b) Show that the system of equations

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

will have (i) unique solution $\lambda = 3,$ if for any value of μ ;

ii) an infinite number of solutions if $\lambda = 3,$ for $\mu = 10$;

iii) no solution if $\lambda = 3,$ for $\mu \neq 10$. 4+6

6. a) Show that

$$\begin{vmatrix} a^2+\lambda & ab & ac \\ ab & b^2+\lambda & bc \\ ac & bc & c^2+\lambda \end{vmatrix}$$ is divisible by and find the remaining factor.

b) Use Cayley-Hamilton theorem to find the inverse of the matrix

$$A = \begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{vmatrix}$$ 5+5

7. If $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, then show that $A^{50} = \begin{pmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 5 \end{pmatrix}$.

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Ex/Met/Math/T/122/2019 (Old)

Bachelor of Engineering in Metallurgical Engineering Examination, 2019

(1st Year, 2nd Semester, Old)

Mathematics IIN

Time : Three hours | Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each part

Notation/Symbols have their usual meanings.

PART - I

(Answer ***any five*** questions)

1. a) Show that the necessary and sufficient condition for two proper vectors to be perpendicular is that their scalar product vanishes.

b) Show that the three points $\hat{i} - 2\hat{j} + 3\hat{k},\ 2\hat{i} + 3\hat{j} - 4\hat{k}$ and $-7\hat{j} + 10\hat{k}$ are collinear.

c) Show that the line joining the mid points of two sides of a triangle is parallel to the third side and half of its length.

3+3+4

2. a) Given that $\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k},$ find $\vec{a} \times \vec{b},$ and the unit vector perpendicular to both $\vec{a}$ and $\vec{b}.$ Also find the sine of the angle between $\vec{a}$ and $\vec{b}.$

b) Prove, by vector method, that the angle in a semi-circle is a right angle. 5+5

3. a) If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$, find λ such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$.

b) If $\vec{u}(t)$ and $\vec{v}(t)$ are two vector functions of the scalar variable t, then show that

$$\frac{d}{dt}(\vec{u}\cdot\vec{v}) = \vec{u}\cdot\frac{d\vec{v}}{dt} + \frac{d\vec{u}}{dt}\cdot\vec{v}$$

Hence show that if $\vec{u}(t)$ is of constant magnitude, then $\vec{u}$ is perpendicular to $\frac{d\vec{u}}{dt}$. 3+7

4. a) Show that, if $\vec{R} = \vec{a}\sin\omega t + \vec{b}\cos\omega t$, where $\vec{a}, \vec{b}$ and ω are constants, then

i) $\dfrac{d^2\vec{R}}{dt^2} = -\omega^2\vec{R}$

ii) $\vec{R}\times\dfrac{d\vec{R}}{dt} = -\omega\,\vec{a}\times\vec{b}$

b) Find the directional derivative of $f(x, y, z) = xy^3 + yz^3$ at the point $(2, -1, 1)$ in the direction of $\hat{i} + 2\hat{j} + 2\hat{k}$. 6+4

3. a) Verify that the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 2 & 3 & 0 \end{bmatrix}$ satisfies its own characteristic equation and hence find A^{-1}.

b) Find the eigen value of the matrix

$$\begin{pmatrix} 9 & -1 & 9 \\ 3 & -1 & 3 \\ -7 & 1 & -7 \end{pmatrix}.$$

6+4

4. a) Find the rank of the matrix

$$\begin{pmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -1 & -3 & 2 \\ 0 & -2 & 2 & 1 \end{pmatrix}.$$

b) If $A = \begin{pmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 5 \end{pmatrix}$, then show that A is idempotent.

6+4

5. a) Verify that $A = \frac{1}{3}\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & -2 & -1 \end{bmatrix}$ is an orthogonal matrix.

[Turn over

PART - II

Answer ***any five*** questions

1. a) Show that

$$\begin{vmatrix} a^2 & bc & c^2+ca \\ a^2+ab & b^2 & ca \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2.$$

b) Find the value of x, which satisfy the equation

$$\begin{vmatrix} x^3-a^3 & x^2 & x \\ b^3-a^3 & b^2 & b \\ c^3-a^3 & c^2 & c \end{vmatrix} = 0.$$

5+5

2. a) If $A = \frac{1}{3} = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$, prove that $A^T = A^{-1}$.

b) Find the solution of the following system of equation by matrix method

$$\begin{aligned} x+y=z&=4 \\ 2x-y+3z&=1 \\ 3x+2t-z&=1. \end{aligned}$$

5+5

5. a) If $\vec{a}$ and $\vec{b}$ are irrotational, then prove that $\vec{a}\times\vec{b}$ is solenoidal.

b) If $u = x^2yz,\ v = xy - 3z^2$, find

i) $\nabla(\nabla u \cdot \nabla v)$

ii) $\nabla \cdot (\nabla u \times \nabla v)$ 4+6

6. a) Find the work done in moving a particle in the force field $\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along the straight line from (0, 0, 0) to (2, 1, 3).

b) Apply Green's theorem to prove that the area enclosed by a plane curve is $\frac{1}{2}\int_C (xdy - ydx)$. Hence find the area of an ellipse whose semi-major and semi-minor axes are of lengths a and b. 4+6

7. a) If and ψ be two scalar point functions, then show that

$$\nabla \cdot (\varphi\nabla\psi - \psi\nabla\varphi) = \varphi\nabla^2\psi - \psi\nabla^2\varphi.$$

b) A vector field is given by

$$\vec{F} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}.$$

Show that $\vec{F}$ is irrotational and find φ such that $\vec{F} = \text{grad}\varphi.$ 4+6