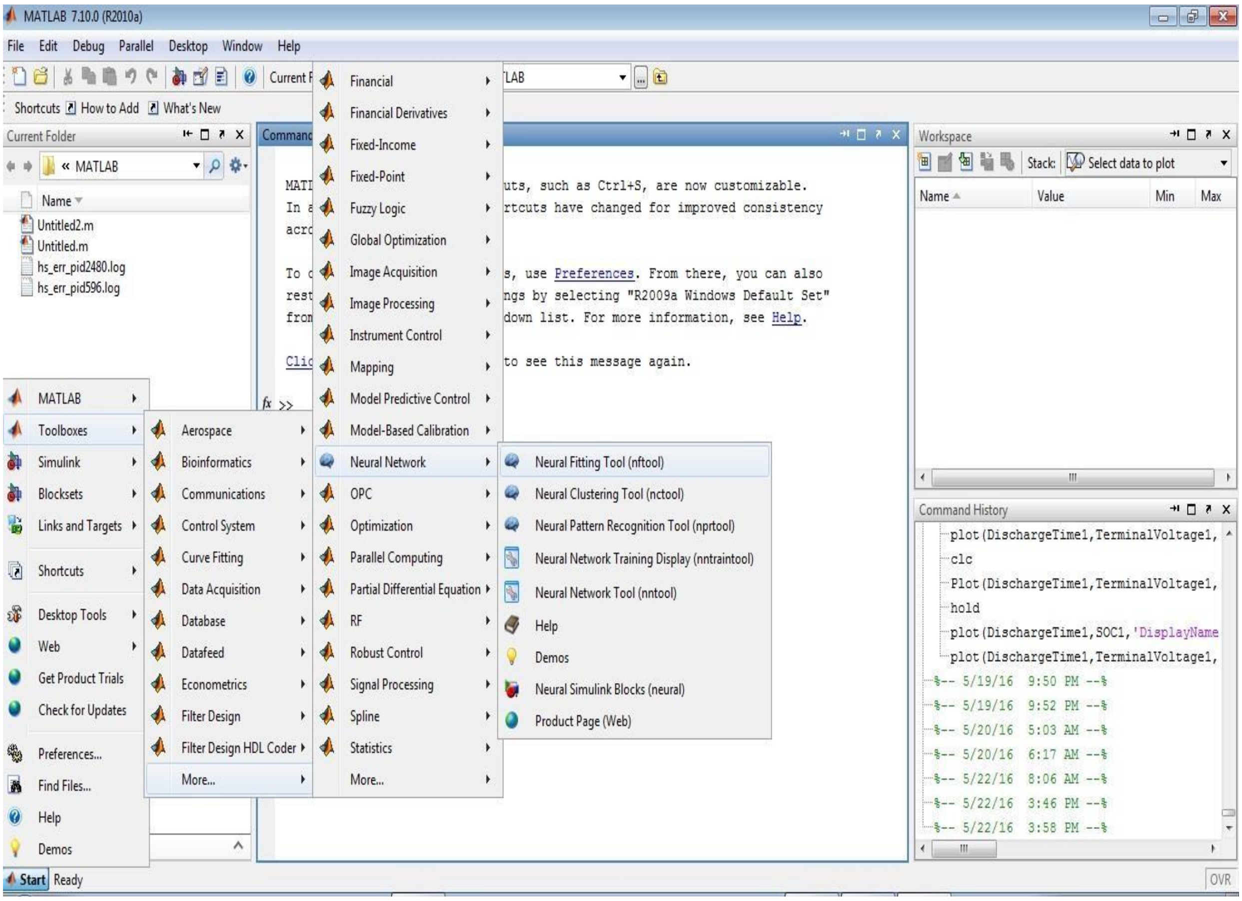
Neural Network Fitting Tool GUI:

To solve the input-output problems, we use the neural network fitting tool (nftool).It is required to map between a data set of inputs and output targets. With the help of neural network fitting tool, we can select data, create and train a network and evaluate its performance based on mean square error and regression analysis.

A two-layer feed-forward network with sigmoid hidden neurons and linear output neurons, with consistent data and enough neurons in it's hidden layers, can fit multi-dimensional mapping problems arbitrarily well. The network will be trained with Levenberg-Marquardt backpropagation algorithm.

