EX/ FTBE/MATH/T/224/2019

BACHELOR OF ENGINEERING IN FOOD TECHNOLOGY AND BIOCHEMICAL ENGG. EXAM. - 2019

(2^{ND}YR. 2^{ND} SEM.)

MATHEMATICS-III

Time: Three hours | Full Marks: 100

GROUP-A(30)

Answer any three questions:

1. (i) $\int_0^{\pi} \frac{x\,dx}{(a^2\cos^2 x + b^2\sin^2 x)^2}$

   (ii) $\int \frac{x^2}{(x^2+a^2)(x^2+b^2)}\,dx$ 6+4
2. The smaller segment of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, cut off by the chord $\frac{x}{a} + \frac{y}{b} = 1$ revolves completely about this chord, find the volume of the solid spindle thus generated. 10
3. Find the volume of the solid generated by revolving cardioide $r = a(1 - \cos\theta)$ about the initial line. 10
4. Find the volume of the solid obtained by the revolution of the cissoid $y^2(2a - x) = x^3$ 10

GROUP-B(30)

Answer any three questions:

5. (a)Solve the following system of equations by Gaussian elimination method: 10

$$3x + 2y + z = 10$$
$$2x + 3y + 2z = 14$$
$$x + 2y + 3z = 14$$

6. The following values of the function $f(x)$for value of x are given: $f(1) = 4, f(2) = 5, f(5) = 6, f(7) = 5$. Find the values of $f(4)$and also the value of x for which$f(x)$ is maximum or minimum. 10
7. Compute by Simpsons one third rule $\int_0^1 (4x - 3x^2)\,dx$ by taking $n = 10$,correct to four decimal places and compare the result with the actual value of the integral. Also find absolute and relative errors. 10
8. (a)Evaluate the missing terms in the following table

X:	0	1	2	3	4	5
F(x):	0	?	8	15	?	35

(b)Prove that $\Delta.\nabla = \Delta - \nabla = \nabla.\Delta$ 8+2

[Turn over

Group-C(10)

Answer any one question:

9. If $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$, find the value of $A^2 - 4A - 5I_3 = 0$, hence obtain a matrix B such that $AB = I_3$. 10

10. Solve the system of equations by Cramer's rule $x + y + z = 6$, $x + 2y + 3z = 14,\ x - y + z = 2$ 10

Group-D(20)

Answer any two questions:

11. (a)If $\vec{e_1}$ & $\vec{e_2}$ be two unit vectors and θ be the angle between them, then show that $2\sin\frac{\theta}{2} = |\vec{e_1} - \vec{e_2}|$.

(b)Given two vectors $\vec{\alpha} = \vec{i} + 2\vec{j} - \vec{k},\ \vec{\beta} = 2\vec{i} - \vec{j} + \vec{k}$; find the vector $\vec{\gamma}$ and the scalar λ which satisfy $\vec{\alpha} \times \vec{\gamma} = \vec{\beta} + \lambda\vec{\alpha}$ and $\vec{\alpha}.\vec{\gamma} = 2$.

(c)If $\vec{\alpha} \times \vec{\beta} + \vec{\beta} \times \vec{\gamma} + \vec{\gamma} \times \vec{\alpha} = \vec{0}$ 3+5+2

12. Find in term of k, the shortest distance between the lines $\rho = \vec{\alpha} + t\vec{\beta}$ and $\rho = \vec{\gamma} + t\vec{\delta}$, where $\vec{\alpha} = (1,2,3), \vec{\beta} = (2,3,4),\ \vec{\gamma} = (k,3,4)$and $\vec{\delta} = (3,4,5)$. For what value of k are the lines coplanar? 10

13. rigid body is spinning with an angular velocity of 5 radians per second about an axis of direction $(0,3,-1)$ passing through the point $A(1,3,-1)$. Find the velocity of the particle at the point $P(4,-2,1)$. 10

Group-E(10)

Answer any one question

14. (a)Show that $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}$ and that the three vectors $\vec{a} \times (\vec{b} \times \vec{c}),\ \vec{b} \times (\vec{c} \times \vec{a}), \vec{c} \times (\vec{a} \times \vec{b})$ are coplanar

(b)Show that $[\vec{a} + \vec{b}\ \ \vec{b} + \vec{c}\ \ \vec{c} + \vec{a}] = 2[\vec{a}\vec{b}\vec{c}]$

(c)Show that $[\vec{a}\,\vec{b}\,\vec{c}]^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{a}.\vec{c} & \vec{b}.\vec{c} & \vec{c}.\vec{c} \end{vmatrix}$ 4+2+4

15. Prove that, by using the Laplace's method $\begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{vmatrix} = (af - be + cd)^2$;10