EX/ FTBE/MATH/T/216/2019

BACHELOR OF ENGINEERING IN FOOD TECHNOLOGY AND BIOCHEMICAL ENGG. EXAMINATION - 2019

(2ND YR. 1ST SEM.)

MATHEMATICS-II

Time: Three hours Full Marks: 100

GROUP-A

Answer any **five** questions $5\times 2 = 10$

1. (a) State Lagrange's Mean value theorem.
(b)State Euler's theorem in several variables.
(c)StateDemoivre's theorem in complex number.
(d)If $f(x) = \frac{|x|}{x}$, the derivative exists at $x = 0$? Justify your answer
(e)State Leibnitz's theorem in the n-th derivative of the product of two functions.
(f)State regular singular point in series solution.
(g) Find the radius of curvature in parametric equations.

GROUP-B

Answer any **Nine** questions $9\times 10 = 90$

2. (a)If $y = e^{\tan^{-1}x} = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots\dots\dots$,find the value of (i) $(1+x^2)y_2 + (2x-1)y_1$; (ii) $(1+x^2)y_{n+2} + \{2(n+1)x-1\}y_{n+1} + n(n+1)y_n$ (iii) $(n+2)a_{n+2} - a_{n+1} + na_n$and (iv) also find $(y_n)_0$
(b)Find the n-th derivative of $y = \frac{\log x}{x}$ 8+2
3. (a)If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then find the value of (i)$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$
(ii) $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u$ and (iii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$
(b) Find the value of i^i. 8+2
4. (a)Using Lagrange's Mean value theorem show that $\frac{x}{1+x} < \log(1+x) < x \ \forall x > 0$
Hence show that $0 < [\log(1+x)]^{-1} - x^{-1} < 1 \ \forall\, x > 0$
(b)Prove that $\sin\left[i\log \frac{a-ib}{a+ib}\right] = \frac{2ab}{a^2+b^2}$ 7+3
5. If ρ_1 and ρ_2 be the radii of curvature at the extremities of two conjugate diameters on an ellipse, then find the value of $\rho_1^{\frac{2}{3}} + \rho_2^{\frac{2}{3}}$ 10
6. (a) If $u = \sin^{-1}\left\{\frac{x^{\frac{1}{3}}+y^{\frac{1}{3}}}{x^{\frac{1}{2}}+y^{\frac{1}{2}}}\right\}^{\frac{1}{2}}$ show that $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right)$
(b)Prove that $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ 7+3

[Turn over

7. Represent (x) , where $f(x) = \cos kx$, on $-\pi \le x \le \pi$ (k not being an integer) in Fourier series. Hence deduce that (i) $\pi \cot k\pi = \frac{1}{k} + 2k \sum_{n=1}^{\infty} \frac{1}{k^2 - n^2}$

(ii) $\frac{\pi}{\sin k\pi} = \sum_{n=0}^{\infty} (-1)^n \left\{ \frac{1}{n+k} + \frac{1}{n+1-k} \right\}$ 10

8. Find the maximum and minimum values of $\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}$, when $lx + my + nz = 0$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. Interpret the result geometrically 10

9. Solve the equation $\frac{d^2y}{dx^2} + (x-1)^2 \frac{dy}{dx} - 4(x-1)y = 0$ in series about the ordinary point $x = 1$. 10

10. Let $f(x) = \cos x, for - \pi \le x \le 0$

$\sin x \ , for\ 0 < x \le \pi$

Obtain the Fourier series of $f(x)\ in\ [-\pi, \pi]$ 10

11. (a)If $f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(\theta x), 0 < \theta < 1$find θ, when $x = 1, f(x) = (1-x)^{\frac{5}{2}}$.

(b) Prove that $\sin ax = ax - \frac{a^3x^3}{3!} + \frac{a^5x^5}{5!} \ldots\ldots\ldots\ldots + \frac{a^{n-1}x^{n-1}}{(n-1)!} \sin\left(\frac{n-1}{2}\pi\right) + \frac{a^nx^n}{n!} \sin\left(a\theta x + \frac{n\pi}{2}\right)$. 3+7

12. Solve the equation $2x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + (1-x^2)y = x^2$ in series. 10