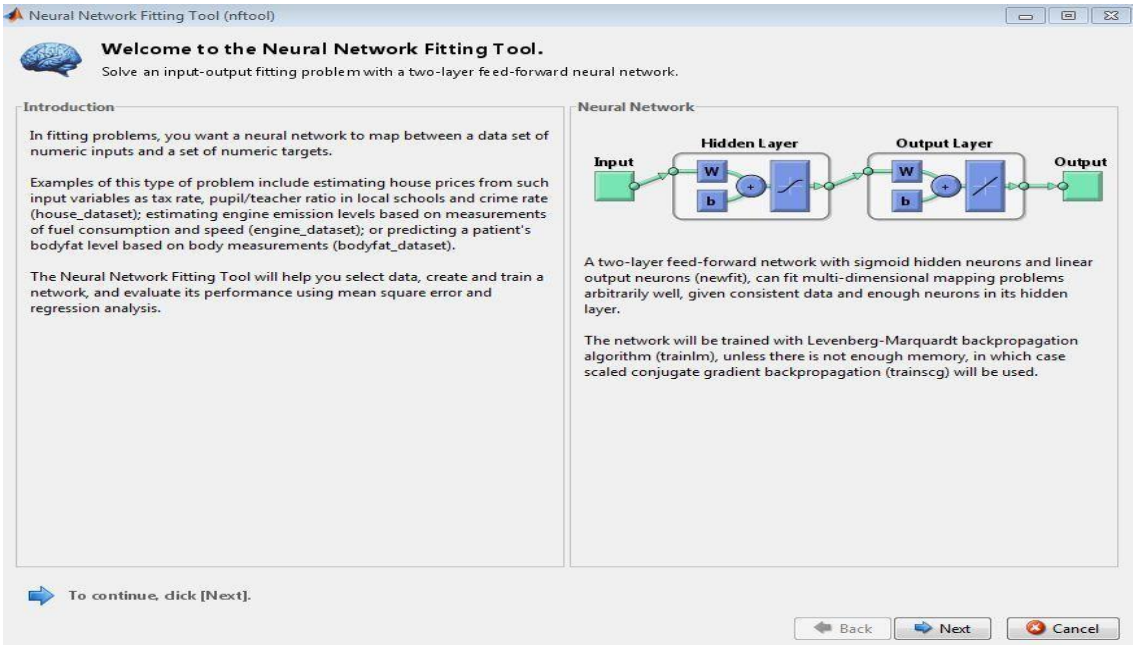
The following procedures to solve an input-output fitting problem in neural network fitting are as follows:

- We first select the neural network fitting tool from neural network toolbox in MATLAB.



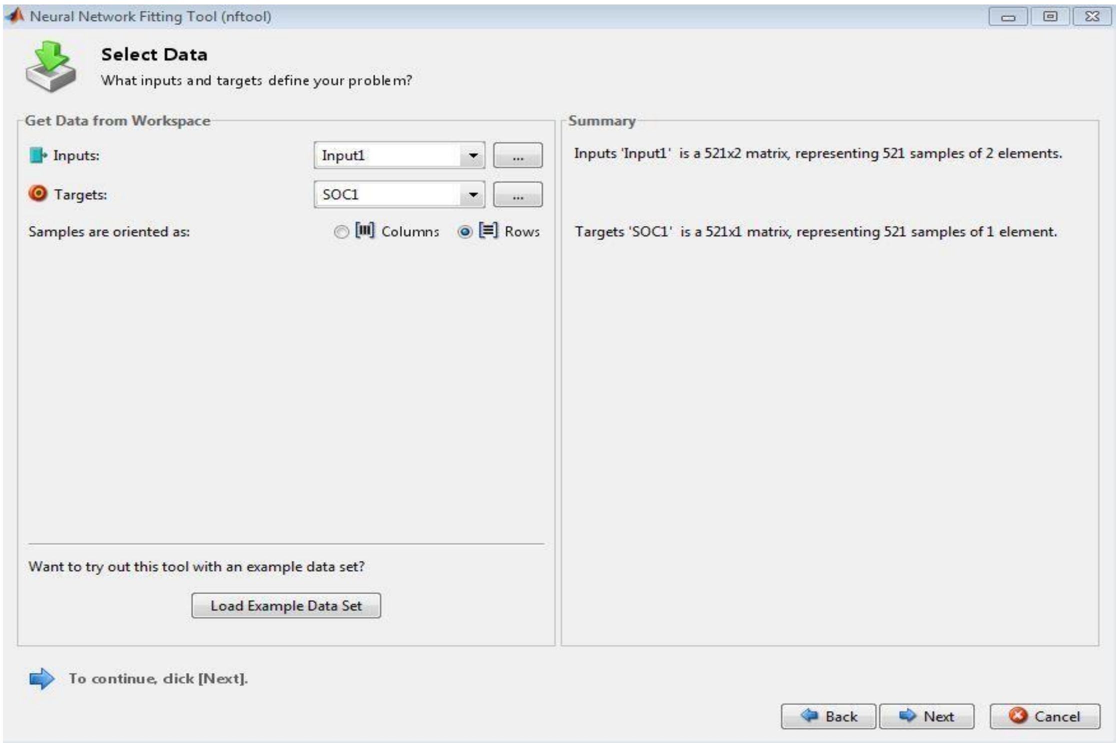
Opening of the Neural network fitting tool

- Select Next after the Neural Network Fitting Tool is opened..



