Ref. No.: EX/MET/MATH/T/214/2019

BACHELOR OF METALLURGICAL ENGINEERING EXAMINATION – 2019

(2^{ND} YEAR 1^{ST} SEMESTER)

MATHEMATICS – IIIN

FULL MARKS : 100 **TIME : 3 HOURS**

Answer Question No. 11 and any *six* questions from the rest.

(Notations have their usual meanings)

1. Solve the following differential equation:

(a) $\left(\frac{2x^2}{y}+\frac{x}{y}\right)dx + 2xdy = 0$

(b) $x^n\frac{dy}{dx}+\frac{n}{x}y = x^m$

(c) $3y^2\frac{dy}{dx} + y^3 = e^{-x}$ 5+5+6

2. Solve the differential equation:

(a) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2e^{3x}$

(b) $(x^2D^2 - xD - 3)y(x) = x^2\, logx$ 8+8

3. (a) Solve the wave equations

$$\frac{\partial^2 u}{\partial t^2}(x,t) = c^2\frac{\partial^2 u}{\partial x^2}(x,t) \text{ for all } 0 < x < 1 \text{ and } t > 0 \text{ where,}$$

$u(0,t) = u(1,t) = 0$ for all $t > 0$,
$u(x,0) = x(1-x)$ for all $0 < x < 1$,
$u_t(x,0) = 0$ for all $0 < x < 1$.

(b) Determine the series solution of the equation about x = 0

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + x^2y = 0.$$ 8+8

4. (a) Obtain the solution of wave equation

$$\frac{\partial^2 u}{\partial t^2}(x,t) = c^2\frac{\partial^2 u}{\partial x^2}(x,t) \text{ for all } 0 < x < 1 \text{ and } t > 0 \text{ where,}$$

$u(0,t) = u(1,t) = 0$ for all $t > 0$,
$u(x,0) = \sin(5\pi x) + 2\sin(7\pi x)$ for all $0 < x < 1$,
$u_t(x,0) = 0$ for all $0 < x < 1$.

(b) Solve the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = sin3x + x^2$ 8+8

5. (a) Use the convolution theorem to evaluate $L^{-1}\left\{\frac{1}{(s^2+1)(s^2+9)}\right\}$.

(b) Solve the following differential equation by using Laplace transformation

y"(t) + y(t) = sin 2t , if y(0) = 0, y'(0) = 1. 8+8

[Turn over

6. (a) If $x+\frac{1}{x}=2\cos\frac{\pi}{7}$, then show that $x^7+\frac{1}{x^7}=-2$.

(b) If z_1, z_2 be non-zero complex numbers, then

$$\arg\,(z_1z_2)=\arg\,z_1+\arg z_2+2k\pi,\quad \text{where}$$

$$k=0\; if\; -\pi<\arg z_1+\arg z_2\le\pi,$$

$$k=1\; if\; \arg z_1+\arg z_2\le-\pi,$$

$$k=-1\; if\; \arg z_1+\arg z_2>\pi.$$

(c) Show that

$$32sin^4\theta\, cos^2\theta = cos6\theta-2cos4\theta-cos2\theta+2.$$ 5+5+6

7. (a) State and Prove De Moivre's theorem.

(b) Find the general solution of $\sinh z=2i$.

(c) Find the general solution of i^i. 7+5+4

8. (a) Determine the series solution of the equation about x = 0

$$2x^2\frac{d^2y}{dx^2}+(2x^2-x)\frac{dy}{dx}+y=0.$$

(b) Find the Laplace transform of

(i) sin2t cos3t

(ii) $(t+2)^2 e^t$. 8+4+4

9. (a) Prove that

$$J_n(x)=\frac{1}{\pi}\int_0^{\pi}cos(n\theta-xsin\theta)d\theta$$

(b) Prove that

$$J_n'(x)=\frac{1}{2}[J_{n-1}(x)-J_{n+1}(x)]$$ 10+6

10. (a) Prove that $\int_{-1}^{1}P_m(x)P_n(x)dx=\begin{cases}\frac{2}{2n+1} & when\; m=n\\ 0 & when\; m\neq n\end{cases}$

(b) Prove that $(n+1)P_{n+1}(x)=(2n+1)xP_n(x)-nP_{n-1}(x)$ 10+6

11. Find out the solution of the differential equation

$$(2xy^4e^y+2xy^3+y)dx+(x^2y^4e^y-x^2y^2-3x)dy=0.$$ 4