b) Show that

$$\int_0^{\pi/2} \sin^m \theta \cos^n \theta d\theta = \frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right).$$

c) Evaluate $\int_0^1 x^3 (1-x^2)^{5/2} dx.$ 3+3+4

13. a) Examine the following sequence for convergence :

$$a_n = \frac{n^2 - 2n}{3n^2 + n}.$$

b) Discuss the convergence of the series :

$$1 + \frac{2!}{2^2} + \frac{3!}{3^2} + \frac{4!}{4^2} + \cdots \infty.$$

c) Test for convergence of the series :

$$\Sigma \frac{(n!)^2}{(2n)!} x^{2n}.$$

3+3+4

BACHELOR OF ENGINEERING IN METALLURGICAL & MATERIAL ENGINEERING EXAMINATION, 2019

(1st Year, 1st Semester, Old)

MATHEMATICS - IN

Time : Three hours Full Marks : 100

(50 marks for each Part)

Use a separate Answer-Script for each Part

PART - I

Answer ***any five*** questions.

1. a) Find the dimensions of a rectangle with perimeter 1000 meters so that the area of the rectangle is maximum.

b) Find the maximum area of a rectangle that can be inscribed in the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$. Assume that the sides of the rectangle are parallel to the axes. 5+5

2. a) Define Continuity of several variables. Consider the two variables real-valued function $f(x, y) = \frac{x^2 + y^2 - x^3 y^3}{x^2 + y^2}$ $\forall (x, y) \in \mathbf{R}^2 \setminus \{(0,0)\}$. Determine whether $\boldsymbol{f}$ cna be redefined as $\boldsymbol{f}^{\wedge}$ at the origin such that $\boldsymbol{f}^{\wedge}$ is continuous $\forall (x, y) \in \mathbf{R}^2$.

b) Define Directional Derivative of a two variables function. Find the directional derivative of $f(x,y)=2x^2-xy+5$ at (1, 1) in the direction of unit vector $\beta=\frac{1}{5}(3,-4)$. 5+5

3. a) Examine the existence of the limit :

$$\lim_{(x,y)\to(0,0)}\frac{x^2}{x^2+y^2}.$$

b) Define : i) Infinite one sided limit, ii) Double limit. 5+5

4. a) If $y=x\log\left(\frac{x-1}{x+1}\right)$, show that

$$y_n=(-1)^{n-2}(n-2)!\left[\frac{x-n}{(x-1)^n}-\frac{x+n}{(x+1)^n}\right].$$

b) Find the nth derivative of $\frac{x}{x^2+x+1}$. 5+5

5. a) Find the Maclaurin series for $\ln(1+x)$ and hence that for

b) Verify Cauchy's MVT for the functions : sin x and cos x in [a, b]. 5+5

6. a) Show that the 1st order partial derivatives exist but the function is not continuous for the function

$$f(x,y)=\begin{cases}\frac{xy}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}.$$

b) $\int_0^\infty \frac{\log(1+x^2)}{1+x^2}dx$ 5+5

10. a) Find the length of the arc of the parabola $x^2=4ay$ measured from the vertex to one extremity of the latus-rectum.

b) Change the order of integration in

$$I=\int_0^{4a}\int_{x^2/(4a)}^{2\sqrt{ax}} dydx,$$

and hence evaluate. 5+5

11. a) Evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dxdy$ by changing to polar coordinates.

b) Establish the relation

$$\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}.$$ 5+5

12. a) Show that

$$\beta(m,n)=2\int_0^{\pi/2}\sin^{2m-1}\theta\cos^{2n-1}\theta d\theta, m,n>0.$$

PART - II

(Notations/Symbols have their usual meanings)

Answer ***any five*** questions.

7. Evaluate the following integrals, if possible :

a) $\int_{-\infty}^{\infty} xe^{-x^2}dx$

b) $\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}}$

c) $\int_{1}^{\infty} \frac{dx}{x^2}$ 4+3+3

8. a) Evaluate $\int_{0}^{\infty} \log\left((x+\frac{1}{x}\right)\cdot\frac{dx}{1+x^2}$.

b) Show that the following integral does not exist in general sense, however, it exists in Cauchy principal value sense :

$\int_{-1}^{2} \frac{dx}{x}$. 6+4

9. Evaluate the following integrals :

a) $\int_{0}^{\pi} \frac{x\sin^3 x}{1+\cos^2 x}dx$

b) Expand $\log_e x$ in powers of $(x-1)$ and hence deduce $\log_e 1.1$ correct to 4 decimal places. 5+5