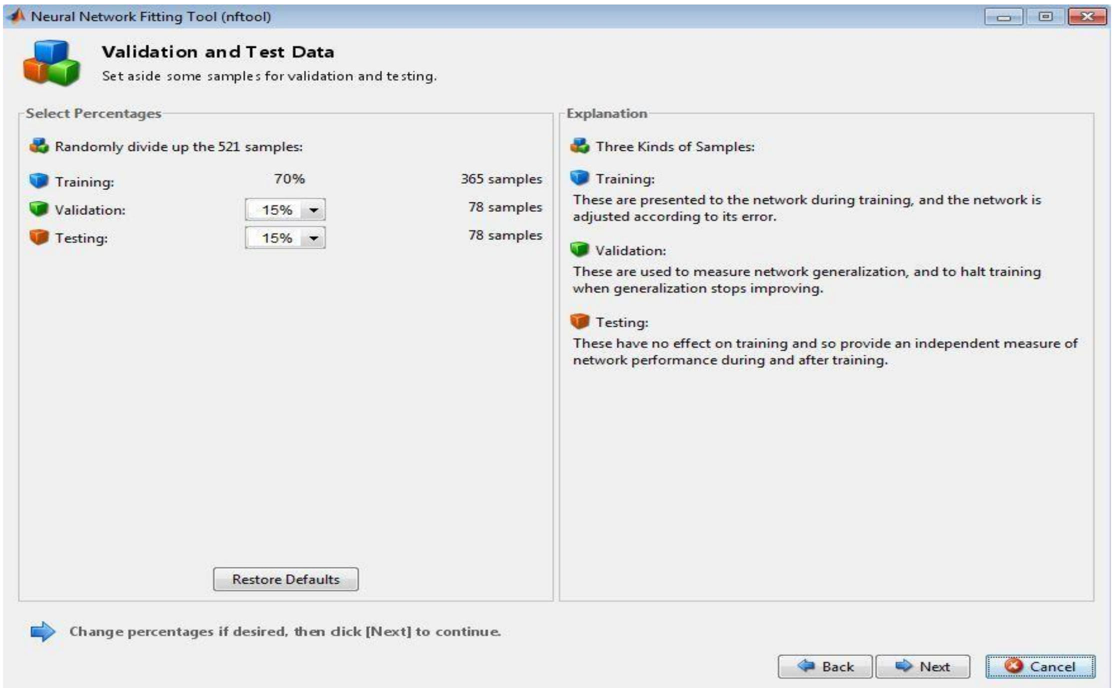
Neural network fitting tool window

- After that, we see the **'Select Data'** window, where we can select the inputs and corresponding target outputs from the previously saved data. After that,we click **Next.**



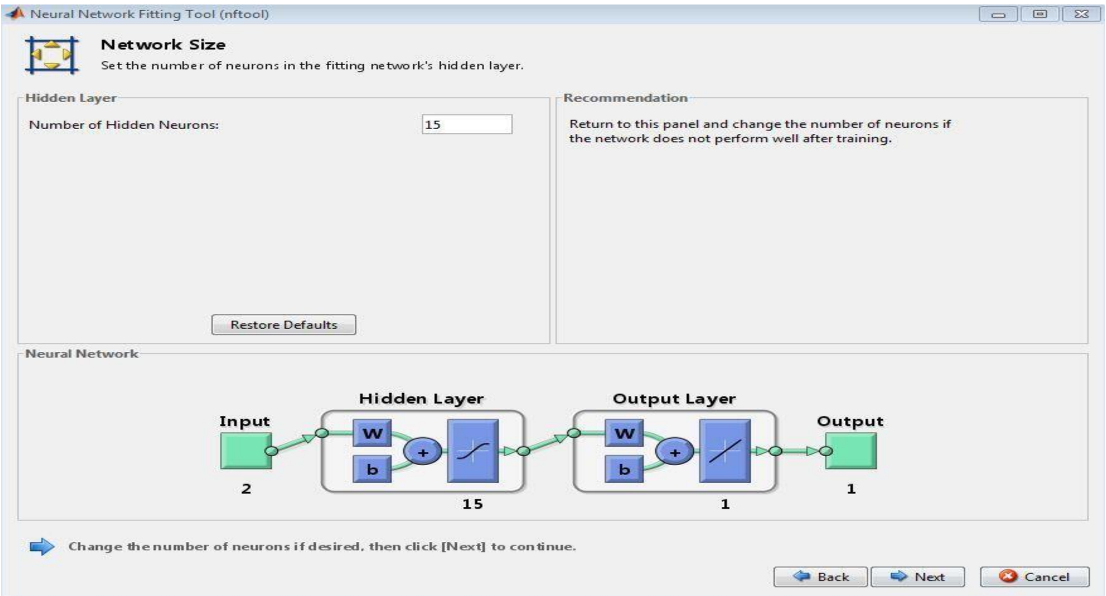
Select data window

- After that, the **'Validation and Test Data'** window. The samples of data is randomly divided up for 'Training' (to optimize the network), 'Validation' (to stop training for best generalization), and 'Testing' (to evaluate the network performance). By default it is 70%, 15%, and 15% of total samples respectively. But, we can change the values as per requirements. After that, we click **Next.**



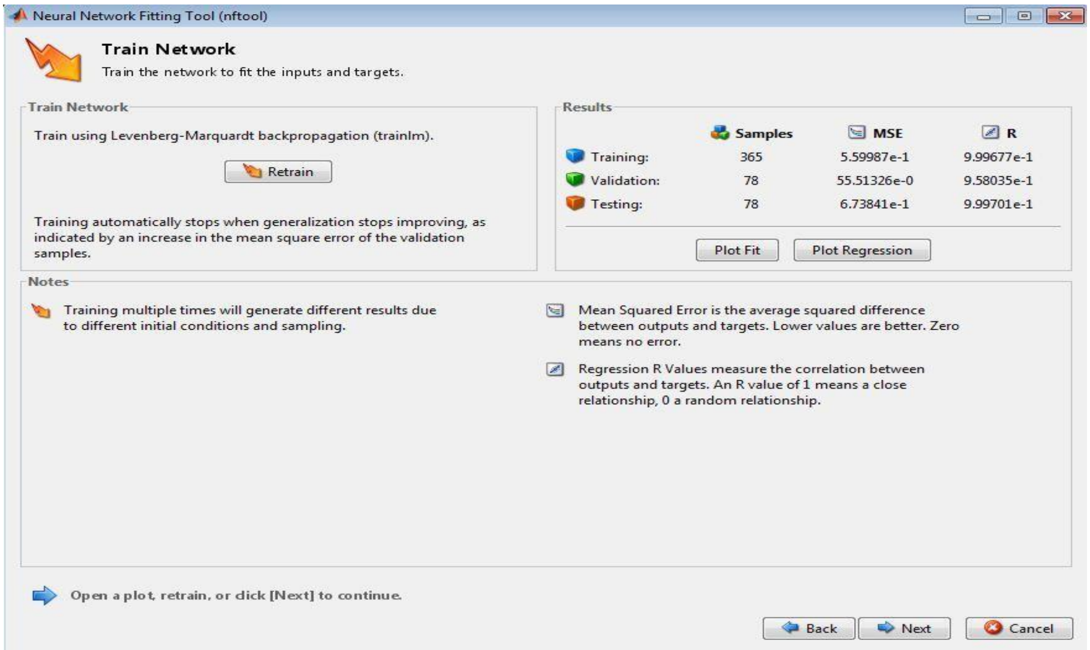
Validation and test data window

- Here we see and can set the the number of neurons in the fitting network's hidden layer. The diagram of the network also appears in this **'Network Size'** window. After selecting the hidden neurons, click **Next.**



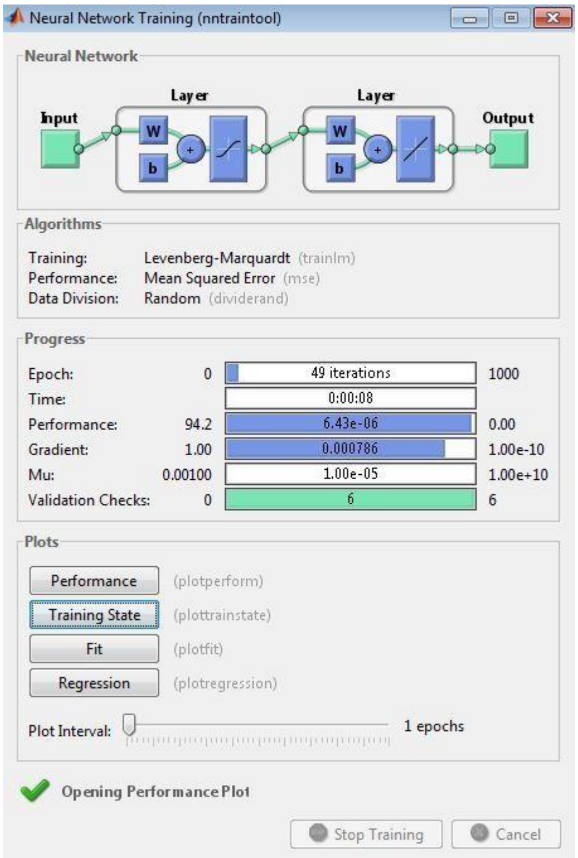
Network size select window

- After that, the 'Train Network' window appears.We now train the network by clicking **Train**.



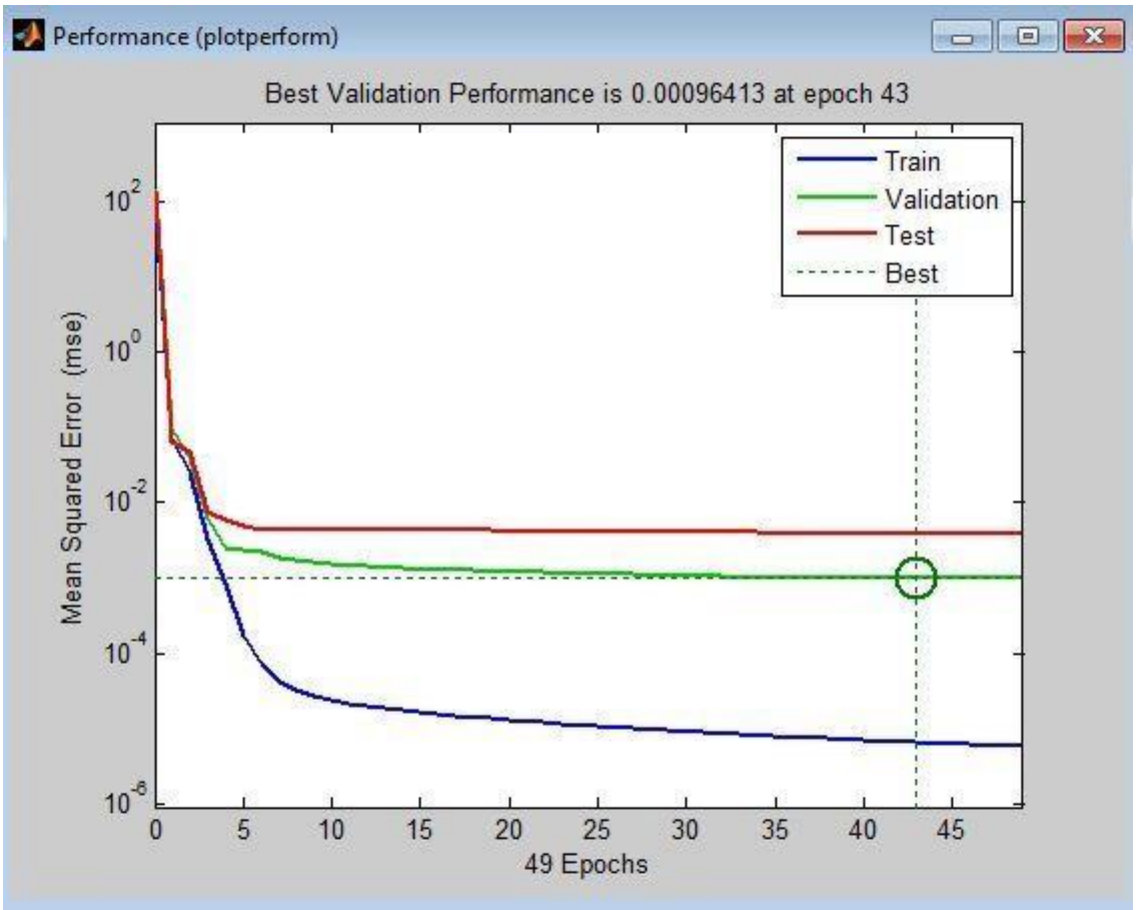
Train network window

Afrer that, we plot the Regression and Fit from this window. If we see that the outputs are not matching the targets accurately, then we retrain the network for accurate results. Another window **'Neural Network Training Display'** tool opens during training.



Neural network training display window

From this window we know about the training. This training stopped when the validation error increased for six iterations, which occurred at iterations 49 is shown in Figure A7. By clicking the Performance in the training window, a plot of the training errors, validation errors and test errors appears, as shown in the Figure A8.



Training error and epochs

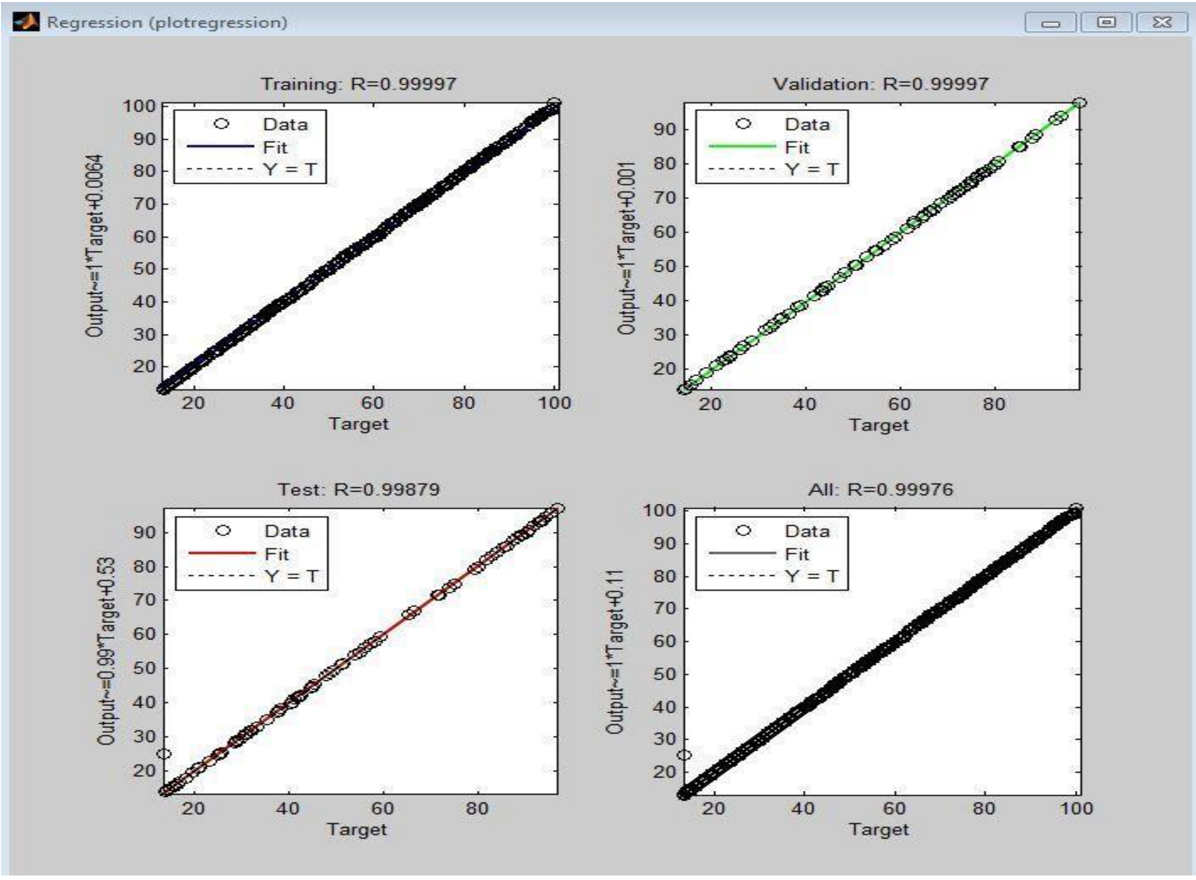
From this case, we see that the result is reasonable, because of the following considerations:

(i) Small final mean square error.

(ii) Similar characteristics present in both test set error and the validation set error.

(iii) We find no occurring of significant overfitting by iteration 43 (where the best validation performance occurs).

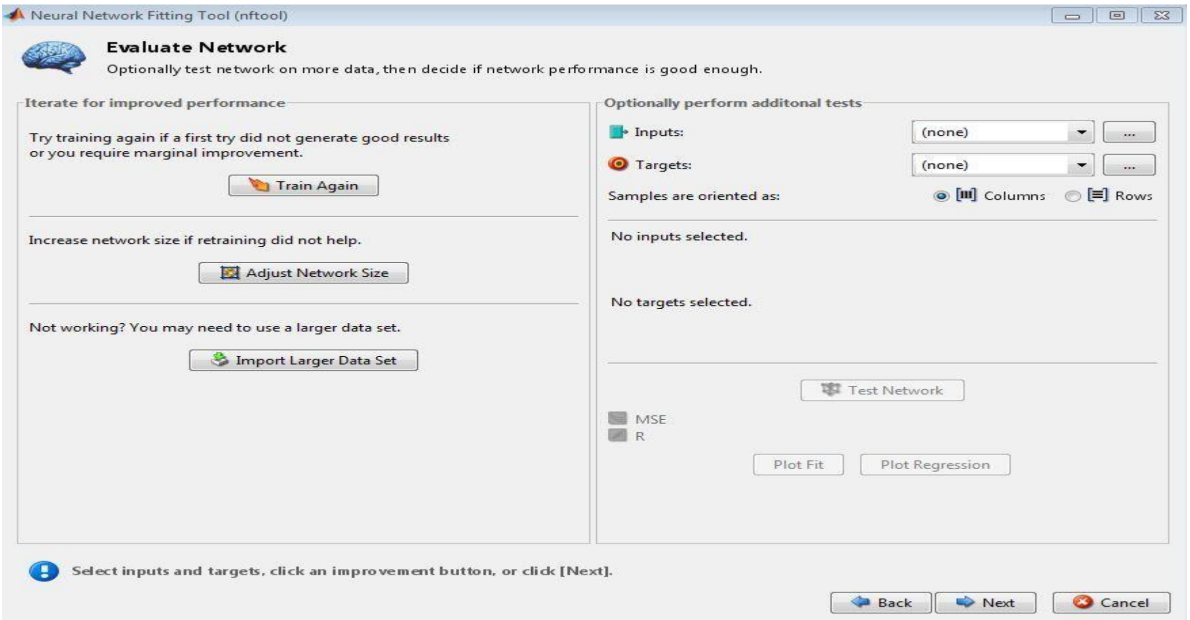
By clicking Regression, we can also perform a linear regression between the network outputs and the corresponding targets . It is clear from the figure below, that the output tracks and the target are very well for training, validation and testing, and the R-value is almost near 1.00 for the total response. Therefore, a most accurate result has been performed.



Regression Plot

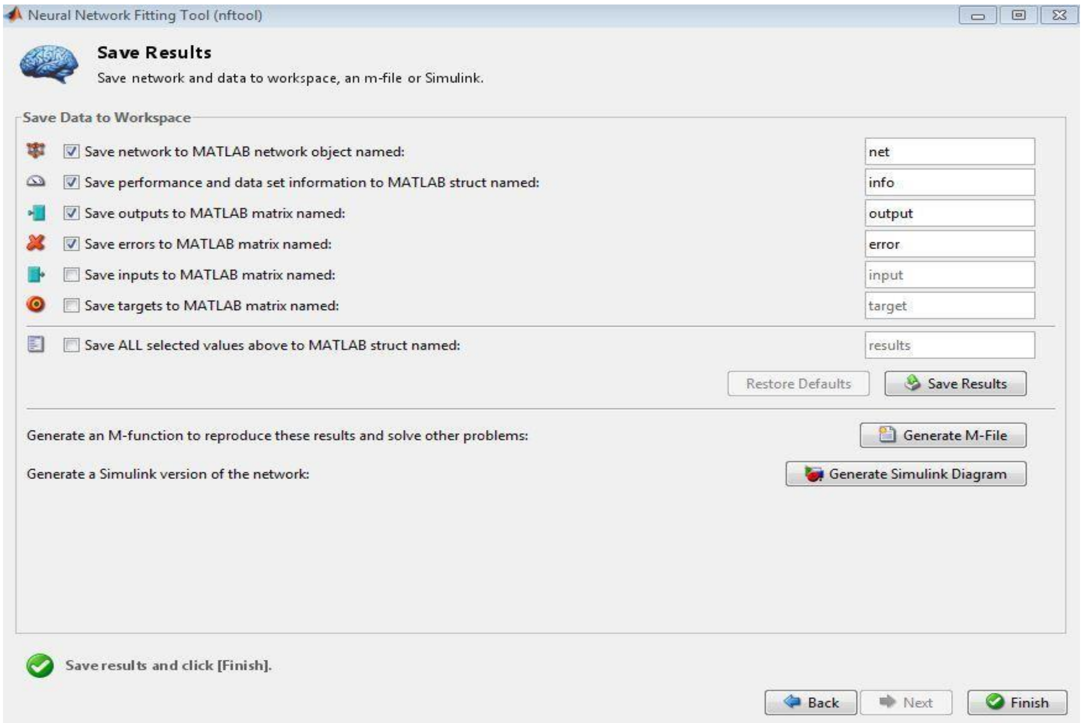
In the "Train Network" window, we click **Next**.

- If we see that the outputs till not give the best performance, **'Evaluate Network'** window helps solving the problem. It imports larger data set, adjust network size and train again to obtain the best result. Then we click **Next.**



Evaluate network window

- Finally we save the network by clicking the Finish.



Save results window

Mean Square Error (MSE):

The average of the square of errors is called Mean Square Error.It is the difference between the estimator and what is estimated.

Here ,the estimator is the mathematical function mapping of a sample data to a parameterof the population from which the data is sampled. The MSE assesses the quality of an **estimator**. Definition of an MSE differs according to whether one is describing an estimator or a predictor.

MSE of Predictor:

If $\hat{Y}$ is a vector of n predictions, and Y is the vector of observed values Ehich corresponds to the inputs of the function, then the MSE of the predictor can be estimated by

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(\hat{Y} - Y)^2$$

i.e., the MSE is the *mean* $(\frac{1}{n}\sum_{i=1}^{n})$ of the *square of the errors* $(\hat{Y} - Y)^2$. This is an easily computable quantity for a particular sample (and hence is sample-dependent)

MSE of Estimator

The MSE of an estimator $\hat{\theta}$ with respect to an unknown parameter θ is defined as

$$\operatorname{MSE}(\hat{\theta}) = \operatorname{E}\left[(\hat{\theta} - \theta)^2\right].$$

Here, MSE is a property of an estimator as the definition depends on unknown parameter.

Regression

We use regression to determine a linear relationship between two variables. It is also used for estimation and prediction.

From the data given by *(x, y)* ,the estimation or prediction of the average value of a variable; say *y*, corresponding to a specified value of the other variable *x*, is called regression. We can also predict the average value of *x* corresponding to a specified value of *y*.

Regression line and linear curve fitting:

It is a suitable curve or a straight line, which fit best the scatter diagram of the bivariate datas given by *(x, y)*, from which the prediction of average value of one variable in respect with other variable is obtained. Regression line is also called linear curve fitting.

The line which is fit and used to predict the average value of *y* depending on *x* is called Regression line of *y* on *x*. Similarly, the line which is used to predict the values of *x* depending on *y* is called Regression line of *x* on *y*.

Regression equation is the line which is used to represent regression equation. It is obtained by a method known as 'Method of Least Square'.

Theorem:(Fitting the Regression Line)

Let (x_1, y_1), (x_2, y_2), (x_3, y_3), ……………… (x_n, y_n) be *n* pairs of observationsgiven by the bivariate data *(x,y)*. Let $\bar{x}$, $\bar{y}$ be the means; σ_x, σ_y be the standard deviations of the values of *x* and *y* respectively; r_{xy} is correlation coefficient. Equation will be:

(i) Regression line of y on x is

$y - \bar{y} = b_{yx}(x - \bar{x});$ where $b_{yx} = r_{xy}\frac{\sigma y}{\sigma x}$

(ii) Regression line of x on y is

$(x - \bar{x}) = b_{yx}(y - \bar{y});$ where where $b_{xy} = r_{xy}\frac{\sigma x}{\sigma y}$

Example:

Let us consider the 10 sets of bivirate data (Target output, Simulated Output):

Target Outputs	98.33	95.00	91.66	85.00	78.33	66.66	60.00	55.00	46.66	43.33

Simulated Outputs	98.38	95.14	9179	84.68	78.01	66.45	60.09	54.92	46.66	43.33

Let x = Target Outputs, y = Simulated Outputs.

Calculation for Regression Lines

x	y	x^2	y^2	xy
98.33	98.38	9668.789	9678.624	9673.705
95	95.14	9025	9051.62	9038.3
91.66	91.79	8401.556	8425.404	8413.471
85	84.68	7225	7170.702	7197.8
78.33	78.01	6135.589	6085.56	6110.523
66.66	66.45	4443.556	4415.603	4429.557
60	60.09	3600	3610.808	3605.4
55	54.92	3025	3016.206	3020.6
46.66	46.66	2177.156	2177.156	2177.156
43.33	43.33	1877.489	1877.489	1877.489
$\sum x = 719.97$	$\sum y = 719.45$	$\sum x^2 = 55579.13$	$\sum y^2 = 55509.17$	$\sum xy = 55544$

The number of pairs = 10

$\bar{x} = \frac{1}{10} \sum x = 71.997$,

$\sigma_x^2 = \frac{1}{n}\sum x^2 - (\frac{1}{n}\sum x)^2$

$= \frac{1}{10} \times 55579.13 - (71.997)^2$

$= 5557.913 - 5183.568$

$= 374.3453$

$\sigma_x = 19.348$

$\bar{y} = \frac{1}{10}\sum y = 71.945$,

$\sigma_y^2 = \frac{1}{n}\sum y^2 - (\frac{1}{n}\sum y)^2$,

$= \frac{1}{10} \times 55509.17 - (71.945)^2$

$= 5550.917 - 5176.083$

$= 374.8342$

$\sigma_y = 19.36063$

Covariance, Con(x,y) $= \frac{1}{n}\sum xy - (\frac{1}{n}\sum x)(\frac{1}{n}\sum y)$

$= \frac{1}{10} \times 55544 - (71.997)(71.945)$

$= 5554.4 - 5179.824$

$= 374.576$

Correlation Coefficient, $r_{xy} = \frac{Cov\ (x,y)}{\sigma x \sigma y} = \frac{374.576}{374.5897} = 0.999963$

Regression Coefficient, $b_{yx} = r_{xy}\frac{\sigma y}{\sigma x}$

$$= 0.999963 \times \frac{19.36063}{19.348}$$

$$= 0.999963 \times 1.000653$$

$$= 1.000616$$

So, the Regression line of y on x is ,

$y - \bar{y} = b_{yx} (x - \bar{x})$

y – 71.945 = 1.000616 (x – 71.997)

y = 1.000616x – 72.04136 + 71.945

$y = 1.000616\, x – 0.09636$

i.e. Outputs = 1.000616 × (Target outputs) – 0.09636

- -